

Prologue by Alejandro Sobrino

In this prologue, which the author of the book has generously commissioned me to pen, I will comment on the text and, more succinctly, on its author.

The book before you is a short, formal, and original one. The fact that it is a concise book is indicated by the number of pages, something typical of many texts on mathematical logic and their authors, mathematicians by training and trade. Let me tell you an anecdote in this connection. Years ago, when I was a scholarship holder from the then Ministry of Education and Science in the Department of Logic of the Faculty of Philosophy at the University of Santiago de Compostela (USC), the professor in our field, Dr. Rafael Beneyto, invited his colleague and professor of algebra, Don Eduardo García-Rodeja, to deliver a talk to us in which he would present his vision of logic. I seem to recall that he offered him several sessions, but Don Eduardo said, with the authority that characterized him, that one was enough. A classroom was reserved for him for a period of three hours, but after thirty minutes, and to the disbelief of those present, he said, “Okay, that’s all, gentlemen,” and without further ado he took questions.

Mr. Eduardo’s talk caused me not only surprise since, after all, I was an inexperienced student who had only taken a formal logic course using the book by Alfredo Deaño,⁵ which was more discursive than technical, but also stunned Professor Beneyto himself, who was accustomed to formal presentations of logic, such as that given in the book by M. Garrido,⁶ on which Beneyto had actively collaborated. Garrido’s text was one of the first in Spain (together with that of M. Sacristán⁷) to touch on *more mathematical* logic, a presentation radically different from the scholastic one, in use and custom until then. The talk gave us a real shock, because we could not identify any way of doing things that was recognizable or that resembled any part of our work. Our logic was Frege’s, and although its creator devised it based on the formalism or ideography with which he spoke, like Don

⁵ Deaño, A. Introducción a la lógica formal. Alianza Editorial, S.A., Madrid, 1974.

⁶ Garrido, M., Lógica simbólica, Tecnos, Madrid, 1995.

⁷ Sacristán, M., Introducción a la lógica y al análisis formal. Ariel, Barcelona. 1964.

Eduardo, of variables, predicates, or calculations, unlike him, Frege discussed the meaning or reference of some of these terms or on their ontological foundation, issues more associated with philosophy.

Mathematical presentations of logic emphasize, not its condition as a model of Boolean algebra, but the structure that underlies it and that is of interest in itself, with its laws and relationships, and where the applications are, be they for example to argumentative correctness or to digital circuits, they are obvious to their authors—as they often say. As Russell said, in this type of presentation “reflection falls on the symbols, since these are tangible”, even if it is not for that that these symbols are there. This arduous task corresponds to philosophy and, although complex, as a good analyst, it seemed quantifiable to him, predicting that “the truly good philosopher is the one who manages to achieve this once every six months for one minute”.

What we find here is a formal, exposed book, although its author has, as can be seen in this very text or in other publications by him, a proven interest in philosophy and in the achievements of logic in language and reasoning, in particular, ordinary language and commonsense reasoning. It shares a way of presenting logic with illustrious predecessors, such as H. B. Curry⁸ or H. Hermes.⁹ They make, in already classic books, an algebraic presentation of logic, which Curry legitimizes, in the introduction to his own text, in the evolution of the object of study of this discipline, first substantiated in syllogistic reasoning to settle disputes in the agora and then in its Boolean mathematization as a purely formal and autonomous discipline, something similar—he says—to what happened with *geometry*, in its etymology and origin, the study of the measurement of the Earth and, in more modern concerns, the branch of mathematics that studies the properties of figures in the plane or in space. Nonetheless, the text presented here has commonalities and differences with the previously mentioned books. The point in common is that language or reasoning is treated in connection with the service of the formalism used, that is to say, the structure of reality, precise or imprecise, is logical, and the logical, Fregean, or fuzzy forms must match that reality. Logic can be imprecise, but if it is logical, it must be clear and precise, and that is shown in the language used. The differentiating factor is that in this text, form is carried out using vague meaning and imprecise reasoning, yes, conveniently selected to fit it.

There is another point that this text shares with the previous ones, which is the special focus that is given to lattice theory. A lattice is a partially ordered set (L, \leq) in which, for every pair of elements, there is a supremum and an infimum. In algebra, a lattice is a set L provided with two binary operations \vee and \wedge such that, for any a, b, c of L the properties of commutativity, associativity, absorption, and idempotence are verified discounting those in which the order relation intervenes. In that case, they are said to define a partial order \leq in L by the following rule: $a \leq b \Leftrightarrow a + b = b$. This confirms that the *less than* or *equal predicate* can be

⁸ Curry, H. B., *Leçons de Logique Algébrique*. Gauthier-Villars, Paris, 1952

⁹ Hermes, H., *La teoría de retículos y su aplicación a la lógica matemática*. Publicaciones del Seminario Matemático de Zaragoza, 1962.

used to represent the heart of logic, which is simply inference, and that makes lattice theory an appropriate tool for an archetypal or structural presentation of it. Despite coupling here with its predecessors, there is something that distinguishes lattice theory, and it is the use herein of mathematical structures that cause the progressive weakening of Boolean algebra, resulting in subsystems of classical system, such as ortho-lattices or algebras of De Morgan, which make it possible to explain more general logical relationships, such as quantum or imprecise implication and of which the classical one becomes a particular case.¹⁰ *Fuzzy* algebras, which the author is especially fond of, are a particular case of De Morgan's and are only lattice when the conjunction is represented by the minimum numerical operation and disjunction by the maximum. The formal skeleton that is achieved is precise, although adaptable, which offers the author the possibility of a *genetic* explanation of logic, showing how these logics evolve from each other in terms of the restrictions that the different types of reasoning demand of the logic formalism.

The text does not include a section on applications, which would allow for the exemplification of these models and show their position in the field of language, everyday rational decisions, or in more technical areas, such as circuit theory. However, apart from whether or not the text deals with this or that topic, a decision that belongs to its author, we have before us an original book that includes a tradition to learn and emancipate from it, spotting new problems and facing them with talent and creativity. It could be said that it fills a gap in the literature on the subject by proposing a *genealogical* presentation of logics, showing how they evolve from one another, branching out from a common and general trunk full of properties, of which Boole's algebra is a paradigm.

The author of this prologue has had philosophical training, not mathematical, with an interest in language, and he is a confessed admirer of the role that form plays in it. In the philosophy of language, the question is often raised as to what came first, syntax (form) or semantics (content). Noam Chomsky, the most inclusive linguist of the last century, bets on the former, so that for him the syntagmatic or architectural structure of sentences determines their meaning, a thesis that he substantiated in a paradigmatic way in syntactically ambiguous sentences. But not everyone agrees with that idea. His critical follower, D. Bickerton, understands that human beings can be explained evolutionarily and, specifically, by what characterizes or defines them, language, and that only an evolutionary advantage could make this faculty appear to stay, identifying its usefulness in the meaning that a word or expression had for someone in a perhaps existentially compromised moment. Semantics is now at the center. But Bickerton also discovered that languages that simply use isolated words side by side, without any hierarchical structure, such as *pidgin* or macaronic languages, eventually become full languages in the next generation, even if their speakers have only heard a simplified and imperfect protolanguage: these are creole languages, which are not different from full languages. This would validate

¹⁰ This book doesn't present a lattice structure as a universal but as a local one; there might be some misprints in the original version of the book.

Chomsky's hypothesis that we are born with an innate language acquisition device, with a mental language in which form dominates. Content comes first, but form emerges later and remains due to its simplicity and the cognitive savings it brings.

In language, the process of acquiring form was complex. British linguistic anthropologist Robin Dunbar surmised that it emerged as an *exaptation* of social calculus. Originally, when hominid groups were small, they came together to groom themselves to get rid of host organisms that caused disease and decimated the population. To avoid freeloaders, they imposed controls such as "when I finish delousing you, you delouse me."

As groups got larger, proximity control disappeared, requiring language as a remote-control mechanism, making it possible to generalize immediate personal verifications to formal rules with spatial or temporal displacement, as occurs in "if *X thumb to Y*, *Y thick to X*." This is the germ of syntax, of the syntagmatic structure that recognizes *Juan reads a book* or *a book is read by Juan* as sentences that are the realization of a single linguistic pattern, that is, of a single deep structure. Syntax is said to arise as a complex adaptation by function change, in the same way that, for example, the bones of vertebrates, which at first served as a calcium reservoir but were used to keep us upright and protect our vital organs once we became bipedal. For Dunbar, form arises from a pattern of social activity that needed to be generalized.

This is an attractive conjecture, though generally accepted. What perhaps enjoys broader consensus is the use of recursive rules, which makes it possible to distinguish human language from animal communication systems and, consequently, is what defines us as a species. Recursive rules make discrete infinity possible, that is, to generate an infinite number of sentences or even an endless sentence, and they make it possible to encapsulate some thoughts in others to express complex scenarios, as happens with compound verb tenses. Thus, the compound conditional makes it possible to describe situations that would have happened but did not, that is, it allows the expression of a future action located in the past. Recursion is considered the core of the language faculty, the internal computational system that allows for abbreviation in rules, which is later diversified in the oral expression of what we communicate.

These hypotheses naturalize language and its form. 'More complex' seems to naturalize logic, even though it has recognized neural correlates in the frontal cortex of the left hemisphere. Hominids are animals with needs, and to satisfy them, we plan actions. It seems that rational thought arises from combining this demand-solution duo. Its achievement is synonymous with effectiveness and durability over time. However, rational thought is one thing, formal reasoning another. While the former appears linked to our immediate survival, the latter arises once we are already a settled species, a tribe that has plans in common. Rational thinking perhaps allowed immediate actions to be taken to defend against predators or to obtain sexual partners. But once we grew in numbers, group behaviors in which we had to synchronize times and agree on objectives gained prominence. Cooperative species that argue publicly may have invented logic as a tool for this purpose, making it possible to confidently explain and predict consequences arising from

our intentional actions or processes in the natural world, explaining or preventing difficulties and complexities. If thought arises from having to face infrastructural or survival problems, logic seems to emerge when these problems become collective; if reason is a natural product, logic is a cultural artifact. Fregean logic, for example, is typical of the mathematical culture of precision and necessity; *fuzzy* logic is characteristic of AI, of imprecision and possibility.

Karl Popper once remarked that “what is important is not so much precision as clarity.” Mathematicians aspire to precision, and according to the philosopher José Ortega y Gasset, “clarity is a form of courtesy that the philosopher owes,” even if it is sometimes barely noticeable. The tension between precision and clarity is evident and is substantiated in the opposition between hard and soft sciences or between exact sciences and the humanities, although it should not be so accentuated because it is possible to be precise or clear in a non-exclusive way, as illustrated very well by the illustrator Galician L. Dávila in one of his cartoons: “Science: 50%; of letters: half.” The formalisms are precise and clear in themselves, though not so much in their application to the world, which they mercilessly stretch to fit their molds. As Quine said, “Science is an extension of common sense that consists in inflating ontology to simplify theory.” In addition, philosophy aspires to be clear, although it rarely succeeds. For the first Wittgenstein that aspiration must be “total clarity.” “How long do you trust me?” as Don Tirso de Molina would say through Don Juan Tenorio, because the language of philosophy is natural language, essentially imprecise and an inexhaustible source of problems due to the width of its interpretive margins; holding it down with formalities is a complex task, sometimes useful, sometimes useless. Finding a balance between formal language and ordinary language remains a challenge, and this book represents a relevant step toward meeting that challenge.

Now, to conclude, a comment about the author, Professor Trillas, who is both a teacher and a friend. Therefore, what I am about to say is biased, though not necessarily subjective. As a teacher, he taught several generations of us the secrets of fuzzy logic, which he introduced and mainly cultivated in Spain, although his research has international reach and acceptance. A man with captivating verbs and exquisite handwriting, his lessons on the blackboard, which he always rushes to, are always a compendium of wisdom, insight, and depth. As an intellectual with a broad outlook who cultivates knowledge in general, he is passionate about not only mathematics and science but also philosophy and literature, with which he peppers his talks, turning them into cultural, as well as technical, presentations. In the style of other great scientists, he is a man of noteworthy phrases, those that combine wisdom and humor, such as “our extraordinary pay is called that way because of how extraordinarily small it is”; “the minister who knows the most mathematics is the one working with the economy: he understands subtraction and division; the rest of the population, just adding and multiplying”; or “in the frontispieces of the faculties of Spain this inscription should appear: here one comes to learn, not to be taught.” As a close friend, he is always willing to share intellectual concerns and even personal belongings, for example, to make his house an inn and habitual lodge for the present writer. When I present him to my students, I always tell them: Here

you have Professor Trillas; take advantage of this opportunity, because it is very unlikely that you will ever meet anyone quite like him. And after so many years of knowing and spending time with him, I still think the same thing: he is unique and it is a privilege to be part of his teachings and his generous friendship.

Thank you for this, teacher, and for such a magnificent book.

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