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An Examination of Alternative LQ-Based Approaches to Computing Regional Input–Output Coefficients

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Abstract
In this paper, we examine alternative methods of computing regional input–output (IO) coefficients, with an emphasis on their relative accuracy and the complexity of the computations required. Our focus is on the well-known FLQ (Flegg's location quotient) approach. Although the FLQ formula often yields satisfactory results, the need to specify values of the unknown parameter δ in this formula presents an obstacle to its implementation. After examining the FLQ's conceptual foundations, we develop a possible new approach that obviates the use of this parameter. Instead, the new formula, the hyperbolic tangent LQ or HTLQ, incorporates an alternative proxy for regional self-sufficiency in terms of a new parameter μ. We argue that, in most cases, it would be reasonable to set $\mu = 0.5$. We test our proposal using the 2005 and 2015 Korean survey-based interregional IO datasets and contrast our estimates with both survey-based values and the results from several other techniques. The results suggest that the new formula can yield more accurate estimates of regional IO coefficients and multipliers, and in a more straightforward way, than is possible with the traditional FLQ.

Keywords Regional input–output tables · Non-survey methods · FLQ · 2D-LQ · HTLQ · SFLQ

1 Introduction

The focus of this paper is on indirect methods of computing regional input–output (IO) coefficients in the common situation where there is an abundance of national data but a paucity of suitable regional data. Our starting point is the well-known FLQ (Flegg's location quotient) approach, in which the relative size of regions plays a key role. After examining the FLQ's conceptual foundations, we explore an alternative treatment of the indirect relationships between regional attributes and economic integration across industries.

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We use a particular form of the logistic function, the hyperbolic tangent, to model such indirect linkages, in the vein of some literature on innovation and technological change (Jarne et al., 2007). We posit a relationship starting at a non-zero value, with a gradient that rises gently as regional self-sufficiency increases.

We show how our method can be implemented without involving parameter estimations. To this end, we choose a proxy μ for regional self-sufficiency and introduce it directly into our calculations. We argue that, in most cases, it would be reasonable to set $\mu = 0.5$.

The remainder of the paper is organized as follows. In the next two sections, we review existing LQ-based methods. In Sect. 4, we re-examine the foundations of the FLQ and identify certain potential limitations. Next, we outline our methodological proposal. In Sect. 6, we present our findings from an application using Korean interregional IO (IRIO) data for 2005 and 2015. In Sect. 7, we use different measures of error to assess the robustness of our findings. Next, we conduct an analysis of important coefficients in terms of the Leontief inverse matrix and associated multipliers. In the penultimate section, we present results for the SFLQ (industry-specific FLQ) method. We conclude with some final remarks.

2 Location Quotients and Regional IO Models

Location quotients (LQs) are still a widely used non-survey technique to regionalize national IO tables and thereby to generate regional IO models. Their principal attraction is the minimal data requirements, namely regional and national output (or employment) by sector.

Many different LQ-based formulae have been developed, which are discussed in detail in Flegg et al. (2021) and in several other papers. The most basic formula is the *simple* LQ:

$$SLQ_i = \frac{\frac{x_i^r}{x_\cdot^r}}{\frac{x_i^n}{x_\cdot^n}} = \frac{\frac{x_i^r}{x_i^n}}{\frac{x_\cdot^r}{x_\cdot^n}} = \frac{wx_i^r}{wx^r} \tag{1}$$

Here x_i^r and x_i^n are the total output (production) of the ith regional and national sector, respectively, while $x_\cdot^r$ and $x_\cdot^n$ are the corresponding regional and national aggregates. wx_i^r represents the weight of regional sector i in the national economy, whereas wx^r represents the weight of region r in the national economy, i.e., its relative size. SLQ_i measures the degree of specialization of region r in sector i relative to the nation.

The regional input coefficients are derived according to the following rule:

$$\hat{a}_{ij}^r = \begin{cases} a_{ij}^n SLQ_i \ \ if \ \ SLQ_i < 1 \\ a_{ij}^n \ \ if \ \ SLQ_i \geq 1 \end{cases} \tag{2}$$

where $\hat{a}_{ij}^r$ is the estimated regional input coefficient and a_{ij}^n is the corresponding observed national input coefficient (excluding inputs purchased from abroad).

However, it has long been known that the SLQ tends to understate a region's imports from other regions; this occurs because the SLQ rules out any 'cross-hauling' (Stevens et al., 1989). Cross-hauling takes place when a region simultaneously imports and exports a given commodity. For a systematic treatment of this issue, see Többen and Kronenberg (2015).

The *cross-industry* LQ was one of the first refinements of the SLQ, as it considers the relative size of both supplying sector i and purchasing sector j. The formula is as follows:

$$CILQ_{ij} = \frac{SLQ_i}{SLQ_j} = \frac{x_i^r / x_i^n}{x_j^r / x_j^n} \tag{3}$$

where the constraints are applied as in (2).[1] Unlike the SLQ, however, the CILQ applies a cell-by-cell adjustment.[2] This means that it does, in principle at least, deal with the problem of cross-hauling. What it does not do is to consider the relative size of a region, $x_\cdot^r/x_\cdot^n$, which cancels out in formula (3). By contrast, this ratio remains a component of the SLQ formula (1).

Nonetheless, Flegg et al. (1995) criticize the SLQ on the grounds that it would tend to understate the imports of relatively small regions owing to the way in which the ratio $x_\cdot^r/x_\cdot^n$ is implicitly incorporated in the SLQ formula. The FLQ aims to correct this shortcoming.

The crucial hypothesis underpinning the FLQ is that a region's propensity to import from other domestic regions is inversely and nonlinearly related to its relative size. By incorporating explicit adjustments for regional size, the FLQ should yield more precise estimates of regional input coefficients and hence multipliers. Along with other non-survey methods, the FLQ aims to offer regional analysts a means by which they can build regional tables that reflect, as closely as possible, each region's economic structure. See, for example, the application to Mexican regions by Dávila-Flores (2015) and to Chilean regions by Mardones and Silva (2021).

The FLQ is defined as follows (cf. Flegg & Webber, 1997):

$$FLQ_{ij} = \begin{cases} \lambda CILQ_{ij} \text{ for } i \neq j \\ \lambda SLQ_i \text{ for } i = j \end{cases} \tag{4}$$

where λ captures a region's relative size. This scalar is defined as follows:

$$\lambda = \left[\log_2\left(1 + \frac{x_\cdot^r}{x_\cdot^n}\right)\right]^\delta \tag{5}$$

Here $0 \leq \delta < 1$ is a parameter that controls the degree of upward convexity in Eq. (5). The larger the value of δ, the lower the value of λ, and the greater the

[1] A parametric version of the CILQ is presented by Sánchez-Chóez et al. (2022).

[2] Garhart and Giarratani (1987) and Ralston et al. (1986) were among the first to argue that adjustments to national coefficients should be cell-specific rather than uniform across rows or columns.

allowance for extra regional imports. The FLQ formula is implemented just like other LQ methods, as in Eq. (2).

A variant of the FLQ is the *augmented* FLQ (AFLQ), which takes regional specialization into account. It is defined as follows (cf. Flegg & Webber, 2000):

$$AFLQ_{ij} = FLQ_{ij}[\log_2(1 + SLQ_j)] \tag{6}$$

which is applicable only when $SLQ_j > 1$. Unlike the other LQ-based formulae, the AFLQ can take values above unity.

The AFLQ approach was successfully employed by Mastronardi et al. (2022) in constructing a bi-regional input–output matrix for Argentina, which separated the city of Buenos Aires from the rest of the country. However, the FLQ could have been used instead in this study since the AFLQ and FLQ often yield very similar outcomes (Flegg et al., 2016; Lampiris et al., 2020).

Several case studies, including Flegg and Tohmo (2016), have demonstrated that the FLQ can yield more accurate results than the SLQ and CILQ. This evidence is corroborated by the Monte Carlo study of Bonfiglio and Chelli (2008). A similar methodological approach is taken by Mardones and Silva (2023).

On the other hand, Lamonica and Chelli (2018) find that the FLQ performs better than the SLQ in smaller regions, yet worse in larger regions. The FLQ is strongly criticized by Fujimoto (2019) on both conceptual and empirical grounds. A response to these criticisms is given in Flegg et al. (2021). Finally, we may note that the FLQ's use in a multiregional context is examined by Hermannsson (2016), Jahn (2017), Jahn et al. (2020), and Garcia-Hernandez and Brouwer (2021).

In a recent contribution, Kwon and Choi (2023) identify an apparent shortcoming of the FLQ. They focus on the CILQ component of the formula and consider situations where $SLQ_i < 1$ and $SLQ_j < 1$. If $SLQ_i < SLQ_j$, then $CILQ_{ij} < 1$ and the national input coefficient will be adjusted downwards, whereas if $SLQ_i > SLQ_j$, no adjustment will be made. The authors argue that a failure to adjust the national coefficient in this case would represent a distortion, which could be corrected by using SLQ_i as the scalar. Proceeding in this way, the authors find that their proposed KFLQ formula yields enhanced results. Also see Tohmo (2025a). However, the CILQ component of the FLQ would no longer represent the relative size of the supplying and purchasing sectors in such cases and its rationale would be lost.

More fundamentally, Pereira-López et al. (2020) propose a two-dimensional approach (2D-LQ) to estimate domestic coefficients at the sub-territorial level. This technique can be extrapolated to other contexts; for instance, generating intermediate flow matrices. In this approach, estimates of regional coefficients are calculated according to the following rule:

$$\widetilde{\mathbf{A}}^R = \widehat{\mathbf{r}}(\alpha)\mathbf{A}^N\widehat{\mathbf{s}}(\beta) \tag{7}$$

where $\mathbf{A}$ is a matrix of intermediate domestic coefficients, and $\widehat{\mathbf{r}}(\alpha)$ and $\widehat{\mathbf{s}}(\beta)$ are diagonal matrices whose main diagonal elements work as weighting factors. Scalars α and β are the parameters influencing row and column rectifications, respectively. As in the CILQ and FLQ, row and column corrections are addressed differently:

$$\mathbf{r}(\alpha) = \begin{cases} (SLQ_i)^{\alpha} \text{ if } SLQ_i \leq 1 \\ \left[\frac{1}{2}\tanh\left(SLQ_i - 1\right) + 1\right]^{\alpha} \text{ if } SLQ_i > 1 \\ \mathbf{s}(\beta) = (wx_j^r)^{\beta} \end{cases} \tag{8}$$

One of the main novelties introduced by the 2D-LQ formulation is a modified hyperbolic tangent curve to describe the indirect relationships between input coefficients and location economies.

Papers presenting the 2D-LQ approach (Martínez-Alpañez et al., 2023; Pereira-López et al., 2020, 2021) report better results than previous LQ-based regionalization methods. Such promising results are obtained at the cost of an additional trouble: providing estimates of the α and β parameters that capture the effect of supply-side location economies. Here it is worth examining the pioneering work of Martínez-Alpañez et al. (2023), who use survey-based Korean multiregional IO tables, which arguably should provide a better basis for evaluating the different methods than do the Eurostat data employed in earlier studies, in which selected individual countries are treated as regions of the European Union.

Martínez-Alpañez et al. find that the performance of the 2D-LQ surpasses that of the FLQ for 14 of the 17 regions in the 2015 Korean dataset. They also assemble a further dataset comprising survey-based tables for thirteen Spanish regions and find that the 2D-LQ outperforms the FLQ in ten of these regions. Even so, for the 2D-LQ to be a useful technique in the typical situation where survey-based data are unavailable, it is crucial to be able to obtain reliable estimates of α and β.

For Korea, Martínez-Alpañez et al. propose two regression equations, using available data on interregional and intraregional transport flows. Unfortunately, these equations yield highly implausible estimates of α and β. For every region, the values of $\widehat{\alpha}$ and $\widehat{\beta}$ obtained are almost identical (Martínez-Alpañez et al., 2023, Table 5), which suggests that $\alpha = \beta$. That, in turn, would reduce the 2D-LQ method to a single dimension. Clearly, more research needs to be undertaken to develop suitable estimating equations, which would require identification of distinct factors determining the value of each parameter.

3 The FLQ + and SFLQ

The main obstacle in applying the FLQ formula is in determining a value for δ in Eq. (5). This is crucial because its value might vary across regions, countries and time. This problem can be addressed by applying the FLQ + approach proposed by Flegg et al. (2021). This approach is discussed next.

The FLQ + procedure involves three steps. The first applies a modified cross-entropy method to regionalize the national IO table. This is designed to account for negative or zero input coefficients. The second step uses the derived regional matrix, along with the national table, to estimate the optimal δ for each region via a simple regression model. In the third step, this estimated δ is used to apply the FLQ formula, thereby computing the final estimates of the regional input coefficients. These estimates are specific to a particular region, country and time. This hybrid approach can easily be adapted to enhance the performance of other pure LQ-based methods such as the AFLQ that depend on one or more unknown parameters.

Nonetheless, the FLQ+approach does not encompass the possibility that δ would vary across sectors too. This possibility is explored by Kowalewksi (2015), whose industry-specific FLQ, the SFLQ, is defined as

$$SFLQ_{ij} \equiv CILQ_{ij} \times [\log_2(1 + E^r/E^n)]^{\delta j} \tag{9}$$

where E^r/E^n is regional size measured in terms of employment. For $i=j$, $CILQ_{ij}$ is replaced by SLQ_i.

To estimate the δ_j, Kowalewksi specifies the model

$$\delta_j = \alpha + \beta_1 CL_j + \beta_2 SLQ_j + \beta_3 IM_j + \beta_4 VA_j + \varepsilon_j \tag{10}$$

where CL_j is the coefficient of localization, which measures the degree of concentration of national industry j, IM_j is the share of imports in total national intermediate inputs, VA_j is the share of value added in total national output and ε_j is an error term. Regional data are needed for SLQ_j, whereas CL_j, IM_j and VA_j require national data.

However, Flegg and Tohmo (2019) observe that analysts using non-survey methods would not know the optimal δ_j required to run a regression of this type for each region. Nevertheless, for analytical purposes, Flegg and Tohmo assume that these optimal values are indeed known. They then respecify the model as

$$\delta_j = \alpha + \beta_1 CL_j + \beta_3 IM_j + \beta_4 VA_j + \varepsilon_j \tag{11}$$

where the dependent variable is now the mean of the optimal δ_j across all 16 Korean regions, while SLQ_j has been omitted as it is a region-specific variable.

Using this model, along with Korean survey-based data for 27 sectors and 16 regions in 2005, Flegg and Tohmo find initially that the SFLQ outperforms the FLQ (with $\delta=0.35$) when estimating the type I output multipliers in one of the regions under consideration but not in the other (Flegg & Tohmo, 2019, Table 6).

However, once degrees of freedom are considered and information criteria are employed, the authors find that the FLQ convincingly outperforms the SFLQ (Flegg & Tohmo, 2019, Table 8). This outcome 'suggests that the enhanced precision gained by capturing the intersectoral dispersion in the values of δ is outweighed by the statistical uncertainty entailed by having to estimate 27 parameters rather than only one' (Flegg & Tohmo, 2019, p. 610). Also see Tohmo (2025b).

A novel and potentially more fruitful approach to applying the SFLQ is proposed by Mardones and Silva (2023). Using real Chilean data and Monte Carlo simulations complemented with partial historical information, they find that the SFLQ is far superior to the best traditional methods, the FLQ and AFLQ (Mardones & Silva, 2023, figs. 1 and 2).[3] Historical information included data on output, along

[3] Also see Mardones and Correa (2025) for another application of this approach.

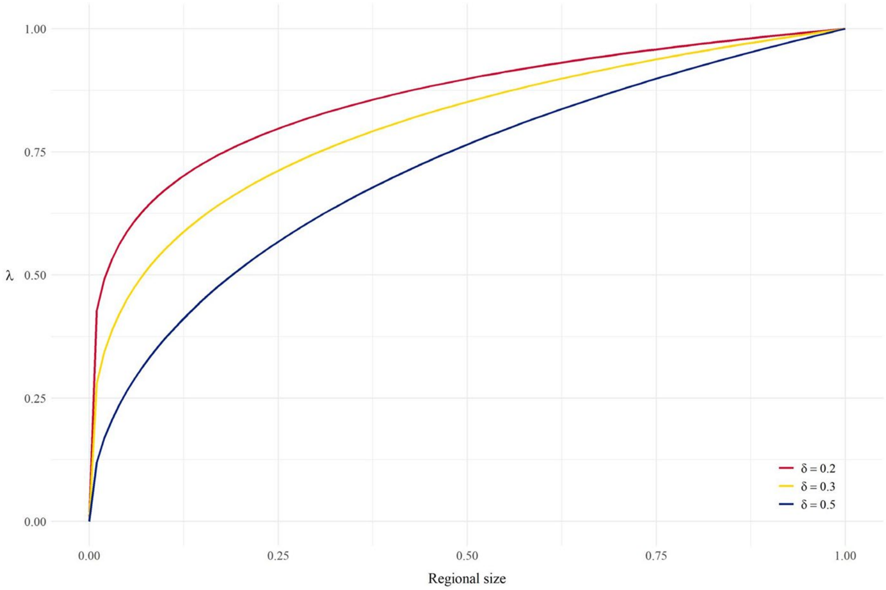

Fig. 1 Concavity (from below) of the FLQ scalar λ for different values of δ. . *Source*: Own elaboration

with regional technical coefficients taken from earlier IO accounts.[4] Even so, while it would be interesting to re-examine this Monte Carlo approach using Korean data, that would be beyond the scope of the present paper.

4 Examining the Foundations of the FLQ

Figure 1 depicts the relationship underlying the FLQ formula between λ, δ and regional size R, which is measured in terms of a region's share of total national output or employment. The graphs are based on a formulation proposed by Flegg and Webber (1997). A key feature of the graphs is that they pass through the points [0,0] and [1,1]. Furthermore, the gradients decline smoothly and continuously as R increases. However, the authors offer no rationale for the shape of the graphs in between [0,0] and [1,1], which is an aspect that is worth reconsidering in two related directions.

First, in the smallest regions, a relatively small rise in R would result in a relatively large increment in λ. There is no obvious reason why that should be so. Indeed, a critical mass of plants and employment might be required before industrial take-off and clustering dynamics appear within a region. Such dynamics would tend to reduce imports from other regions and thus increase the regional input coefficients. Furthermore,

[4] This approach resembles that of Mínguez et al. (2009), where historical data are used to update IO matrices, as well as to regionalize them.

especially in the case of relatively large regions in relatively small nations, it might be desirable to have some control over how far the estimated regional input coefficients could diverge from their national counterparts as R varies.

Secondly, regional size might not be the best proxy to use to adjust CILQs. Even in small regions, some sectors—especially those producing less tradable goods and services—might have input coefficients close to the national average (a_{ij}^n), owing to high regional supply/demand ratios for those commodities (Haddad, 2014). Conversely, even in large regions, sectors producing highly tradable commodities might differ in terms of their regional input structure from the national average. Therefore, assuming a strong linear relationship between regional and national input coefficients might not be appropriate.

5 Methodological Proposal

Computational capacity is now more or less ubiquitous and is still increasing. In the field of regional IO analysis, this makes it possible to complement theory-based regionalization techniques with data-driven research. In this paper, we assess an alternative method of adjusting CILQs, by using a generalized hyperbolic tangent transformation. We contrast our proposal with well-established regionalization approaches, using data from the South Korean interregional IO accounts, which is one of the most comprehensive datasets available worldwide.

The FLQ method, as described in Sect. 2, adjusts CILQs according to a region's size within the national economy. However, the way in which the FLQ models the indirect relationship between regional size and input coefficients is worth examining.

More specifically, we propose using a hyperbolic tangent (*tanh*) function to adjust CILQs. The hyperbolic tangent is one of the many parametric specifications of the logistic function:

$$f(x) = \frac{k}{1 + be^{-ax}}, k, a, b \in \mathfrak{R} \tag{12}$$

where k is the maximum value, a is the logistic growth rate and b adjusts the value of $f(0)$, the intersection with the ordinate axis. For the hyperbolic tangent, we set $k = 2$, $a = 2$ and $b = 1$. x stands for the CILQ (Fig. 2).

To generalize the CILQ transformation and to account for possible effects of regional size, we introduce a single parameter μ that regulates the maximum value k. Next, we displace the hyperbolic tangent across the ordinate and abscissa axes. Formally:

$$HTLQ_{ij} = \begin{cases} 0 \ if \ CILQ_{ij} = 0 \\ \frac{2\mu}{1+e^{-2[(CILQ_{ij}-1)+1]}} = \mu\left[\tanh\left(CILQ_{ij} - 1\right) + 1\right] \ 0 < CILQ_{ij} \leq 1 \\ 1 \ if \ CILQ_{ij} > 1 \end{cases} \tag{13}$$

Notable features of this function are as follows. First, it is convex (from below), with a continuously rising gradient and a positive intercept, which is in line with the threshold effect posited above. We implicitly assume a minimum supply structure in the

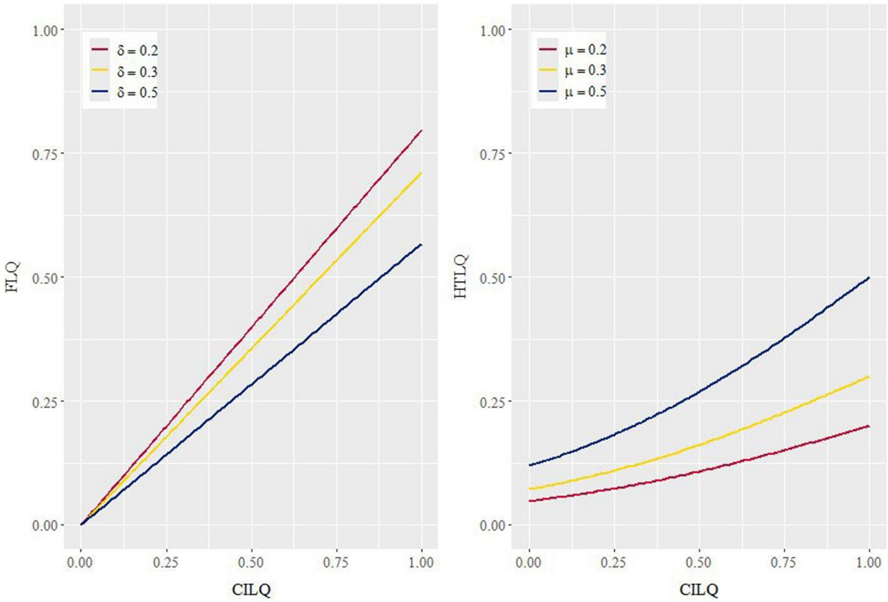

Fig. 2 FLQ and HTLQ for CILQ values in the interval [0, 1] and three different δ and μ values. (For a hypothetical region accounting for 25% of national employment *Source*: Own elaboration

region once an industry exists and restrict the upper bound of the regional-size effect. When $CILQ_{ij} = 0$, $HTLQ_{ij} = 0$, to account for missing industries in a region.

Secondly, we hypothesize that a parameter μ non-linearly regulates the relationship between regional size and the CILQ. Trivially, if a region does not exist, we set $\mu = 0$, so $HTLQ_{ij} = 0$. As μ increases ($0 < \mu < 1$), the minimum adjustment for small CILQ values and the upper bound for $CILQ_{ij} = 1$ rise as well, since μ is the value that our formula takes when $CILQ_{ij} = 1$. That is, the bigger the region, the larger are both the minimum supply structure and the upper bound of the regional-size effect. Moreover, if there were only one region in the country, we would have $\mu = 1$ and $CILQ_{ij} = 1$. Thus $HTLQ_{ij} = 1$ and the regional and national input coefficients would coincide. In contrast, δ in the FLQ formula changes the slope defining the linear relationship between the CILQ and the FLQ. This slope is given by the value of the scalar λ, Eq. (5).

6 Empirical Application

6.1 Methods and Data

Our aim was to extract a regional model for each Korean region from the national matrices using our HTLQ methodology and to assess its performance

Table 1 STPE results for alternative regionalization techniques, coefficients, year 2005. *Source*: Own elaboration

	Region (2005)	Regional size (%)	CILQ	FLQ (δ)	AFLQ (δ)	2D-LQ (α;β)	HTLQ (μ)	HTLQ ($\mu = 0.5$)
1	Gyeonggi	20.14	69.90	41.56	42.73	37.49	39.16	39.63
				(0.56)	(0.62)	(0.68; 0.42)	(0.55)	
2	Seoul	18.20	54.36	51.11	57.42	41.80	68.94	69.29
				(0.19)	(0.71)	(0.72; 0.25)	(0.58)	
3	North Gyeongsang	8.42	82.39	61.06	67.54	53.64	54.43	54.44
				(0.69)	(0.81)	(0.32; 0.29)	(0.49)	
4	South Gyeongsang	7.35	73.56	55.56	60.53	52.45	51.66	51.66
				(0.45)	(0.53)	(0.52; 0.30)	(0.50)	
5	Ulsan	7.11	106.77	79.79	83.60	64.28	61.43	64.11
				(0.56)	(0.89)	(0.16; 0.28)	(0.37)	
6	South Jeolla	6.46	84.98	75.68	75.22	53.80	61.93	63.23
				(0.38)	(0.95)	(0.40; 0.22)	(0.40)	
7	South Chun-gcheong	6.33	112.65	69.24	66.41	53.05	53.14	56.72
				(0.64)	(0.67)	(0.16; 0.32)	(0.39)	
8	Incheon	5.49	116.02	57.61	58.69	50.30	46.68	50.92
				(0.54)	(0.70)	(0.32; 0.36)	(0.40)	
9	Busan	5.06	85.35	52.99	52.59	49.70	54.66	55.71
				(0.40)	(0.44)	(1.04; 0.23)	(0.42)	
10	North Chun-gcheong	2.92	100.61	68.98	70.35	61.99	62.13	64.76
				(0.50)	(0.52)	(0.40; 0.32)	(0.36)	
11	Daegu	2.88	84.62	57.78	58.34	50.50	55.23	56.11
				(0.30)	(0.34)	(0.88; 0.25)	(0.42)	
12	North Jeolla	2.74	107.54	69.88	70.15	58.24	62.49	68.61
				(0.46)	(0.54)	(0.52; 0.29)	(0.32)	
13	Gangwon	2.16	97.97	71.99	72.15	58.16	73.62	75.94
				(0.18)	(0.33)	(0.64; 0.15)	(0.38)	
14	Gwangju	2.15	112.48	72.81	72.55	60.65	65.47	74.26
				(0.37)	(0.44)	(0.64; 0.28)	(0.30)	
15	Daejeon	1.93	152.24	87.61	87.27	74.61	74.96	97.41
				(0.67)	(0.75)	(0.84; 0.36)	(0.19)	
16	Jeju	0.67	94.19	83.92	86.23	53.10	79.24	86.54
				(0.20)	(0.33)	(0.96; 0.13)	(0.30)	

relative to that of four alternative methods, namely the CILQ, FLQ, AFLQ and 2D-LQ.

We began by aggregating the published South Korean IRIO data for 2005 and 2015 up to 31 sectors. Despite possible biases induced by aggregation (Lahr & Stevens, 2002), that was the only way we could find to establish straightforward

Table 2 STPE results for alternative regionalization techniques, coefficients, year 2015. *Source*: Own elaboration

	Region (2015)	Regional size (%)	CILQ	FLQ (δ)	AFLQ (δ)	2D-LQ (α;β)	HTLQ (μ)	HTLQ ($\mu = 0.5$)
1	Gyeonggi	22.85	61.53	39.41	40.44	34.23	33.25	34.26
				(0.55)	(0.58)	(0.44; 0.42)	(0.55)	
2	Seoul	18.97	56.93	56.59	61.66	41.99	70.25	71.30
				(0.16)	(0.58)	(0.76; 0.17)	(0.62)	
3	North Gyeongsang	7.00	76.31	55.67	61.52	44.65	45.46	45.52
				(0.35)	(0.41)	(0.28; 0.29)	(0.48)	
4	South Gyeongsang	6.93	67.13	55.82	63.74	46.83	45.19	45.93
				(0.27)	(0.52)	(0.36; 0.23)	(0.56)	
5	Ulsan	6.32	99.33	73.55	75.35	56.51	52.68	53.94
				(0.65)	(0.73)	(0.04; 0.30)	(0.43)	
6	South Jeolla	4.89	75.21	65.82	72.03	46.69	54.22	54.28
				(0.29)	(0.30)	(0.52; 0.19)	(0.49)	
7	South Chun-gcheong	6.96	94.44	67.93	68.15	58.66	57.42	59.13
				(0.64)	(0.69)	(0.08; 0.37)	(0.41)	
8	Incheon	4.96	100.01	51.33	52.64	48.56	49.46	50.73
				(0.48)	(0.55)	(0.80; 0.26)	(0.42)	
9	Busan	4.73	68.68	49.95	49.93	42.74	46.64	47.86
				(0.28)	(0.33)	(0.92; 0.20)	(0.58)	
10	North Chun-gcheong	3.47	100.86	72.38	73.68	62.01	64.18	65.50
				(0.48)	(0.62)	(0.12; 0.34)	(0.41)	
11	Daegu	2.82	77.21	58.77	58.16	44.53	51.39	51.41
				(0.29)	(0.31)	(1.04; 0.19)	(0.51)	
12	North Jeolla	2.82	82.42	62.71	64.11	50.30	55.35	55.40
				(0.33)	(0.33)	(0.64; 0.19)	(0.49)	
13	Gangwon	1.97	88.31	67.63	64.74	52.46	67.15	67.51
				(0.40)	(0.42)	(0.88; 0.19)	(0.46)	
14	Gwangju	2.07	91.12	69.87	72.12	52.27	59.53	59.73
				(0.35)	(0.41)	(1.08; 0.18)	(0.46)	
15	Daejeon	1.92	124.34	79.82	78.86	59.84	64.35	63.38
				(0.44)	(0.47)	(1.04; 0.25)	(0.41)	
16	Jeju	0.81	77.16	71.21	71.37	48.13	69.39	69.43
				(0.19)	(0.27)	(0.68; 0.13)	(0.48)	
17	Sejong	0.50	183.03	87.88	88.30	72.63	76.65	109.98
				(0.59)	(0.60)	(0.60; 0.29)	(0.19)	

Table 3 Ranking of alternative regionalization techniques: coefficients, year 2005, STPE. *Source*: Table 1

	Region (2005)	Regional size (%)	CILQ	FLQ	AFLQ	2D-LQ	HTLQ
1	Gyeonggi	20.14	5	3	4	**1**	2
2	Seoul	18.20	3	2	4	**1**	5
3	North Gyeongsang	8.42	5	3	4	**1**	2
4	South Gyeongsang	7.35	5	3	4	2	**1**
5	Ulsan	7.11	5	3	4	2	**1**
6	South Jeolla	6.46	5	4	3	**1**	2
7	South Chungcheong	6.33	5	4	3	**1**	2
8	Incheon	5.49	5	3	4	2	**1**
9	Busan	5.06	5	3	2	**1**	4
10	North Chungcheong	2.92	5	3	4	**1**	2
11	Daegu	2.88	5	3	4	**1**	2
12	North Jeolla	2.74	5	3	4	**1**	2
13	Gangwon	2.16	5	2	3	**1**	4
14	Gwangju	2.15	5	4	3	**1**	2
15	Daejeon	1.93	5	4	3	**1**	2
16	Jeju	0.67	5	3	4	**1**	2

comparisons across time. We subsequently aggregated all trade flows into national IO models.

Our detailed results for the two years are presented in Tables 1 and 2, which display the values of the following statistic for each region and method:

$$STPE = 100\frac{\sum_{i=1}^{n}\sum_{j=1}^{n}\left|\widetilde{a}_{ij}^{r} - a_{ij}^{r}\right|}{\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}^{r}} \tag{14}$$

where *STPE* is the standardized total percentage error, $\widetilde{a}_{ij}^{r}$ stands for the estimated regional input coefficients and a_{ij}^{r} for their true counterparts.

For the parametric methods (FLQ, AFLQ, 2D-LQ and HTLQ), we took the parameter value that minimized the *STPE* for regional input coefficient matrices over the range [0, 1], with steps of 0.01. In contrast to Flegg et al. (2021), we assigned values of δ by using information from the observed regional coefficient matrices, instead of estimating these values via a cross-entropy method.

We also examine a scenario in which we set $\mu = 0.5$ for all regions in the HTLQ calculations. Parameter μ linearly regulates the range of values that $HTLQ_{ij}$ can take. The higher the value of μ, the wider the range. μ equals the maximum value of $HTLQ_{ij}$ when $CILQ_{ij} = 1$.

In the absence of further information, we propose a μ value that yields half of the maximum range. Our choice is consistent with Haddad (2014) and Haddad et al. (2017), who set an upper limit of 0.5 for industries producing tradable commodities.

Furthermore, we also limit the range of adjustments to national coefficients, thereby preventing "outlier" values that might impair the accuracy of our estimates.

6.2 Detailed Results

Our full set of detailed results is presented in Tables 1 and 2 but it is helpful to start by considering the rankings of the parametric methods in terms of the *STPE*, as shown in Tables 3 and 4. Not surprisingly, a key finding is the excellent performance of the 2D-LQ approach, for which we use much more information to optimize its two parameters. It gives the best results for 13 of the 16 regions in 2005 and takes second place in the other regions. This excellent performance is mirrored in 2015, where it is best for 13 of the 17 regions.

Nevertheless, there is a potentially serious problem inherent in the 2D-LQ method: it involves two unknown parameters, α and β, whose optimal values can vary noticeably across regions, as is evident in the results displayed in Tables 1 and 2.

As regards the other methods, the CILQ is clearly (apart from Seoul) the worst approach in both years. We may note too that the FLQ outperforms the AFLQ in 11 out of 16 cases in 2005 and in 13 out of 17 cases in 2015. This outcome suggests either that sectoral specialization is not an important factor for most regions or that the specialization term $[\log_2(1 + SLQ_j)]$ in Eq. (6) does not adequately capture the effects of specialization.

Table 4 Ranking of alternative regionalization techniques: coefficients, year 2015, STPE. *Source*: Table 2

	Region (2015)	Regional size (%)	CILQ	FLQ	AFLQ	2D-LQ	HTLQ
1	Gyeonggi	22.85	5	3	4	2	**1**
2	Seoul	18.97	3	2	4	**1**	5
3	North Gyeongsang	7.00	5	3	4	**1**	2
4	South Gyeongsang	6.93	5	3	4	2	**1**
5	Ulsan	6.32	5	3	4	2	**1**
6	South Jeolla	4.89	5	3	4	**1**	2
7	South Chungcheong	6.96	5	3	4	2	**1**
8	Incheon	4.96	5	3	4	**1**	2
9	Busan	4.73	5	4	3	**1**	2
10	North Chungcheong	3.47	5	3	4	**1**	2
11	Daegu	2.82	5	4	3	**1**	2
12	North Jeolla	2.82	5	3	4	**1**	2
13	Gangwon	1.97	5	4	2	**1**	3
14	Gwangju	2.07	5	3	4	**1**	2
15	Daejeon	1.92	5	4	3	**1**	2
16	Jeju	0.81	5	3	4	**1**	2
17	Sejong	0.50	5	3	4	**1**	2

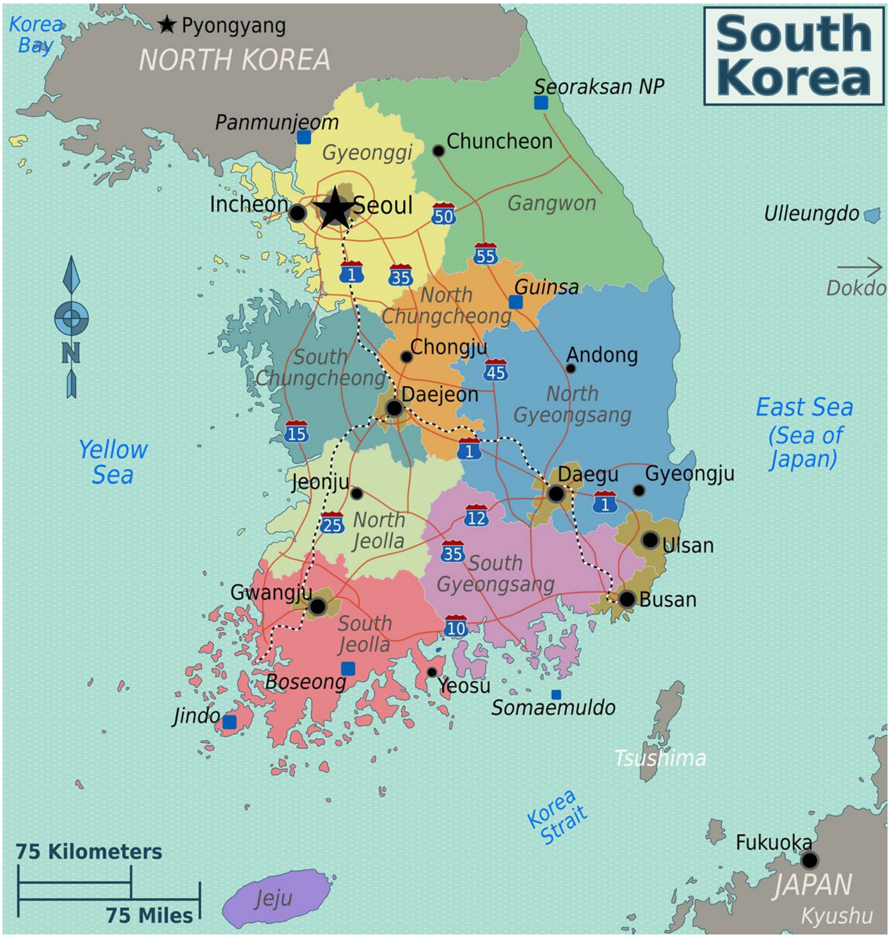

Fig. 3 South Korean regions. *Source*: South Korea regions map.png author: Peter Fitzgerald, NordNord-West; licensed under the Creative Commons Attribution-Share Alike 4.0 International; available in Wikimedia Commons

Table 5 Means and coefficients of variation of parameters. *Source*: Own elaboration

Formula	FLQ	2D-LQ		HTLQ
Parameter	δ	α	β	μ
Mean 2005	0.44	0.58	0.28	0.40
Mean 2015	0.40	0.60	0.25	0.47
V (%) 2005	38	47	27	25
V (%) 2015	38	57	32	20

V coefficient of variation

The comparative outcomes for the FLQ and HTLQ are also very interesting. Tables 1 and 2 reveal that the HTLQ gives better results than the FLQ for 13 of the 16 regions in 2005. The only exceptions are Seoul, Busan and Gangwon. The location of these regions is indicated in Fig. 3.

In 2015, the HTLQ outperforms the FLQ in all regions apart from Seoul, where its performance is very poor. Even so, in terms of rankings, it is evident that the HTLQ is a big improvement on the FLQ. What is striking about the outcomes in the two years is the similarity of the rankings, notwithstanding the gap of ten years.

Table 5 provides some useful extra information regarding the parametric methods. For instance, the mean values of α, β and δ are very similar in the two years, while there is more difference in the mean values of μ. In terms of dispersion, this is much greater for α than it is for β. α and β are the influential parameters in the correction by rows and columns, respectively. This finding can be confirmed by examining the detailed results in Tables 1 and 2. α also fluctuates substantially more than does δ. Finally, one can see that parameter μ for the HTLQ exhibits markedly less dispersion than does δ for the FLQ.

As regards the 2D-LQ, it is worth noting that our results are at variance with the findings of Pereira-López et al. (2020). Based on an analysis of Eurostat data for six European countries in 2005, they found that the range of suitable values of α and β was relatively small. Furthermore, they suggested that analysts would not go far wrong by choosing an α of 0.1 or 0.15 and a β in the range 0.8–1.2. The reasons for these strikingly different findings are hard to explain.

Figures 6 and 7 (See Appendix) depict the relationship between the values of parameters and the resulting STPE for all regions in years 2005 and 2015. Almost invariably across both regions and time, we observe that the relationships are U-shaped: values close to 0 and 1 tend to yield the highest errors.

The range of optimal δ and μ values is rather similar. The minimum and maximum values for δ are 0.16 (Seoul, 2015) and 0.69 (North Gyeongsang, 2005), whereas those for μ are 0.19 (Sejong, 2015) and 0.62 (Seoul, 2015). Interestingly, the maximum and minimum values for μ occur in the second-largest and the smallest region in our dataset.

Furthermore, we observe that the errors for our fixed HTLQ scenario ($\mu = 0.5$) are consistently lower than if we fix the FLQ's parameter at $\delta = 0.3$, which is a value suggested in the literature (Jahn et al., 2020). What is more, for $\mu = 0.5$, HTLQ outperforms the FLQ in 10 (2005) and 14 (2015) regions even when we use the optimum δ (See Figs. 4 and 5). This result suggests that HTLQ might be able to perform at least as well as other more data-demanding competitors, by using an uninformed one-value-fits-all parameter.

Should these results be confirmed in further testing, this would suggest that choosing $\mu = 0.5$ might be reasonable in the absence of additional regional information. Indeed, setting $\mu = 0.5$ restricts the number of national coefficients a_{ij}^{n} that are downscaled to values close to zero.

In addition, no matter how specialized a region is in a supplying or purchasing industry, choosing $\mu = 0.5$ ensures the existence of interregional or international imports for

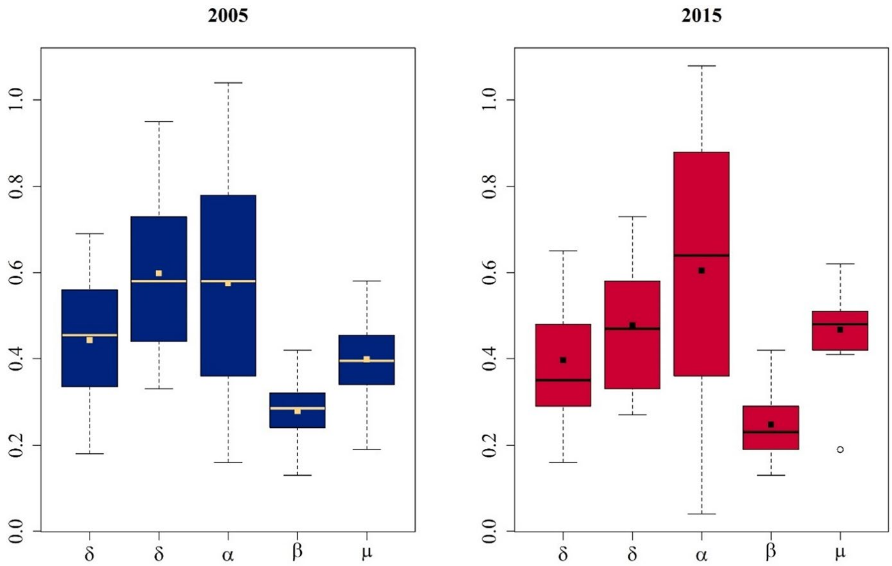

Fig. 4 Box and whiskers plot for parameter values. Squares mark means across South Korean regions *Source*: Own elaboration

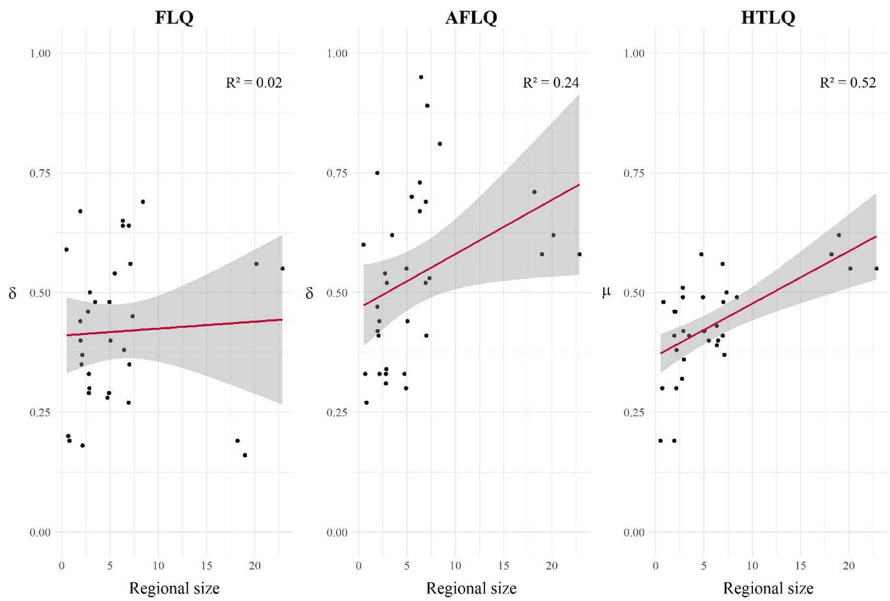

Fig. 5 Scatter plots of optimal parameters and regional size for FLQ, AFLQ and HTLQ, 2005 and 2015 (The grey shadow illustrates confidence intervals at the 95% level.) *Source*: Own elaboration

a fraction of its inputs, as we should expect for industries producing tradable goods and services.

6.3 Parameter Stability, and Correlations with Regional Size

Figure 4 provides some useful extra information regarding the stability of parameters in our selection of methods. The first δ in the abscissa axis relates to the FLQ, the second one to the AFLQ. The mean values of α, β and the FLQ's δ are very similar in the two years, whereas there is a more noticeable difference in the mean values of μ and the AFLQ's δ. Even so, these differences are under 0.15 (0.12 for the AFLQ and 0.06 for the HTLQ).

As regards dispersion across regions, this is clearly more pronounced for the 2D-LQ's row-adjusting parameter α. It is also evident that α fluctuates much more than does δ for the FLQ and AFLQ. It is interesting that μ exhibits much less dispersion than does δ for the FLQ and AFLQ in both years, and likewise for α. Note: we are ignoring the small administrative city of Sejong in these comparisons.

Based on the above findings, it appears reasonable to suggest that one could apply our HTLQ approach by: (i) using optimum parameters from previous years and/or (ii) by using a single parameter for all regions. Of course, the generality of our findings would need to be confirmed by a fresh set of data.

The optimal values of μ, shown in parentheses in Tables 1 and 2, are also worth examining as regards regional size R. For the initial year 2005, we can discern a tendency for μ to decline as regions get smaller: for the four largest regions, the mean value of μ is 0.530; for the next seven regions, it is 0.394; while, for the four smallest regions, it is 0.298. By contrast, the values of μ for the year 2015 do not vary so greatly across regions: the mean is 0.553 for the four largest regions, whereas it is 0.464 for the next seven and 0.460 for the five smallest regions. We are again ignoring Sejong here.

A simple regression analysis allows comparisons across single-parameter methods (FLQ, AFLQ and HTLQ) regarding the relationship between optimal parameters and R. The scatter plots in Fig. 5 reveal that μ exhibits by far the strongest correlation ($p \approx 0.001$) with R of the three parameters. This result confirms that μ is a good proxy for regional size. There is a rather weaker positive correlation between the AFLQ's optimal δ and R ($p \approx 0.05$).

By contrast, the optimal values for the FLQ's parameter δ show no significant correlation with R in our dataset. This is as expected since, on a priori grounds, δ should not vary with R. In the FLQ, regional size is considered via the term $\log_2(1+R)$.

7 Sensitivity Analysis

In this section, we examine the robustness of our findings by employing different summary measures of error.

The following alternatives to the *STPE* (See Eq. 14) are considered here:

$$MAD = \frac{1}{n^2} \sum_{j=1}^{n} \left| \widetilde{a}_{ij}^{r} - a_{ij}^{r} \right| \tag{15}$$

$$MAPE = \frac{100}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \left| \widetilde{a}_{ij}^{r} - a_{ij}^{r} \right| / \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}^{r} \tag{16}$$

$$U = 100 \sqrt{ \sum_{i=1}^{n} \sum_{j=1}^{n} (a_{ij}^{r} - \widetilde{a}_{ij}^{r})^2 / \sum_{i=1}^{n} \sum_{j=1}^{n} (a_{ij}^{r})^2 } \tag{17}$$

where $n = 31$ is the number of sectors. *MAD* measures the mean absolute deviation of the estimated coefficients from the observed values. This statistic is proposed by Wiebe and Lenzen (2016) and employed by Flegg et al. (2021). As a relative measure, *MAPE* is a refinement of *MAD* that can be employed to compare accuracy across studies. Since *MAPE* equals *STPE* divided by 961, it is simply a matter of convenience which measure is used. *STPE* and *MAPE* would yield identical rankings of techniques, as would *MAD*.

Theil's index *U* is an attractive measure since it considers simultaneously two sources of simulation errors: bias and dispersion. However, a possible demerit is that the use of squared differences has the effect of emphasizing any large positive or negative errors. We should expect *U* to yield similar but not identical rankings of techniques as the other measures.

The results for *STPE*, *MAD* and *MAPE* in 2015 are displayed in Tables 2, 6 and 7. It can be verified that the rankings are unaffected by a change in the error criterion. Moreover, the optimal values of the parameters are unchanged.

As expected, some changes in rankings arise once Theil's criterion *U* is used to assess accuracy. The 2D-LQ is still the best method for 14 out of 17 regions according to all four criteria. However, this overall outcome masks a very minor and offsetting change in the rankings of two regions when Theil's measure is applied. More fundamentally, whereas the *STPE*, *MAD* and *MAPE* judge the HTLQ to be superior to the FLQ in all but one region, Table 8 reveals that this is only true for 9 out of 17 regions according to *U*. Most of the changes in ranking occur in the smallest regions.

It is worth noting, finally, that there is a marked fall in the size of the optimal values of δ for the FLQ once Theil's criterion is employed. This effect is especially noticeable in the smaller regions and may be a consequence of the squaring of errors, which can exaggerate the weighting of atypically large negative or positive errors.

8 Multipliers and Important Coefficients

In their classic study, Jensen and West (1980) demonstrate that the relatively largest coefficients in a coefficient matrix exert the most influence on the size of output and other multipliers; conversely, that relatively small coefficients have minimal impact on the size of these multipliers. Moreover, the more interconnected

Table 6 MAPE results for alternative regionalization techniques, coefficients, year 2015 *Source*: Own elaboration

	Region (2015)	Regional size (%)	CILQ	FLQ (δ)	AFLQ (δ)	2D-LQ (α;β)	HTLQ (μ)	HTLQ ($\mu = 0.5$)
1	Gyeonggi	22.85	0.06403	0.04101	0.04208	0.03561	0.03460	0.03570
				(0.55)	(0.58)	(0.44; 0.42)	(0.55)	
2	Seoul	18.97	0.05924	0.05889	0.06416	0.04369	0.07310	0.07419
				(0.16)	(0.58)	(0.76; 0.17)	(0.62)	
3	North Gyeongsang	7.00	0.07940	0.05793	0.06402	0.04646	0.04731	0.04737
				(0.35)	(0.41)	(0.28; 0.29)	(0.48)	
4	South Gyeongsang	6.93	0.06986	0.05809	0.06632	0.04873	0.0341	0.04780
				(0.27)	(0.52)	(0.36; 0.23)	(0.56)	
5	Ulsan	6.32	0.10336	0.07653	0.07855	0.05880	0.05482	0.05613
				(0.65)	(0.73)	(0.04; 0.30)	(0.43)	
6	South Jeolla	4.89	0.07826	0.06849	0.07495	0.04859	0.05642	0.05648
				(0.29)	(0.30)	(0.52; 0.19)	(0.49)	
7	South Chungcheong	6.96	0.09828	0.07069	0.07092	0.06104	0.05975	0.06153
				(0.64)	(0.69)	(0.08; 0.37)	(0.41)	
8	Incheon	4.96	0.10407	0.05341	0.05477	0.05053	0.05147	0.05279
				(0.48)	(0.55)	(0.80; 0.26)	(0.42)	
9	Busan	4.73	0.07147	0.05198	0.05195	0.04448	0.04853	0.04980
				(0.28)	(0.33)	(0.92; 0.20)	(0.58)	
10	North Chungcheong	3.47	0.10496	0.07532	0.07667	0.06453	0.06678	0.06816
				(0.48)	(0.62)	(0.12; 0.34)	(0.41)	
11	Daegu	2.82	0.08035	0.06115	0.06052	0.04633	0.05347	0.05350
				(0.29)	(0.31)	(1.00; 0.19)	(0.51)	
12	North Jeolla	2.82	0.08577	0.06526	0.06671	0.05234	0.05760	0.05765
				(0.33)	(0.33)	(0.64; 0.19)	(0.49)	
13	Gangwon	1.97	0.09189	0.07037	0.06737	0.05469	0.06988	0.07025
				(0.40)	(0.42)	(0.88; 0.19)	(0.46)	
14	Gwangju	2.07	0.09482	0.07270	0.07505	0.05439	0.06195	0.06215
				(0.35)	(0.41)	(1.08; 0.18)	(0.46)	
15	Daejeon	1.92	0.12939	0.08305	0.08206	0.06227	0.06696	0.06907
				(0.44)	(0.47)	(1.04; 0.25)	(0.41)	
16	Jeju	0.81	0.08029	0.07410	0.07427	0.05009	0.07221	0.07225
				(0.19)	(0.27)	(0.68; 0.13)	(0.48)	
17	Sejong	0.50	0.19046	0.09145	0.09188	0.07557	0.07976	0.11445
				(0.59)	(0.60)	(0.60; 0.29)	(0.19)	

Table 7 MAD results for alternative regionalization techniques, coefficients, year 2015 *Source*: Own elaboration

	Region (2015)	Regional size (%)	CILQ	FLQ (δ)	AFLQ (δ)	2D-LQ (α;β)	HTLQ (μ)	HTLQ ($\mu = 0.5$)
1	Gyeonggi	22.85	0.00502	0.00321 (0.55)	0.00330 (0.58)	0.00279 (0.44; 0.42)	0.00271 (0.55)	0.00279
2	Seoul	18.97	0.00386	0.00384 (0.16)	0.00418 (0.58)	0.00285 (0.76; 0.17)	0.00476 (0.62)	0.00483
3	North Gyeong-sang	7.00	0.00550	0.00401 (0.35)	0.00443 (0.41)	0.00322 (0.28; 0.29)	0.00327 (0.48)	0.00328
4	South Gyeong-sang	6.93	0.00506	0.00421 (0.27)	0.00480 (0.52)	0.00353 (0.36; 0.23)	0.00341 (0.56)	0.00346
5	Ulsan	6.32	0.00656	0.00486 (0.65)	0.00498 (0.73)	0.00373 (0.04; 0.30)	0.00348 (0.43)	0.00356
6	South Jeolla	4.89	0.00529	0.00463 (0.29)	0.00507 (0.30)	0.00328 (0.52; 0.19)	0.00381 (0.49)	0.00382
7	South Chun-gcheong	6.96	0.00667	0.00481 (0.64)	0.00481 (0.69)	0.00414 (0.08; 0.37)	0.00406 (0.41)	0.00418
8	Incheon	4.96	0.00638	0.00327 (0.48)	0.00336 (0.55)	0.00310 (0.80; 0.26)	0.00315 (0.42)	0.00324
9	Busan	4.73	0.00515	0.00375 (0.28)	0.00375 (0.33)	0.00321 (0.92; 0.20)	0.00350 (0.58)	0.00359
10	North Chun-gcheong	3.47	0.00626	0.00449 (0.48)	0.00457 (0.62)	0.00385 (0.12; 0.34)	0.00398 (0.41)	0.00407
11	Daegu	2.82	0.00529	0.00403 (0.29)	0.00398 (0.31)	0.00305 (1.00; 0.19)	0.00352 (0.51)	0.00352
12	North Jeolla	2.82	0.00562	0.00427 (0.33)	0.00437 (0.33)	0.00343 (0.64; 0.19)	0.00377 (0.49)	0.00378
13	Gangwon	1.97	0.00570	0.00436 (0.40)	0.00418 (0.42)	0.00338 (0.88; 0.19)	0.00433 (0.46)	0.00436
14	Gwangju	2.07	0.00568	0.00436 (0.35)	0.00450 (0.41)	0.00326 (1.08; 0.18)	0.00371 (0.46)	0.00372
15	Daejeon	1.92	0.00683	0.00439 (0.44)	0.00433 (0.47)	0.00329 (1.04; 0.25)	0.00354 (0.41)	0.00365
16	Jeju	0.81	0.00452	0.00417 (0.19)	0.00418 (0.27)	0.00282 (0.68; 0.13)	0.00407 (0.48)	0.00407
17	Sejong	0.50	0.00737	0.00354 (0.59)	0.00356 (0.60)	0.00293 (0.60; 0.29)	0.00309 (0.19)	0.00443

Table 8 Theil's *U* results for alternative regionalization techniques, coefficients, year 2015. *Source* Own elaboration

	Region (2015)	Regional size (%)	CILQ	FLQ (δ)	AFLQ (δ)	2D-LQ (α;β)	HTLQ (μ)	HTLQ ($\mu = 0.5$)
1	Gyeonggi	22.85	49.30	45.20 (0.26)	53.46 (0.42)	41.28 (0.40; 0.25)	41.79 (0.71)	49.46
2	Seoul	18.97	48.56	48.27 (0.05)	68.97 (0.22)	38.00 (0.56; 0.11)	65.04 (0.54)	65.25
3	North Gyeongsang	7.00	57.59	49.85 (0.21)	73.83 (0.62)	48.42 (0.20; 0.16)	44.95 (0.65)	49.26
4	South Gyeongsang	6.93	57.34	50.50 (0.17)	77.46 (0.64)	51.09 (0.20; 0.17)	46.35 (0.66)	51.00
5	Ulsan	6.32	90.85	80.01 (0.63)	84.50 (0.96)	66.54 (-0.08; 0.27)	65.94 (0.49)	65.96
6	South Jeolla	4.89	47.24	45.32 (0.07)	63.28 (0.09)	42.96 (0.64; 0.11)	54.55 (0.68)	58.87
7	South Chungcheong	6.96	72.82	59.78 (0.38)	64.97 (0.47)	54.55 (0.08; 0.20)	53.17 (0.60)	54.98
8	Incheon	4.96	78.87	60.03 (0.28)	68.03 (0.34)	54.00 (0.40; 0.20)	54.72 (0.54)	55.11
9	Busan	4.73	59.62	49.19 (0.19)	53.42 (0.24)	45.73 (1.00; 0.14)	50.73 (0.61)	53.06
10	North Chungcheong	3.47	73.44	63.66 (0.39)	70.11 (0.52)	56.99 (0.48; 0.19)	58.81 (0.56)	59.42
11	Daegu	2.82	52.96	49.80 (0.05)	60.88 (0.17)	46.30 (1.04; 0.11)	53.82 (0.65)	57.24
12	North Jeolla	2.82	64.71	57.09 (0.24)	62.58 (0.33)	47.84 (0.68; 0.16)	51.79 (0.63)	54.61
13	Gangwon	1.97	56.55	52.56 (0.08)	54.87 (0.23)	46.98 (1.08; 0.13)	66.56 (0.65)	68.83
14	Gwangju	2.07	60.87	55.51 (0.08)	77.90 (0.11)	54.67 (0.68; 0.11)	57.65 (0.59)	58.89
15	Daejeon	1.92	74.79	71.72 (0.08)	76.18 (0.30)	59.11 (0.88; 0.16)	63.02 (0.56)	63.59
16	Jeju	0.81	46.57	44.29 (0.06)	49.10 (0.13)	41.34 (1.68; 0.14)	61.92 (0.60)	63.29
17	Sejong	0.50	71.43	70.40 (0.02)	84.84 (0.14)	53.14 (1.68; 0.14)	70.95 (0.61)	72.11

the economy under review, the greater the importance of the larger coefficients. For instance, metropolitan economies would tend to be more interconnected than, say, rural regions and small urban areas. In general, Jensen and West suggest that research resources should be focused on the successful estimation of the larger coefficients.

Here we examine how far the FLQ and HTLQ methods can capture what Hewings and Romanos (1981) call ‘inverse important coefficients’, whereby an input coefficient a_{ij} is said to be *inverse important* if an error of α% in that coefficient produces a corresponding error of β% in one or more entries of the Leontief inverse.

In this analysis, α and β were set equal to 30% and 20%, respectively, as suggested by Hewings and Romanos. This means that an input coefficient is held to be inverse important if a perturbation of 30% generates a change of at least 20% in one or more entries in the Leontief inverse.

More formally, following Flegg et al. (2021), we assume that only one coefficient, a_{ij}, of the matrix **A** is perturbed. Let a_{ij}^{p} denote this perturbed value of a_{ij}, i.e. $a_{ij}^{p} = a_{ij}(1+\alpha/100)$. This shock generates a perturbed matrix $\mathbf{L^p} = [L_{ks}^{p}]$ of the Leontief matrix $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1} = [L_{ks}]$, k, $s = 1, 2, \ldots, 31$, whose elements are determined as follows:

$$L_{ks}^{p} = L_{ks} + [L_{ki} L_{sj} a_{ij} (a/100)]/[1 - L_{ji} a_{ij} (a/100)] \tag{18}$$

where the indices i and j relate to the perturbed coefficient in **A**, while k and s pertain to the Leontief inverse **L** and the corresponding perturbed matrix $\mathbf{L^p}$.

The relative error of each coefficient of the Leontief matrix reads as

$$\frac{L_{ks}^{P} - L_{ks}}{L_{ks}} = \frac{L_{ki} L_{sj} a_{ij} (a/100)}{L_{ks}[1 - L_{ji} a_{ij} (a/100)]} \tag{19}$$

We then say that the coefficient a_{ij} is inverse important if there exists at least one pair (k, s) such that the following inequality holds:

$$\left| \frac{L_{ki} L_{sj} a_{ij} (a/100)}{L_{ks}[1 - L_{ji} a_{ij} (a/100)]} \right| \geq b/100 \tag{20}$$

Tables 9 and 10 report STPE values for the Type I output multipliers generated by the FLQ and HTLQ methods when using their optimal parameters. The columns headed ‘Multipliers’ show the values generated by the Leontief inverse that yields minimum STPE. By contrast, the columns headed ‘Important multipliers’ show the values generated by the Leontief inverse when only inverse-important coefficients are considered. Finally, the ratios of the STPE values in the respective columns are presented. For instance, for Seoul in 2015, the STPE for important multipliers is 65% of that for the complete set of HTLQ-based multipliers. Clearly, important multipliers are markedly more accurately estimated than are multipliers in general.

Also, as would be expected, STPE is invariably much lower for multipliers than for coefficients. Again, taking Seoul in 2015 and the HTLQ to illustrate, STPE is 70.25 for coefficients but 15.32 for multipliers.

Table 9 STPE results for output multipliers, FLQ and HTLQ methods, year 2005 *Source*: Own elaboration

Region and size (%)		FLQ			HTLQ		
		Multipliers (δ)	Important multipliers	Ratio	Multipliers (μ)	Important multipliers	Ratio
1 Gyeonggi	22.85	9.43	5.70	0.60	8.82	5.08	0.58
		(0.53)			(0.56)		
2 Seoul	18.97	10.53	6.69	0.64	15.74	8.97	0.57
		(0.19)			(0.61)		
3 North Gyeongsang	7.00	13.62	9.45	0.69	11.95	7.20	0.60
		(0.69)			(0.55)		
4 South Gyeongsang	6.93	12.15	7.82	0.64	11.21	6.79	0.61
		(0.41)			(0.53)		
5 Ulsan	6.32	15.42	10.99	0.71	12.33	8.09	0.66
		(0.73)			(0.39)		
6 South Jeolla	4.89	16.91	12.63	0.75	13.82	9.21	0.67
		(0.62)			(0.46)		
7 South Chungcheong	6.96	13.25	8.99	0.68	10.41	6.73	0.65
		(0.59)			(0.41)		
8 Incheon	4.96	10.75	7.88	0.73	8.70	6.07	0.70
		(0.55)			(0.40)		
9 Busan	4.73	11.26	7.91	0.70	11.96	7.90	0.66
		(0.36)			(0.45)		
10 North Chungcheong	3.47	13.55	8.90	0.66	12.40	8.10	0.65
		(0.41)			(0.42)		
11 Daegu	2.82	11.05	7.65	0.69	10.90	7.08	0.65
		(0.30)			(0.45)		
12 North Jeolla	2.82	12.04	8.25	0.68	11.26	7.33	0.65
		(0.44)			(0.36)		
13 Gangwon	1.97	14.68	10.23	0.70	15.67	9.97	0.64
		(0.19)			(0.41)		
14 Gwangju	2.07	11.67	8.13	0.70	10.76	6.93	0.64
		(0.36)			(0.33)		
15 Daejeon	1.92	12.05	8.92	0.74	10.56	7.84	0.74
		(0.60)			(0.22)		
16 Jeju	0.81	13.06	10.04	0.77	12.95	8.31	0.64
		(0.20)			(0.34)		
Mean		12.59	8.76	0.69	11.84	7.60	0.64
Standard Deviation		1.95	1.68	0.04	2.02	1.22	0.04
Median		12.10	8.57	0.70	11.61	7.58	0.65
Min		9.43	5.70	0.60	8.70	5.08	0.57
Max		16.91	12.63	0.77	15.74	9.97	0.74

Table 10 STPE results for output multipliers, FLQ and HTLQ methods, year 2015 *Source*: Own elaboration

Region and size (%)		FLQ			HTLQ		
		Multipliers (δ)	Important multipliers	Ratio	Multipliers (μ)	Important multipliers	Ratio
1 Gyeonggi	22.85	9.79	6.83	0.70	8.14	5.63	0.69
		(0.54)			(0.58)		
2 Seoul	18.97	11.77	9.44	0.80	15.32	9.99	0.65
		(0.26)			(0.61)		
3 North Gyeong-sang	7.00	12.48	9.02	0.72	9.81	7.07	0.72
		(0.39)			(0.51)		
4 South Gyeongsang	6.93	12.96	8.74	0.67	10.31	7.22	0.70
		(0.31)			(0.58)		
5 Ulsan	6.32	14.59	11.05	0.76	10.53	7.78	0.74
		(0.65)			(0.45)		
6 South Jeolla	4.89	14.59	11.27	0.77	11.73	8.22	0.70
		(0.31)			(0.51)		
7 South Chun-gcheong	6.96	15.21	11.31	0.74	13.17	9.81	0.75
		(0.54)			(0.44)		
8 Incheon	4.96	9.78	7.78	0.80	9.70	7.54	0.78
		(0.48)			(0.43)		
9 Busan	4.73	11.10	8.26	0.74	10.41	7.14	0.69
		(0.28)			(0.59)		
10 North Chun-gcheong	3.47	14.33	10.56	0.74	12.78	9.03	0.71
		(0.47)			(0.43)		
11 Daegu	2.82	12.43	9.20	0.74	10.84	7.65	0.71
		(0.24)			(0.52)		
12 North Jeolla	2.82	13.23	9.66	0.73	12.33	8.56	0.69
		(0.33)			(0.49)		
13 Gangwon	1.97	13.37	10.28	0.77	13.44	9.40	0.70
		(0.39)			(0.47)		
14 Gwangju	2.07	13.24	10.27	0.78	11.28	7.98	0.71
		(0.35)			(0.48)		
15 Daejeon	1.92	13.29	10.42	0.78	10.97	7.99	0.73
		(0.44)			(0.41)		
16 Jeju	0.81	12.83	10.53	0.82	12.40	8.60	0.69
		(0.24)			(0.47)		
17 Sejong	0.50	11.09	8.70	0.78	9.66	7.57	0.78
		(0.58)			(0.20)		
Mean		12.71	9.61	0.76	11.34	8.07	0.71
Standard Deviation		1.59	1.28	0.04	1.75	1.10	0.03
Median		12.96	9.66	0.76	10.97	7.98	0.71

Table 10 (continued)

Region and size (%)	FLQ			HTLQ		
	Multipliers (δ)	Important multipliers	Ratio	Multipliers (μ)	Important multipliers	Ratio
Min	9.78	6.83	0.67	8.14	5.63	0.65
Max	15.21	11.31	0.82	15.32	9.99	0.78

An important facet of the multiplier results is that they demonstrate the superior performance of the HTLQ relative to the FLQ across all regions apart from Seoul, thereby confirming the outcomes for coefficients.[5]

9 The SFLQ Revisited

Unlike the FLQ, the SFLQ permits the values of δ to vary freely across sectors. Here we explore the consequences of relaxing the FLQ's assumption of a constant δ.

Table 11 displays the results of using the SFLQ formula (9) to estimate a δ for each sector and year, along with the associated STPE and summary statistics. The tabulated values are averages across all regions. The criterion employed was minimum STPE for the Type I output multiplier for each sector. Sectoral output rather than employment was used in the computations.

It is evident from Table 11 that δ varies greatly across sectors, although more so in 2005 than in 2015. This is confirmed by the coefficient of variation, which shows that this variability fell from 90% in 2005 to 54% in 2015. The summary statistics also record a rise in the mean δ over this period. A clear difference between the mean and median is apparent in both periods, which points to a lack of symmetry in both distributions.

An interesting facet of the results is the very low values of δ in sectors 6, 16, 19 and 26 in both years. This outcome reflects the negligible role of imports in these sectors. It is also worth noting that sectors 1 and 27 have near-zero values of δ in 2005 but not in 2015.

Tables 12 and 13 present results for the two years, tabulated by region, showing regional mean δ and associated summary statistics. The results confirm the substantial amount of variability in the sectoral values of δ for each region considered separately, although this phenomenon is more pronounced in 2005 than in 2015.

Finally, Table 14 shows results, again tabulated by region and year, for regional mean STPE. The purpose of this table is to enable a comparison to be made of the accuracy achieved via our HTLQ approach with what, in principle at least, might be obtained by using the SFLQ. In fact, from Table 1, the mean value of STPE across all regions for the HTLQ in 2005 is 60.3%, while the corresponding figure for 2015

[5] Busan and Gangwon offer some trivial exceptions to this statement.

Table 11 SFLQ estimation by sector. *Source* Own elaboration

Sectors		2005		2015	
		STPE	δ	STPE	δ
1	Agricultural, forest, and fishery goods	3.783	0.01	0.169	0.78
2	Mined and quarried goods	0.162	0.16	0.142	0.47
3	Food, beverages and tobacco products	0.310	0.55	0.211	0.35
4	Textile and leather products	1.000	1.00	0.031	0.63
5	Wood and paper products, printing and reproduction of recorded media	0.005	0.37	0.101	0.44
6	Petroleum and coal products	6.926	0.01	5.483	0.01
7	Chemical products	0.081	0.55	0.157	0.41
8	Non-metallic mineral products	0.003	0.31	0.191	0.68
9	Basic metal products	0.020	0.34	0.009	0.42
10	Fabricated metal products, except machinery and furniture	0.079	0.38	0.112	0.37
11	Computing machinery, electronic equipment and optical instruments	0.017	0.84	0.080	0.37
12	Electrical equipment	0.077	0.42	0.190	0.42
13	Machinery and equipment	0.050	0.66	0.228	0.39
14	Transport equipment	1.409	1.00	0.264	0.52
15	Other manufactured products & Manufacturing services and repair services of industrial equipment	0.209	0.37	0.137	0.34
16	Electricity, gas, and steam supply & Water supply, sewage and waste treatment and disposal services	0.228	0.05	0.078	0.03
17	Construction	0.351	0.03	0.286	0.40
18	Wholesale and retail trade and commodity brokerage services	0.025	0.21	0.043	0.38
19	Transportation	0.136	0.03	2.022	0.01
20	Food services and accommodation	0.065	0.22	0.159	0.39
21	Communications and broadcasting	0.156	0.23	0.074	0.58
22	Finance and insurance	0.374	0.23	0.007	0.25
23	Real estate services	0.048	0.11	0.231	0.06

Table 11 (continued)

Sectors		2005		2015	
		STPE	δ	STPE	δ
24	Professional, scientific, and technical services	0.148	0.48	0.150	0.43
25	Business support services	0.269	0.33	0.278	0.15
26	Public administration, defense, and social security services	0.080	0.04	0.034	0.04
27	Education services	0.353	0.02	0.190	0.16
28	Health and social care services	0.310	0.08	0.139	0.29
29	Art, sports, and leisure services	0.008	0.15	0.199	0.39
30	Other services	0.014	0.09	0.059	0.41
31	Others	0.242	0.46	0.126	0.57
Mean		1.059	0.31	0.373	0.36
Standard deviation		1.378	0.28	1.010	0.20
Median		0.148	0.23	0.150	0.47
Coefficient of variation		1.302	0.90	2.703	0.54
Min		0.003	0.01	0.007	0.01
Max		6.926	1.00	5.483	0.78

Table 12 SFLQ results for sectoral δ tabulated by region, year 2005. *Source*: Own elaboration

	Region (2005)	Regional size (%)	Mean	SD	Median	Min	Max
1	Gyeonggi	20.14	0.41	0.23	0.47	0.01	0.92
2	Seoul	18.20	0.36	0.34	0.27	0.01	1.00
3	North Gyeongsang	8.42	0.48	0.30	0.44	0.01	1.00
4	South Gyeongsang	7.35	0.41	0.20	0.43	0.01	0.75
5	Ulsan	7.11	0.50	0.31	0.48	0.01	1.00
6	South Jeolla	6.46	0.63	0.31	0.68	0.01	1.00
7	South Chungcheong	6.33	0.43	0.21	0.45	0.01	0.77
8	Incheon	5.49	0.36	0.19	0.42	0.01	0.73
9	Busan	5.06	0.27	0.20	0.27	0.01	1.00
10	North Chungcheong	2.92	0.32	0.16	0.35	0.01	0.62
11	Daegu	2.88	0.30	0.16	0.31	0.01	0.64
12	North Jeolla	2.74	0.28	0.17	0.32	0.01	0.66
13	Gangwon	2.16	0.29	0.21	0.26	0.01	0.81
14	Gwangju	2.15	0.41	0.22	0.39	0.01	1.00
15	Daejeon	1.93	0.39	0.22	0.38	0.01	1.00
16	Jeju	0.67	0.31	0.28	0.23	0.01	1.00
Mean			0.38	0.23	0.38	0.01	0.87
SD			0.10	0.06	0.11	0.00	0.15
Median			0.38	0.22	0.38	0.01	0.96
Min			0.27	0.16	0.23	0.01	0.62
Max			0.63	0.34	0.68	0.01	1.00

SD standard deviation

from Table 2 is 56.6%. By contrast, Table 14 records an error for the SFLQ of under 1% in both years.

We now need to consider the feasibility of capturing at least some of the potential enhancements in efficiency. In the absence of survey data, one could try to use regression analysis to estimate a δ for each sector, yet this approach is very problematic, as explained in Sect. 3.

A more fruitful approach might be to incorporate prior information about the degree of sectoral self-sufficiency, as proposed by Flegg et al. (2016) in a study of the province of Córdoba, Argentina. They note that many of Córdoba's service sectors, such as hotels and restaurants, and also transport, storage and communications, are highly location-specific.

Consequently, Flegg et al. impose the restriction $\hat{a}_{ij}^{r} = \mathrm{a}_{\mathrm{ij}}^{\mathrm{n}}$ on 12 of the 30 sectors, on the basis that these sectors predominantly produce non-tradable goods and services, so that no allowance is required for regional imports. This restriction results in a marked improvement in the results for the FLQ. However, the authors note that a δ in the range 0.3–0.4 is then needed for the remaining 18 sectors, which is much higher than the $\delta = 0.1$ required when all 30 sectors are analyzed together.

Table 13 SFLQ results for sectoral δ tabulated by region, year 2015. *Source*: Own elaboration

	Region (2015)	Regional size (%)	Mean	SD	Median	Min	Max
1	Gyeonggi	22.85	0.29	0.21	0.28	0.01	0.74
2	Seoul	18.97	0.98	0.06	1.00	0.75	1.00
3	North Gyeongsang	7.00	0.34	0.20	0.36	0.01	0.69
4	South Gyeongsang	6.93	0.25	0.16	0.26	0.01	0.56
5	Ulsan	6.32	0.32	0.20	0.35	0.01	0.62
6	South Jeolla	4.89	0.38	0.26	0.39	0.01	0.88
7	South Chungcheong	6.96	0.20	0.15	0.19	0.01	0.52
8	Incheon	4.96	0.36	0.20	0.39	0.01	0.78
9	Busan	4.73	0.37	0.21	0.37	0.01	1.00
10	North Chungcheong	3.47	0.22	0.15	0.23	0.01	0.56
11	Daegu	2.82	0.35	0.19	0.36	0.01	1.00
12	North Jeolla	2.82	0.12	0.11	0.11	0.01	0.41
13	Gangwon	1.97	0.29	0.18	0.29	0.01	0.76
14	Gwangju	2.07	0.54	0.19	0.51	0.25	1.00
15	Daejeon	1.92	0.50	0.22	0.45	0.17	1.00
16	Jeju	0.81	0.29	0.27	0.20	0.01	1.00
17	Sejong	0.50	0.11	0.13	0.09	0.01	1.00
Mean			0.35	0.18	0.34	0.08	0.80
SD			0.20	0.05	0.20	0.19	0.21
Median			0.32	0.19	0.35	0.01	0.78
Min			0.11	0.06	0.09	0.01	0.41
Max			0.98	0.27	1.00	0.75	1.00

SD standard deviation

Although beyond the scope of the present paper, it would be interesting to examine the impact on our results of using national input coefficients for selected sectors in place of LQ-based estimates.

10 Concluding Remarks

In this paper, we re-examine the foundations of the FLQ formula and propose a simpler alternative formulation that does not depend explicitly on the relative size of regions. Instead, the new formula, the HTLQ, contains a parameter μ that is a proxy for a region's degree of self-sufficiency, which would depend on a region's size, along with other factors.

Using survey-based South Korean interregional data, we found that the HTLQ outperformed the FLQ in 13 of the 16 regions in 2005. In 2015, it did so in all 17 regions apart from Seoul, where its accuracy was very poor.

Nonetheless, once the *STPE* was replaced by Theil's criterion *U*, the HTLQ outperformed the FLQ in only 9 of these 17 regions. This lack of robustness is a

Table 14 SFLQ results for sectoral STPE tabulated by region. *Source*: Own elaboration

Region	2005	2015
1 Gyeonggi	0.98	1.73
2 Seoul	1.92	9.10
3 North Gyeongsang	0.53	0.51
4 South Gyeongsang	0.42	0.91
5 Ulsan	1.55	1.16
6 South Jeolla	1.15	1.05
7 South Chungcheong	0.56	2.44
8 Incheon	0.69	0.37
9 Busan	1.26	0.39
10 North Chungcheong	0.60	1.07
11 Daegu	0.54	0.88
12 North Jeolla	0.67	2.52
13 Gangwon	1.04	0.88
14 Gwangju	0.44	0.17
15 Daejeon	0.82	0.23
16 Jeju	0.55	1.12
17 Sejong		0.11
Overall mean	0.86	0.35

little concerning, although it should be borne in mind that U would tend to exaggerate the importance of any atypically large simulation errors, and this may be the explanation here.

Furthermore, a caveat should be noted regarding Seoul. Its importance derives from the fact that it is the second-largest region, along with its status as the capital city. In this region, the FLQ outperformed the HTLQ.

In choosing between the FLQ and HTLQ, it is important to consider their data requirements. For the FLQ, one would need regional sectoral output or employment data, which should not be problematic. Obtaining suitable values for the unknown parameter δ would be more challenging, although such values could be estimated via the FLQ + method. Moreover, these estimates would be specific to a particular region, country and time.

By contrast, the HTLQ has an unknown parameter μ. Here we argued that, in most cases, it would be reasonable to use a single value of $\mu = 0.5$, which would greatly simplify the computations.

We also performed an analysis of important coefficients and the associated type I output multipliers. The results revealed that the HTLQ clearly outperformed the FLQ, even when the analysis was restricted to the larger input coefficients.

In addition, we considered the 2D-LQ method as a possible alternative to the FLQ and HTLQ, especially since it gave the best results for 13 of the 16 regions in 2005 and took second place in the others. This outstanding performance was mirrored in 2015. However, these promising results were obtained by using the optimal values of the unknown parameters α and β. We argued that it would be very challenging indeed to obtain reliable estimates of α and β.

In another type of analysis, we employed the SFLQ (industry-specific FLQ) to examine the extent to which the optimal values of δ in the FLQ varied across sectors. We identified a substantial amount of such variation and noted the consequent loss of accuracy by not taking it into account. Nevertheless, bearing in mind the great difficulty in finding the necessary data to implement the SFLQ method, we suggested that analysts using the FLQ should consider using different values of δ for particular types of sector, e.g. non-competitive sectors.

It is worth noting, finally, that our focus has been on relatively straightforward approaches that can be implemented using readily available data. Of course, more complex approaches, requiring data less readily available, do exist. Such approaches should produce more accurate results but at the cost of additional complexity and more demanding data requirements. A good example is the regression study by Lahr et al. (2020), which is discussed by Flegg et al. (2021). A key issue is whether the regression results obtained from one dataset can be transferred to another context.

Appendix

See Figs. 6 and 7.

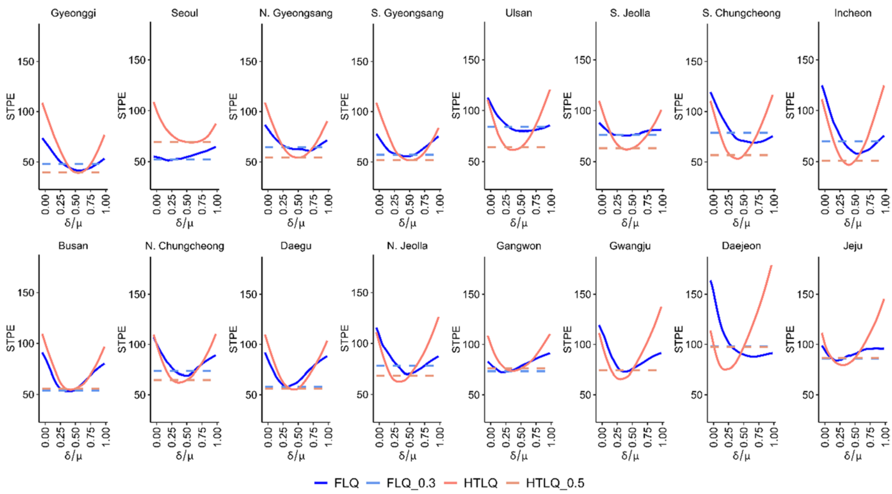

Fig. 6 Elasticity between STPE and parameters δ (FLQ) and μ (HTLQ). South Korean regions, 2005. *Source* Own elaboration

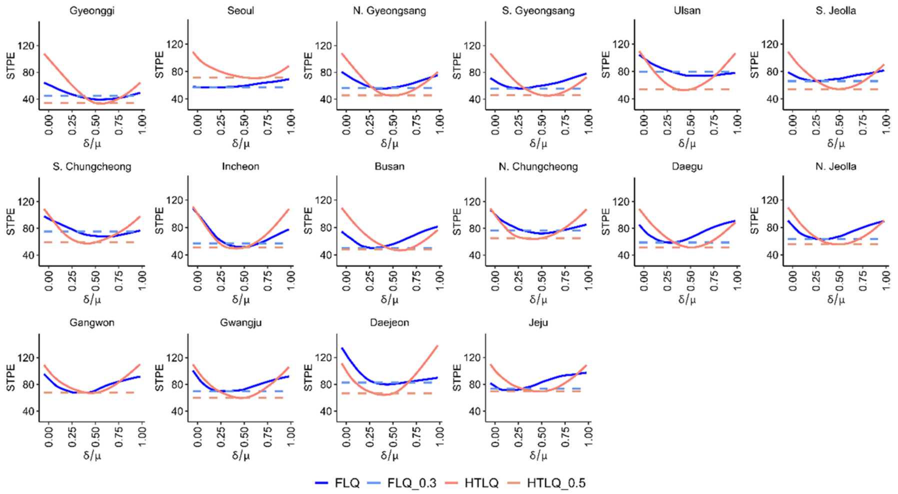

Fig. 7 Elasticity between STPE and parameters δ (FLQ) and μ (HTLQ). South Korean regions, 2015. Source Own elaboration. Note: The term elasticity in these figures refers to the relation between the delta/mu parameters and the STPE errors in the A matrices

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Data Availability The basic data were downloaded from the Bank of Korea's website, bok.or.kr. For instance, for 2015, the link is: https://www.bok.or.kr/eng/bbs/E0000634/view.do?nttId=10059403&menuNo=40006&pageIndex=12

Declarations

Competing interests The authors have no relevant financial or non-financial interests to disclose.

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References

Bonfiglio, A., & Chelli, F. (2008). Assessing the behaviour of non-survey methods for constructing regional input–output tables through a Monte Carlo simulation. *Economic Systems Research, 20*, 243–258. https://doi.org/10.1080/09535310802344315

Dávila-Flores, A. (2015). *Modelos interregionales de insumo producto de la economía mexicana*. Universidad Autónoma de Coahuila.

Flegg, A. T., Lamonica, G. R., Chelli, F. M., Recchioni, M. C., & Tohmo, T. (2021). A new approach to modelling the input–output structure of regional economies using non-survey methods. *Journal of Economic Structures, 10*, 12. https://doi.org/10.1186/s40008-021-00242-8

Flegg, A. T., Mastronardi, L. J., & Romero, C. A. (2016). Evaluating the FLQ and AFLQ formulae for estimating regional input coefficients: Empirical evidence for the province of Córdoba, Argentina. *Economic Systems Research, 28*, 21–37. https://doi.org/10.1080/09535314.2015.1103703

Flegg, A. T., & Tohmo, T. (2016). Estimating regional input coefficients and multipliers: The use of FLQ is not a gamble. *Regional Studies, 50*, 310–325. https://doi.org/10.1080/00343404.2014.901499

Flegg, A. T., & Tohmo, T. (2019). The regionalization of national input–output tables: A study of South Korean regions. *Papers in Regional Science, 98*, 601–620. https://doi.org/10.1111/pirs.12364

Flegg, A. T., & Webber, C. D. (1997). On the appropriate use of location quotients in generating regional input-output tables: Reply. *Regional Studies, 31*, 795–805. https://doi.org/10.1080/713693401

Flegg, A. T., & Webber, C. D. (2000). Regional size, regional specialization and the FLQ formula. *Regional Studies, 34*, 563–569. https://doi.org/10.1080/00343400050085675

Flegg, A. T., Webber, C. D., & Elliott, M. V. (1995). On the appropriate use of location quotients in generating regional input–output tables. *Regional Studies, 29*, 547–561. https://doi.org/10.1080/00343409512331349173

Fujimoto, T. (2019). Appropriate assumption on cross-hauling national input–output table regionalization. *Spatial Economic Analysis, 14*, 106–128. https://doi.org/10.1080/17421772.2018.1506151

Garcia-Hernandez, J. A., & Brouwer, R. (2021). A multiregional input–output optimization model to assess impacts of water supply disruptions under climate change on the Great Lakes economy. *Economic Systems Research, 33*, 509–535. https://doi.org/10.1080/09535314.2020.1805414

Garhart, R. E., & Giarratani, F. (1987). Nonsurvey input-output estimation techniques: Evidence on the structure of errors. *Journal of Regional Science, 27*(2), 245–253. https://doi.org/10.1111/j.1467-9787.1987.tb01157.x

Haddad, E. A., El-Hattab, F., & Ail, A. A. (2017). *A practitioner's guide for building the interregional input–output system for Morocco, 2013 Research Paper* (17/02; OCP Policy Center Research Papers).

Haddad, E. (2014). Trade and interdependence in Lebanon: An interregional input–output perspective. *Journal of Development and Economic Policies, 16*(1), 5–45.

Hermannsson, K. (2016). Beyond intermediates: The role of consumption and commuting in the construction of local input–output tables. *Spatial Economic Analysis, 11*, 315–339. https://doi.org/10.1080/17421772.2016.1177194

Hewings, G. J. D., & Romanos, M. C. (1981). Simulating less-developed regional economies under conditions of limited information. *Geographical AnalySis, 13*, 373–390.

Jahn, M. (2017). Extending the FLQ formula: A location quotient-based interregional input–output framework. *Regional Studies, 51*, 1518–1529. https://doi.org/10.1080/00343404.2016.1198471

Jahn, M., Flegg, A. T., & Tohmo, T. (2020). Testing and implementing a new approach to estimating interregional output multipliers using input–output data for South Korean regions. *Spatial Economic Analysis, 15*, 165–185. https://doi.org/10.1080/17421772.2020.1720918

Jarne, G., Sanchez-Choliz, J., & Fatas-Villafranca, F. (2007). "S-shaped" curves in economic growth. A theoretical contribution and an application. *Evolutionary and Institutional Economics Review, 3*, 239–259. https://doi.org/10.14441/eier.3.239

Jensen, R. C., & West, G. R. (1980). The effect of relative coefficient size on input–output multipliers. *Environment and Planning A, 12*, 659–670.

Kowalewksi, J. (2015). Regionalization of national input–output tables: Empirical evidence on the use of the FLQ formula. *Regional Studies, 49*, 240–250. https://doi.org/10.1080/00343404.2013.766318

Kwon, H., & Choi, S.-G. (2023). An alternative approach to estimating regional input–output tables: The KFLQ method. *Annals of Regional Science*. https://doi.org/10.1007/s00168-023-01211-8

Lahr, M. L., Ferreira, J. P., & Többen, J. R. (2020). Intraregional trade shares for goods-producing industries: RPC estimates using EU data. *Papers in Regional Science, 99*, 1583–1605. https://doi.org/10.1111/pirs.12541

Lahr, M. L., & Stevens, B. H. (2002). A study of the role of regionalization in the generation of aggregation error in regional input-output models. *Journal of Regional Science, 42*, 477–507. https://doi.org/10.1111/1467-9787.00268

Lamonica, G. R., & Chelli, F. M. (2018). The performance of non-survey techniques for constructing sub-territorial input–output tables. *Papers in Regional Science, 97*, 1169–1202. https://doi.org/10.1111/pirs.12297

Lampiris, G., Karelakis, C., & Loizou, E. (2020). Comparison of non-survey techniques for constructing regional input–output tables. *Annals of Operations Research, 294*, 225–266. https://doi.org/10.1007/s10479-019-03337-5

Mardones, C., & Correa, M. (2025). Methodological proposal to approximate the sectoral impacts of a carbon tax at the regional level—The case of Chile. *Economic Systems Research*, *37*(1), 52–75.

Mardones, C., & Silva, D. (2021). Estimation of regional input coefficients and output multipliers for the regions of Chile. *Papers in Regional Science, 100*, 875–889. https://doi.org/10.1111/pirs.12603

Mardones, C., & Silva, D. (2023). Evaluation of non-survey methods for the construction of regional input–output matrices when there is partial historical information. *Computational Economics, 61*, 1173–1205. https://doi.org/10.1007/s10614-022-10241-x

Martínez-Alpañez, R., Buendía-Azorín, J. D., & Sánchez-de-la-Vega, M. D. (2023). A new improvement proposal to estimate regional input–output structure using the 2D-LQ approach. *Economies, 11*, 20. https://doi.org/10.3390/economies11010020

Mastronardi, L. J., Romero, C. A., & González, S. N. (2022). Interregional analysis using a bi-regional input-output matrix for Argentina. *Investigaciones Regionales – Journal of Regional Research, 53*, 135–156. https://doi.org/10.38191/iirr-jorr.22.014

Mínguez, R., Oosterhaven, J., & Escobedo-Cardeñoso, F. (2009). Cell-corrected RAS method (CRAS) for updating or regionalizing an input–output matrix. *Journal of Regional Science, 49*(2), 329–348. https://doi.org/10.1111/j.1467-9787.2008.00594.x

Pereira-López, X., Carrascal-Incera, A., & Fernández-Fernández, M. (2020). A bidimensional reformulation of location quotients for generating input–output tables. *Spatial Economic Analysis, 15*, 476–493. https://doi.org/10.1080/17421772.2020.1729996

Pereira-López, X., Sánchez-Chóez, N. G., & Fernández-Fernández, M. (2021). Performance of bidimensional location quotients for constructing input–output tables. *Journal of Economic Structures, 10*, 7. https://doi.org/10.1186/s40008-021-00237-5

Ralston, S. N., Hastings, S. E., & Brucker, S. M. (1986). Improving regional I–O models: Evidence against uniform regional purchase coefficients across rows. *The Annals of Regional Science, 20*(1), 65–80. https://doi.org/10.1007/BF01283624

Sánchez-Chóez, N. G., Pereira-López, X., & Fernández-Fernández, M. (2022). A modification of the cross-industry location quotient for projecting sub-territorial input-output tables. *Revista De Economía Mundial*. https://doi.org/10.33776/rem.vi62.5130

Stevens, B. H., Treyz, G. I., & Lahr, M. L. (1989). On the comparative accuracy of RPC estimating techniques. In R. E. Miller, K. R. Polenske, & A. Z. Rose (Eds.), *Frontiers of input–output analysis* (pp. 245–257). Oxford University Press.

Többen, J., & Kronenberg, T. H. (2015). Construction of multi-regional input–output tables using the charm method. *Economic Systems Research, 27*, 487–507. https://doi.org/10.1080/09535314.2015.1091765

Tohmo, T. (2025a). The KFLQ revisited: Estimating regional input–output tables for regions in South Korea. *Annals of Regional Science*, *74*, 13. https://doi.org/10.1007/s00168-024-01345-3

Tohmo, T. (2025b). Estimating SFLQ-based regional input–output tables for South Korean regions. *National Accounting Review*, *7*(1), 125–142. https://doi.org/10.3934/NAR.2025006

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Wiebe, K. S., & Lenzen, M. (2016). To RAS or not to RAS? What is the difference in outcomes in multi-regional input−output models? *Economic Systems Research, 28*, 383–402. https://doi.org/10.1080/09535314.2016.1192528

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