AGORA — Papeles de Filosofía— (2001), 20/1: 175-179 *ISSN 0211-6642*

THE VIEW OF LOGIC AS MODEL:
Comments on Stewart Shapiro's paper

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In his paper Shapiro intends to clarify the relation between a logical system —understood as a triple composed of a language, a calculus and a semantics— and human reasoning. Shapiro's paper is not a paper to discuss the different possibilities in which such a relation could be understood (Cook, R., 2001) but to evaluate one of them: the view of logic as model. His purpose is to appraise whether the view of logic as model provides an adequate account of logic. The result of the assessment, in his own words, "is perhaps disappointing" because what the analysis puts forward is that the view of logic as model does not satisfy the two conditions he establishes (without further argument) as necessary conditions for any adequate account of logic. He takes it that an account of logic would be adequate only if it allowed us:

1st) To fix those cases in which we are in front of a gap between the model and the practice and those in which we are facing a mistake, in other words, to account for the normativity involved in logic; and,
2nd) To conclude that classical logic is the underlying logic of mathematics.

In these comments we will abound on the limitations of the view of logic as model in relation to the first necessary condition and argue that there is a chance that the second condition should not be considered a necessary one according to the view of logic as model.

In Shapiro's paper there is a principal argument that contains two minor ones. The first minor argument aims at evaluating whether the view of logic as model allows us to explain the normativity involved in logic. The second appraises whether the view of logic as model allows us to conclude that classical logic is the underlying logic of classical mathematics.

The two minor arguments go as follows:

1st) ARGUMENT:

He assumes that the sort of account he is interested in is the one proposed by Corcoran, the view of logic as model (Corcoran, 1973); This account is a descriptive account of a normative discipline according to Burguess

 Aceptación: Diciembre 2001

(Burguess, 1992). Shapiro follows Burguess to differentiate between a normative account of a normative discipline (an attempt to say what the norms should be, never mind what the norms are) and a descriptive account of a normative discipline (an attempt to uncover or describe the norms that underlie a given practice).

According to Corcoran's proposal a logical system, <L,D,S>, is a mathematical model of natural language, L, of the process of showing an argument to be valid, D, and, of the ways someone uses to come to know that an argument is not valid, S.

According to Shapiro, when one works with models, it is the nature of the enterprise that there is a gap between the model and the modeled.

Following Corcoran, Shapiro maintains that the modeled phenomenon is *mathematical practice*; in particular, *correct* mathematical practice is what logic aims at modeling. So far, so good. But now, the issue is whether we can use these logical models to correct mathematical practice; In other words, does this account of logic allow us to be normative?

The argument goes on to introduce two hypothetical situations:

- case (1), "the toy example case", an example in which a mathematician claims an inference that violates a well-known logical rule to be correct, denying that she is misapplying that rule and refusing to worry about which would be the underlying logic for the inference she allows; and,
- case (2) Maddy's wild example. Shapiro intends the two examples to show that from our premises an absurd conclusion obtains.

Hence, Shapiro's verdict is that the view of logic as model does not allow us to differentiate between a gap in the model and a mistake. In other words, it does not allow us to turn into a normative mode, the one logicians would feel more comfortable to adopt in situations such as the ones described in case 1 or in case 2.

The issue here is who or what defines what is correct mathematical practice. As far as I can see, the view of logic as model is compatible with the community of mathematicians being the one that establishes what is correct practice, hence when the mathematician disagrees there are no clear grounds for protest. This is precisely what Shapiro's examples put forward. Shapiro remarks that maybe naturalism has gone too far, to mean that remaining descriptive seems quite unacceptable to the logician. Quite clearly Shapiro is pointing at a limitation of the account provided by the view of logic as model. But, does this imply a rejection of the view of logic as model? It seems he does not want to come to this conclusion since he considers "there is job to do".

Hence, maybe something can be added to the view of logic as model so that objectivity is not left out. Since the purpose of mathematical practice is to establish mathematical truths, in a Quinean mood the logician could argue that, apart from what the community of mathematicians may say, if any of the changes considered above (cases 1 and 2) were allowed, not only mathematical practice but also mathematical results and the results of any other discipline that used those mathematical results to formulate her own results would change dramatically. Consequently, allowing for 'logical correctness' to be inter-subjectively re-defined in the way put forward by Maddy's wild example or in the one considered in the toy example seems not to be possible in fact.

If mathematical practice is understood in this sense, (mathematical practice is good as far as it allows us to obtain mathematical truths) then, once logicians have made explicit which is the underlying logic of a given mathematical practice, there is a justification for using that theory of logic to evaluate the processes of attaining new mathematical results. (For the sake of the argument we are leaving aside Penelope Maddy's critique to Quine).

In this way, we achieve certain objectivity at a given level. But, don't we get a regress? (The regress Wright describes in his 1986 paper.) Isn't it the case that in the frame depicted above the question that immediately prompts is about the logic mathematicians use to evaluate whether mathematical results have been useful in other areas of discourse?

Hence, the view of logic as model does not allow us to account for the normativity of logic. Maybe, the reason why it does not is the one pointed out by Boghossian (Boghossian, 2000): If we accept the ban on the use of a logical principle in reconstructing our warrant for that principle, we would have to conclude that there can be no such reconstruction.

Another topic that Shapiro does not address in his paper but which would be very interesting to analyze in the light of the view of logic as model is that of the apriority of logic. In his 2000 paper Shapiro maintains that some sort of apriority is involved in a Quinean account of logic, the issue is then how this fits in with the view of logic as model, a view which, in principle, seems to rule out any aprioristic account of logic.

2nd ARGUMENT

Now, let us consider the other issue, the second minor argument. Once again the view of logic as model is assessed; this time the intended conclusion being that revisionism is unacceptable. In other words, the view of logic-as-model should allow for a good argument to maintain that classical logic is *the* underlying logic for classical mathematics. But, Shapiro's undesired conclusion is that the view of logic as model does not allow us to rule out either the revisionism proposed by the relevantist, or the revisionism set forth by the intuitionist. In other words, Shapiro's verdict is that the view of logic as model does not allow us to formulate a conclusive argument to establish that classical logic is the underlying logic for classical mathematics.

A possible consideration here is whether what Shapiro intends the view of logic as model to establish (a good argument for classical logic) is not going beyond what has traditionally been asked to a model. A model is a fruitful way of representing a phenomenon; the notion of model (therefore, the view of logic as model) as such does not imply that the model corresponds to something real. In sciences in which models are part of the methodology, it has sometimes been the case that there were two alternative models that, at a given historical interval, were underdetermined by data. Maybe this is the case with classical and intuitionistic logic: we count on alternative models of mathematical practice.

In order to see this we need to go into the notion of mathematical practice. The notion of mathematical practice not only implies assuming a given set of correct rules of inference (as Shapiro seems to be doing). Clearly it also involves ontological, epistemological and semantic assumptions. Hence, providing an account of classical mathematical practice should imply giving an explanation of all those aspects involved in it.

If 'mathematical practice' is understood in that sense, then intuitionistic logic can be seen as an alternative model since: i) though there are theorems of classical logic that are not theorems of intuitionistic logic, we count on intuitionistic translations of those theorems of classical logic that are theorems of intuitionistic logic; ii) Bishop (Bishop, 1967) has shown that that part of analysis that is necessary for classical mathematics can be accounted for in constructive terms (that implies in intuitionistic terms); iii) intuitionism provides a better account of the epistemology of mathematics than the classical view; iv) constructive methods seem to agree very well to the way the mathematician works.

If we accept that classical logic and intuitionistic logic are alternative models for classical mathematical practice, then some interesting questions need further attention:

1st) Whether, as Resnik claims (Resnik, 1985), this implies that logic is not objective (he adopts a Benacerrafian line of argument, as Shapiro points out);

2nd) Which would be the nature of the data that could possible show a given logic incorrect (could they be empirical data? Is logic empirically defeasible?);

3rd) Whether the fact that we count on alternative models has to do with the fact that they pay more or less attention to a given sort of reasoning though the same set of mathematical truths obtains. From this point of view, maybe we could say that classical logic would account for a way of reasoning that proves most adequate to reason in the mathematics of infinite, while intuitionistic logic proves most adequate to develop, for instance, theoretical formalisms for computer science.

Shapiro's purpose in this paper was to analyze whether the view of logic as model allowed us to establish good arguments for the normativity of logic and to infer that classical logic is *the* underlying logic of mathematics. What his analysis of both points puts forward is that it would be very interesting to see what global picture of logic obtains from an analysis of the topics addressed above more faithful to the notion of model.

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