ON THE NUMBER OF LICENSES WITH SIGNALLING

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We analyse a two-period licensing game in which a non-producing upstream patent holder licenses an innovation that lasts for two periods to either one or two downstream users. Licensing is made through a payment based on a two-part tariff, namely a fixed fee plus a royalty per output unit. Regarding the innovation value when commercialized by each user (high or low), we compare a symmetric information context where such value is publicly known with a situation in which users have private information about the value, but with their period-1 output signalling that value. We find that the patent holder is more likely to prefer to grant two licenses under signalling than under symmetric information, which highlights the benefits of resorting to market competition between users to reduce the amount of informational rents.

1 Introduction

Consider a non-producing upstream patent holder owning an innovation that lasts for two periods, after which it becomes obsolete and no longer has economic value. Since the patent holder is unable to commercialize the innovation, he is obliged to sell it to just one or several downstream firms capable of making the innovation profitable by developing a new product. In such a context, the licensing literature has shown that, if users have private information about the economic value of the innovation, the patent holder may sell more licenses than if such value is publicly known (Schmitz, 2002, 2007). What we explore in this paper is whether a similar result would emerge in a signalling scenario, in which each innovation user

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publicly signals, through its production level, the value it obtains from the innovation.

We built a two-period licensing game in which each potential user of the innovation has private information about the marginal cost of production when the innovation is developed (high- or low-cost of production), and where this information may be disclosed through the output produced at first period. We consider two options for the patent holder, each with an issuing cost per license: either to grant just one license, in which case the downstream market becomes a monopoly, or to grant several (two) licenses, in which case the users engage in Cournot competition. We also assume that when a duopoly emerges, the two users produce a homogeneous good. Finally, the payment in exchange for each license takes the form of a two-part tariff: a fixed fee plus a royalty rate per output unit.

Under these circumstances, and if information about the user's production cost when marketing the innovation is incomplete but symmetric, the existence of a single user or several users in the product market depends on the trade-off between two well-known effects: the sampling effect and the rent dissipation effect. Issuing several licenses makes it more likely that there will be a low-cost user, with the value of the innovation increasing in line with the number of licenses. However, more licenses also reduce the patent holder's expected payoff, by increasing competition between users. The application of a royalty rate in the licensing contract thus allows the patent holder to countervail the dissipation effect of competition so that it will always prefer to grant two licenses rather than just one if the cost of issuing each license is zero or sufficiently low.

We also show that, under asymmetric information, a signalling effect also arises (in the separating equilibrium), whereby the users' production level may be distorted for opportunistic reasons. In fact, to minimize the payment to the patent holder in period 2, a single user of the innovation has an incentive to signal a low value of the innovation (a high production cost). Since this incentive applies equally when the user is inefficient or efficient, in the former case he wants to reduce period-1 output to a point where he has no interest to move when efficient. This output distortion in period 1, in turn, reduces the patent holder's expected revenue with respect to the symmetric information scenario.

On the other hand, when the innovation is commercialized through two licenses, competition between users also leads each to signal a low production cost to reduce the rival's market share. Although the 'vertical' incentive to be perceived as a high-cost user by the patent holder is still dominant, the above effect countervails this and thus mitigates output distortion in period 1. As under symmetric information, competition benefits the patent holder because of the sampling effect, whereas the rent dissipation effect can be offset by charging a per-unit royalty in the two-part tariff. However, competition has an additional effect in a signalling context, which is that competition is much more useful under signalling than under symmetric information, since the royalty (of the two-part tariff) alone does not countervail the additional negative repercussions of signalling on patent holder income.

In particular, we find that, in the perfect Bayesian separating equilibrium of least cost, the patent holder strictly prefers to issue two licenses at licensing cost values for which a single license would be optimally granted under symmetric information. Thus, resorting to competition in awarding the innovation is more likely under signalling than under symmetric information. The intuition of this finding is that the conjunction of royalties and downstream competition allows the upstream patent holder to reduce the informational rents granted to efficient users in period 1 of the contract.

The formal analysis of licensing was initiated by Arrow (1962), who argued that a competitive industry compared to a monopoly provides more incentives to innovate. Then, the literature has extensively addressed licensing under symmetric and asymmetric information regarding the value of the innovation. The focus has been placed on the relative merits of various contractual arrangements (Beggs, 1992; Kamien, 1992; Antelo, 2013), as the patent holder is either one of the incumbent firms in the industry or an outsider innovator. However, the effects of adverse selection and signalling on the number of awarded licenses have tended to be overlooked.

An exception is Schmitz (2002), who built an adverse selection model[1] in which the user is better informed than the licensor about the innovation value and examined the consequences of this informational regime on the number of licenses granted. He showed that, in some states of the world, an efficient mechanism requires the granting of two licenses rather than just one. Our results complement this finding in that we consider a signalling framework where the users of the innovation, as a means to modify the license renewal payment in period 2, can signal the innovation value through period-1 output. We find that, compared to the incomplete but symmetric information context, the patent holder is more likely to prefer to issue two licenses under signalling than under symmetric information, given that the use of competition countervails the prejudicial effects of users' opportunistic behaviour on patent holder's income.

Finally, it is worth noting that, in Schmitz (2002)'s framework, more licenses need not imply more downstream competition because there is always the possibility that an user will fail to develop the innovation into a new product. In the absence of that possibility, the patent holder has no incentive but to grant more than one license. Our approach differs from that of Schmitz (2002) in four aspects. Firstly, we incorporate signalling; secondly, we allow for two-part licensing tariffs; thirdly, we assume that each

[1]Schmitz (2007) undertook a similar analysis for a moral hazard setting.

user is a successful producer; and finally, the concession of more licenses leads unequivocally to more competition in the product market.

The rest of the paper is structured as follows. Section 2 describes the model; Sections 3 and 4 present the main results in a symmetric and an asymmetric information scenario, respectively; and Section 5 concludes. Proofs are relegated to the Appendix.

2 The Model

We consider the case of an upstream patent holder owning an innovation that will become worthless after two production periods, $t=1,2$. Lacking production capacity, the patent holder is obliged to license the innovation to users capable of using it to manufacture a new product. In each period t the market demand for such a product is publicly known and is given by

$$p_t(q_t)=\begin{cases} 1-q_t, & \text{if } q_t<1, \\ 0, & \text{if } q_t\geq 1, \end{cases} \tag{1}$$

where q_t denotes total output produced by the user (or users) of the innovation in period t depending on the number of licenses granted by the patent holder (one or two) in period 1. If two licenses have been issued, users compete *à la* Cournot in the product market and the final good is homogeneous.

At the beginning of period 1, and depending on whether we consider a context with incomplete but symmetric information or a scenario with asymmetric information, the user's marginal cost of production (or the innovation's economic value) can be publicly known to all parties or privately known to each user and unknown to the patent holder (and perhaps to the rival user). Those who do not know the innovation value have only a prior assessment of this value. In particular, the marginal cost of production for each user is represented by an independent random variable $\tilde{c}$ distributed as

$$\tilde{c}=\begin{cases} 0, & with\ probability\ \mu, \\ c, & with\ probability\ 1-\mu, \end{cases} \tag{2}$$

where $0<\mu<1$.[2] Throughout the paper we assume that $0<c<1/2$, so that when two licenses are granted, the existence of an equilibrium with positive production by an user is ensured irrespective of the true cost type and of any party's beliefs about this cost. We also assume that all players intervening in the licensing game are risk-neutral and that there is no discount factor on profits obtained by each user.

[2]The same results would follow if we assumed uncertainty to lie rather in the size of market demand.

Two-part contracts to sell the innovation adopt the form of a fixed fee plus a per-unit royalty. The royalty is assumed to be non-negative. We also assume that, for each license issued, the patent holder incurs in a fixed cost (a showing cost), $S > 0$. Given that we are considering two-part licensing contracts, allowing for a positive showing cost per license is crucial to explore how, in a licensing-signalling game, downstream market competition is a useful device to reduce the informational rents obtained by those efficient users of the innovation.[3] Finally, there is no renegotiation of contracts.

3 Symmetric Information

As a benchmark, we start by examining patent holder behaviour regarding the number of licenses to be granted when information about the value of the innovation for each user is uncertain but symmetric. In this case, the timing of the licensing game is as follows. At the beginning of period 1 neither users nor the patent holder know the cost realization from using the innovation. The patent holder offers a two-part licensing contract to one or several users as a function of its or their types. This contract lasts for two periods.[4] Nature then selects the user type (efficient or inefficient) and this type is publicly observed. Finally, each user produces the corresponding output level in period 1 and period 2.

In the first period the patent holder must decide whether to sell either one license lasting two periods or two licenses lasting two periods each. The contracts are contingent on the innovation value which will be known to both parties just once the contract is signed and the patented innovation is transferred.

In this symmetric information context, the shape of the optimal licensing contracts for the patent holder is recorded in the following proposition.

Proposition 1: The optimal contract to license the innovation under symmetric information is as follows:

(i) If a single license is granted in period 1, then $\{f_M, r_M\} = \left\{\frac{(1-\tilde{c})^2}{4}, 0\right\}$, for $\tilde{c} = \{0, c\}$, where subscript M stands for a monopolistic user, f denotes the fixed fee and r the per-unit royalty rate. The same contract leads to renewal of the license in period 2.

(ii) If two licenses are granted in period 1, the contract offered to each user i ($i = A, B$) is

[3]This result also holds when licensing contracts are based exclusively on fixed fees, in which case the use of several licenses is more likely under signalling than under symmetric information even if $S = 0$, as is shown in Antelo and Sampayo (2014).

[4]Alternatively, the parties could sign two contracts covering one period each, one for the first period when information is incomplete, and another for the second period, once they know the true value of the innovation. It is easy to see that both alternatives are equivalent.

$$\{f_{iD}, r_{iD}\} = \begin{cases} \left\{0, \dfrac{1-2c}{2}\right\}, & \text{if } \tilde{c}_i = c \text{ and } \tilde{c}_j = 0, \\ \left\{\dfrac{(1-c)^2}{16}, \dfrac{1-c}{4}\right\}, & \text{if } \tilde{c}_i = c \text{ and } \tilde{c}_j = c, \\ \left\{\dfrac{1}{16}, \dfrac{1}{4}\right\}, & \text{if } \tilde{c}_i = 0 \text{ and } \tilde{c}_j = 0, \\ \left\{\dfrac{1}{4}, 0\right\}, & \text{if } \tilde{c}_i = 0 \text{ and } \tilde{c}_j = c, \end{cases} \quad (3)$$

where subscript D denotes a duopoly and $i, j = A, B; i \neq j$. The same contract holds for period 2.

Proof: (i) The profit-maximizing production of a single user per period is

$$q_M = \frac{1 - r - \tilde{c}}{2}, \quad (4)$$

where $\tilde{c} = \{0, c\}$. In turn, the patent holder solves the problem

$$\max_{\{f_M \geq 0, r_M \geq 0\}} \; 2(f_M + r_M q_M) = \max_{r_M \geq 0} \; 2\{(1 - r_M - \tilde{c} - q_M)q_M + r_M q_M\}, \quad (5)$$

where q_M is given by (4). The solution to problem (5) is $r_M = 0$ and, consequently, $f_M = \frac{(1-\tilde{c})^2}{4}$.

(ii) If the patent holder decides to sell two licenses in period 1, the resulting market is a duopoly with the two users engaging in Cournot competition. In each period, the profit-maximizing output for each user i ($i = A$, B) is

$$q_{iD} = \frac{1 - 2r_i - 2\tilde{c}_i + r_j + \tilde{c}_j}{3}. \quad (6)$$

Hence, the fixed and variable payments to be paid by each user are obtained by solving the problem

$$\begin{aligned} &\max_{\{f_{iD} \geq 0, r_{iD} \geq 0\}_{i,j=A,B}} \sum_{i,j=A,B} 2(f_{iD} + r_{iD} q_{iD}) \\ &= \max_{\{r_{iD}\}_{i,j=A,B} \geq 0} \sum_{i,j=A,B} 2\left[\left(1 - r_{iD} - \tilde{c}_i - q_{iD} - q_{jD}\right) q_{iD} + r_{iD} q_{iD}\right], \end{aligned} \quad (7)$$

where q_{iD} is given by (6) and $i, j = A, B; i \neq j$. The solution to problem (7) is given by (3). It should be noted that, when both users of the innovation are of the same type, the solution to problem (7) is non-unique and is given by

$\{f_{iD}, r_{iD}\} = \left\{\frac{(1-2r_{iD}-\tilde{c}+r_{jD})^2}{9}, \frac{1-\tilde{c}}{2} - r_{jD}\right\}$; in (3) we report the symmetric solution, $r_{iD}=r_{jD}$. ■

Contracts stated in Proposition 1 may be understood as a unique payment to the patent holder (in which case the amount paid by each user is twice the amount described in each contract) or as the payment in period 1 and one more payment (of the same amount) in period 2 to retain the right to use the innovation in such period.

Our result when the patent holder issues a single license parallels the standard finding in the literature: A pure fixed fee is preferred rather than a two-part tariff so as not to distort the user's production. However, when two licenses are granted, the use of a two-part tariff allows the patent holder to replicate the state contingent profits obtained under a fixed-fee contract to a single user, as can be easily verified from Proposition 1. This allows the patent holder to overcome the rent dissipation effect arising from market competition. In fact, there is a trade-off in the licensing process between rent dissipation and sampling effects. Under the former, the concession of several licenses reduces each user's profits and hence the patent holder's expected payoff, whereas under the latter, there is an increased likelihood of low-cost users as more licenses are granted and, thus, profits for each user and the patent holder are higher under our assumed payment scheme. By obtaining royalties in addition to a fixed fee, the patent holder can thus enjoy the sampling effect while countervailing rent dissipation.

This suggests that if there were no cost for issuing licenses, $S=0$, the patent holder would always prefer to sell two licenses of the innovation rather than just one. In order to highlight the role of competition in ameliorating the costs of asymmetric information and signalling (in terms of revenue losses), we assumed the existence of a positive cost per license, $S > 0$. This will allow us to see how more competition is able to reduce informational rents so that the patent holder will be willing to accept the higher costs associated with two licenses. To this end, we first determine the cut-off value for the issuing cost that leads the patent holder to be indifferent between selling one or two licenses under symmetric information.

Proposition 2: Let $S^* = \frac{1}{4}\mu(1-\mu)c(2-c)$ be the cost of issuing each license that leaves the patent holder indifferent between selling one or two licenses under symmetric information. Then, it will choose to grant two licenses if $S < S^*$ but a single license if $S > S^*$.

Proof: Substituting the optimal contract $\{f_M(\tilde{c}), r_M\}$ from Proposition 1(i) into the objective function in (5), the patent holder's payoff from a single license is

$$E\pi_{MM}^{\mathrm{PH}} = \frac{1}{2}\left[\mu + (1-\mu)(1-c)^2\right] - S, \tag{8}$$

where superscript PH stands for the patent holder and subscript MM indicates that a monopoly exists in both production periods. If, instead, the patent holder grants two licenses lasting two periods each, the optimal contract is given by (3), once the objective function (7), that defines the state contingent patent holder's profits, is taken into account. The patent holder's two-period payoff resulting from selling two licenses is thus

$$E\pi_{DD}^{\mathrm{PH}} = \frac{1}{2}\left[\mu(2-\mu) + (1-\mu)^2(1-c)^2\right] - 2S, \tag{9}$$

where subscript DD denotes that a duopoly exists in each period. The proposition follows immediately after comparing (8) and (9). ■

4 Asymmetric Information and Signalling

Things may change radically regarding the number of licenses granted if the patent holder does not know the users' type at the same time as the users themselves. In a perfect Bayesian separating equilibrium, a user's type will dictate different outputs in period 1 and, based on this, the other parties will update their beliefs at the beginning of period 2 in such a way as to be able to distinguish between users.

Let us assume that each user has private information about its own production cost, which is known with certainty from the outset.[5] Prior to period-1 production, the patent holder (and the rival user, if any) only knows that this production cost is a random variable, which is distributed as stated in (2). The patent holder can also observe at the end of period 1 the user's production in that period and use that information (and Bayes' rule) to revise its beliefs about the production cost. The patent holder offers a one-period contract to each user at the beginning of period 1 in exchange for a payment based on a two-part tariff. After period-1 production of each user is observed by third parties, these infer the production cost and the patent holder offers another contract (at the start of period 2) to the user or users in exchange for a different two-part tariff. Note that, unlike under symmetric information, it is now suboptimal for the patent holder to offer a two-period contract at the start of period 1, since that would preclude using first-period production as a signal to identify the user's type.

Because the patent holder cannot distinguish between user types before observing period-1 production, payment in the licensing agreement for the first period cannot be a function of the type. After observing period-1 production,

[5]This assumption is intended to reflect a common situation: the patent holder remains largely unaware of the innovation's downstream development; the user, with more experience in the field, is better positioned to recognize the innovation's true value.

however, the patent holder can craft a period-2 licensing agreement whose payment arrangements reflect the true production costs. This setup leads to a signalling game in which the user(s) have incentives to behave opportunistically.

4.1 One License

We prove, in Proposition 3, that a single user of the innovation has a vertical incentive to be perceived as inefficient in period 1 by the patent holder in order to reduce the payment for license renewal in period 2; hence, both an efficient and inefficient user have an incentive to signal a high production cost.

Proposition 3: If the patent holder grants one license, then, in the perfect Bayesian separating equilibrium of least cost, the period-1 output of the inefficient user is lower than it would be if symmetric information signalled a low-value innovation.

Proof: See Appendix. ■

To signal a low value of the innovation to the patent holder, the inefficient user needs to produce less output in period 1 than he would produce under symmetric information, something that he would not find profitable if he were efficient.

In the following proposition we characterize the optimal period-1 contract offered by the patent holder to a single user of the innovation. Clearly, the period-2 contract is the same as the symmetric information contract stated in Proposition 1(i).

Proposition 4: If a single license is sold, the optimal contract offered by the patent holder for period 1 is given by

$$\{f_{1M}, r_{1M}\} = \begin{cases} \left\{ \dfrac{\left(1-\sqrt{c(2-c)}\right)\left(1-c-\frac{1}{2}\left(1-\sqrt{c(2-c)}\right)\right)}{2}, 0 \right\}, & \text{if } \mu \leq \min\ \{\mu_M(c), \mu^*(c)\}, \\ \left\{ \dfrac{\left(c-1+\mu\sqrt{c(2-c)}\right)\left(3c-1-(2-\mu)\sqrt{c(2-c)}\right)}{4}, c-(1-\mu)\sqrt{c(2-c)} \right\}, & \text{if } \mu_M(c) \leq \mu \leq \mu^{**}(c), \\ \left\{ \dfrac{1}{4}, 0 \right\}, & \text{if } \mu \geq \max\ \{\mu^*(c), \mu^{**}(c)\}. \end{cases} \tag{10}$$

Proof: See the Appendix. ■

In Proposition 4, the region defined by condition $\mu \leq \min \{\mu_M(c), \mu^*(c)\}$ is the (c, μ)-region where the patent holder sets fixed fees such that both types of user will produce a positive amount in the first period.[6] Likewise, in the region defined by condition $\mu_M(c) \leq \mu \leq \mu^{**}(c)$ both types of user also produce a positive amount in the first period but royalties are included in the contract. Finally, in the region defined by condition $\mu \geq \max \{\mu^*(c), \mu^{**}(c)\}$ there are no royalties in the contract and only the efficient user produces a positive amount in the first period. Figure 2 in the Appendix shows all these regions. The blank area represents the region where it is optimal to set fees that allow the user to produce irrespective of efficiency and where the royalty rate can be either positive or zero depending on the (c, μ)-values. Subarea A's royalty rate is zero and in subarea B is positive and equals the value given in the second line of equation (10). In the shaded area the user will only produce during period 1 if he is efficient.

In summary, in the separating equilibrium of the licensing game between the patent holder and a single user there are two ways in which a bad user of the innovation can signal low efficiency in period 1. One is with no production and the other is with positive production but lower than under symmetric information (in which case the optimal contract can have either a zero or positive royalty rate depending on the parameter values). In the first case, where production is zero, we have a corner equilibrium, in which the patent holder is able to extract all period-1 profits from the efficient user—without granting informational rents but at the cost of having zero profits if the user becomes inefficient. In the second case, we have an interior equilibrium, in which the efficient user of the innovation enjoys informational rents in the first period.

4.2 Two Licenses

We continue to assume that users have private information about their own costs, known with certainty from the outset, when they put the innovation into practice. The timing of the game is the same as for a single license except that now there is Cournot competition in the product market. Both the patent holder and competing user j ($j = A, B; j \neq i$) can observe user i's production level in period 1 and use that information (and Bayes' rule) to update their beliefs about user i's production costs. Two one-period contracts are now simultaneously offered by the patent holder at the beginning of period 1 in exchange for a payment based on a two-part tariff. Proposition 5 states the existence of a separating equilibrium of this game.

Proposition 5: If the patent holder issues two licenses of the innovation, there is a least cost separating equilibrium whereby the period-1 output of each inefficient user is lower and that of each efficient user is higher than under symmetric information.

[6]As is shown below, in period 2 both types of user always produce a positive quantity in a separating equilibrium.

Proof: See Appendix. ■

Although each user now has opposing incentives to be perceived as efficient and inefficient by the rival and the patent holder, respectively, the latter dominates because is a first-order effect. Hence, as in the case of a single license, each inefficient user reduces his output in period 1 to signal a low-value innovation. In turn, the efficient user plays according to his best response and sets a quantity higher than the symmetric information quantity, not because he is acting strategically but as a consequence of the fact that the inefficient user chooses a lower quantity than his symmetric information quantity.

Proposition 6 details the patent holder's optimal contract for period 1. The contract for period 2 is that corresponding to symmetric information as given in proposition 1(ii).

Proposition 6: When two licenses of the innovation are issued, the period-1 optimal contract for the patent holder is given by

$$\{f_{1D}, r_{1D}\} = \begin{cases} \left\{ \dfrac{(2-(2+\mu)\Lambda)\left(1-c-\mu\left(\frac{1}{3}+\frac{1}{6}(1-\mu)\Psi\right)-(2-\mu)\left(\frac{1}{3}-\frac{1}{6}(2+\mu)\Lambda\right)\right)}{6}, 0 \right\}, & \text{if } \mu \leq \mu''_D(c), \\ \left\{ \begin{array}{l} \dfrac{3c+2\mu(1-\mu)\Psi+2\mu(2+\mu)\Lambda-3}{144} \times \\ \left(6c+3\mu(1+3c)+2\mu\left(4\mu-3\mu^2-1\right)\Psi+2\mu\left(2\mu+3\mu^2-8\right)\Lambda-6\right), \\ \dfrac{1+3c+2(1-\mu)\Psi-2\left(\mu-\mu^2-2\right)\Lambda}{4} \end{array} \right\}, & \text{if } \mu''_D(c) \leq \mu \leq \mu_D(c), \\ \left\{ \dfrac{2+\mu-(1-\mu^2)\Psi}{4(1+\mu)^2}, \dfrac{2\mu-1+(1-\mu^2)\Psi}{2(1+\mu)} \right\}, & \text{if } \mu \geq \mu_D(c), \end{cases} \tag{11}$$

where $\Lambda = \sqrt{c-\mu c+\mu c^2}$ and $\Psi = \sqrt{2c-c^2-\mu c}$.

Proof: See Appendix. ■

Figure 3 in the Appendix shows the (c, μ)-region where the royalty paid by each user is either zero or positive The locus $\mu''_D(c)$—also characterized in the Appendix—represents the values of parameters c and μ where, since the patent holder's revenue is the same, it is indifferent between a positive and a zero royalty rate. Finally, the locus $\mu_D(c)$ reflects the values of parameters c and μ for which the patent holder is indifferent between allowing the inefficient user to produce or not.

We finally show how granting two licenses to exploit the innovation and using royalties in the licensing contracts may allow the patent holder to countervail opportunistic behaviour by users in order to be considered as inefficient users, with royalties serving to reduce the profit dissipation effect. Indeed, from Proposition 2 we know that, for a value of the licensing cost S as $S^* = \frac{1}{4}\mu(1-\mu)c(2-c)$, the patent holder is indifferent between selling one or two licenses when there is symmetric information. However, Proposition 7 below, as compared with Proposition 2, states that, for $S = S^*$, the patent holder would prefer to issue two licenses under asymmetric information.

Proposition 7: For the same licensing cost that leaves the patent holder indifferent between selling one or two licenses under symmetric information, it will choose to sell two licenses under asymmetric information.

Proof: See Appendix. ■

To understand the relevance of this result, note that under symmetric information it is the availability of a royalty—as a device to increase licensing expected revenue—which makes the use of a single license unappealing for commercializing the innovation when there is no cost per license issued. In fact, royalties and competition become complementary devices to the patent holder since the appropriate use of royalties will avoid any rent dissipation coming from a stronger competition between users. Selling a single license becomes attractive for the patent holder only when the cost of issuing each license, as borne by the patent holder, is sufficiently high. In fact, as indicated by Proposition 2, when the cost of issuing each license is $S > S^*$, then it would be better for the patent holder to sell only one license, for all admissible values of parameters μ and c.

However, things change in a signalling context. In this case, royalties cannot countervail the additional dissipation of rents coming from informational asymmetry and output distortion resulting from signalling. Therefore, competition—i.e. the concession of two licenses—is more useful to the patent holder than it would be under symmetric information, not only because of the sampling effect but also because it allows the patent holder to extract part of (or sometimes even all) the users' informational rents in period 1. This is illustrated by the fact that, under signalling, the patent holder is willing to assume a higher cost per license issued, i.e. a higher cost of competition, precisely because his revenue is large enough to compensate this cost.

To check the robustness of our result, assume that the cost per issued license amounts to $\hat{S} = \mu^2 \frac{1}{4} + \mu(1-\mu) + (1-\mu)^2 \frac{(1-c)^2}{4}$, where we have $\hat{S} > S^*$. This cost would make the patent holder's business unprofitable if two licenses were granted under symmetric information as indicated in (9). As for asymmetric information, Fig. 1 shows that patent holder profits would be even higher for two licenses for some range of the parameter values.

The above analysis reveals that, overall, it is more likely that two licenses would be granted in equilibrium under asymmetric information and signalling than in equilibrium under symmetric information.

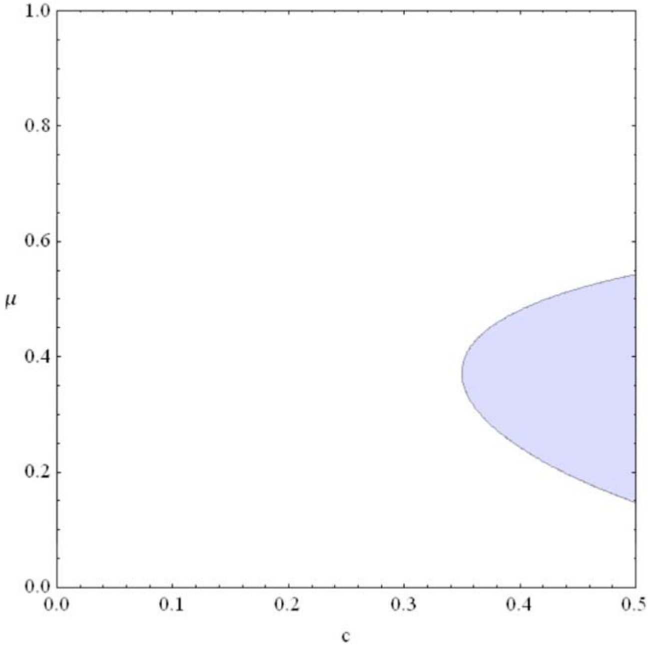

FIG. 1. The Number of Licenses Under Asymmetric Information for $S=\hat{S}$
The shaded area represents the parameters range where patent holder revenue from granting two licenses is higher than from granting only one, provided the cost of issuing two licenses is equal to its revenue under symmetric information. [Colour figure can be viewed at wileyonlinelibrary.com]

5 CONCLUSIONS

In a licensing framework in which each user of the innovation has private information about the value of the innovation and signals it through the output produced, we have shown that issuing two licenses may alleviate the effects of opportunistic behaviour by users. We computed a perfect Bayesian separating equilibrium where, for a certain range of parameters defining the level of inefficiency of a bad user and the probability of a user being efficient, the patent holder sells only one license under symmetric information but two under signalling. In this differential case, competition serves to reduce the informational rents that the patent holder was obliged to grant to efficient users in order to induce them to reveal their private information. This reduction in the amount of informational rents, added to the sampling effect of granting several licenses, outweighs the rent dissipation effect of issuing two licenses.

Appendix[7]

Proof of Proposition 3

The patent holder chooses period-2 fees based on his beliefs about the licensee costs by solving the following problem

$$\pi_{2M}^{\mathrm{PH}}\left(\tilde{c}^{b}\right)=\max_{r_{2M}\geq 0}\left[f_{2M}\left(\tilde{c}^{b}\right)+r_{2M}q_{M}\right]=\max_{r_{2M}\geq 0}\left[\left(1-r_{2M}-\tilde{c}^{b}-q_{2M}\right)q_{2M}+r_{2M}q_{2M}\right], \tag{A1}$$

where $\tilde{c}^{b}\in\{0,c\}$ denotes the patent holder beliefs about the user's production costs, $q_{2M}=\frac{1-r_{2M}-\tilde{c}^{b}}{2}$ is the quantity that the patent holder believes the user will produce as a monopolist in period 2, superscript PH denotes patent holder, and subscript M stands for monopoly. The solution to problem (A1) matches that of symmetric information but depends on patent holder beliefs, $r_{2M}\left(\tilde{c}^{b}\right)=0$ and $f_{2M}\left(\tilde{c}^{b}\right)=\frac{\left(1-\tilde{c}^{b}\right)^{2}}{4}$. Therefore the patent holder profits are $\pi_{2M}^{PH}\left(\tilde{c}^{b}\right)=f_{2M}\left(\tilde{c}^{b}\right)$.

Let $v_{2}^{L}\left(\tilde{c}|\tilde{c}^{b}\right)$ denote the monopolist's profit in period 2, where $\tilde{c}\in\{0,c\}$ and $\tilde{c}^{b}\in\{0,c\}$ are, respectively, the user's production costs and patent holder beliefs about them. The per-period profit-maximizing quantity for the monopolist is $q_{2M}=\frac{1-r_{2M}\left(\tilde{c}^{b}\right)-\tilde{c}}{2}$ and therefore

$$v_{2}^{L}\left(\tilde{c}|\tilde{c}^{b}\right)=v_{2}^{L}(\tilde{c})=\frac{(1-\tilde{c})^{2}}{4}. \tag{A2}$$

Let $\pi_{2}^{L}\left(\tilde{c}|\tilde{c}^{b}\right)=v_{2}^{L}\left(\tilde{c}|\tilde{c}^{b}\right)-f_{2M}\left(\tilde{c}^{b}\right)$ denote the period-2 user's profits net of fixed licensing fees, where $v_{2}^{L}\left(\tilde{c}|\tilde{c}^{b}\right)$ is given by (A2) and, as we saw before, $f_{2M}\left(\tilde{c}^{b}\right)$ is also given by (A1) when $\tilde{c}=\tilde{c}^{b}$. These profits are given by

$$\pi_{2}^{L}\left(\tilde{c}|\tilde{c}^{b}\right)=\begin{cases}\dfrac{c(2-c)}{4}, & \text{if } \tilde{c}=0 \text{ and } \tilde{c}^{b}=c,\\ 0, & \text{if } \tilde{c}=0 \text{ and } \tilde{c}^{b}=0,\\ 0, & \text{if } \tilde{c}=c \text{ and } \tilde{c}^{b}=c,\\ -\dfrac{c(2-c)}{4}, & \text{if } \tilde{c}=c \text{ and } \tilde{c}^{b}=0.\end{cases} \tag{A3}$$

From (A3) it is clear that $\pi_{2}^{L}(\tilde{c}|c)\geq\pi_{2}^{L}(\tilde{c}|0)$ for all $\tilde{c}\in\{0,c\}$, which means that both types of user would prefer to be perceived as inefficient.

In a separating equilibrium, beliefs are correct and the efficient user is charged the highest possible fee in period 2 (which implies a fixed fee equal to the efficient user's profits and a zero royalty rate). A costly signal of her low cost is therefore of no use to her and so she produces the profit-maximizing quantity in each period, $q_{1M}(0)=\frac{1-r_{1M}}{2}$, where r_{1M} denotes the royalty charged by the patent holder in period 1—which does not depend on the user's type because it cannot be observed—and $q_{2M}(0)=\frac{1}{2}$; thus

[7] To facilitate the explanation, we will refer to efficient and inefficient licensees as 'she' and 'he' respectively.

$v_2^L(0|0)=\frac{1}{4}$ and $\pi_2^L(0|0)=0$. The inefficient user can pay a lower fee if he uses his period-1 output, $q_{1M_s}(c)$, to signal low efficiency. This user would then be charged a fee that allows him to also earn zero net profits in period 2, $\pi_2^L(c|c)=0$ (where the variable fee is zero). For there to be an equilibrium, the incentive compatibility constraint of each user must be satisfied. Hence, the inefficient user's profits computed over two periods under signalling must be no less than the maximum profits obtained from sending no signal and be treated as an efficient producer in period 2, i.e.[8]

$$(1-r_{1M}-c-q_{1M_s}(c))q_{1M_s}(c)+\pi_2^L(c|c) \geq \frac{(1-r_{1M}-c)^2}{4}-\frac{c(2-c)}{4}. \tag{A4}$$

For the efficient user, profits from sending no signal must be no less than sending a signal and paying the fee that is charged to an inefficient user in period 2

$$\frac{(1-r_{1M})^2}{4}+\pi_2^L(0|0) \geq (1-r_{1M}-q_{1M_s}(c))q_{1M_s}(c)+\frac{c(2-c)}{4}. \tag{A5}$$

To write inequalities (A4) and (A5) we are assuming that patent holder revised beliefs that the user is efficient for all possible quantities in period 1 are

$$\hat{\mu}_{PH}=\begin{cases} 0, & \text{if } q_1 \leq q_{1M_s}(c), \\ 1, & \text{otherwise.} \end{cases} \tag{A6}$$

Finally, the value of $q_{1M_s}(c)$ that maximizes the inefficient user's profits and that verifies constraints (A4) and (A5)—the former with strict inequality—is $q_{1M_s}(c)=\frac{1-r_{1M}}{2}-\frac{\sqrt{c(2-c)}}{2}$.

With no signalling, an inefficient user in period 1 would produce the profit-maximizing output, namely $q_{1M}(c)=\frac{1-r_{1M}-c}{2}$. Hence, the efficient user produces $q_{1M}(0)=\frac{1-r_{1M}}{2}$ and the inefficient user produces a lower output than under no signalling, $q_{1M_s}(c)=\frac{1-r_{1M}}{2}-\frac{\sqrt{c(2-c)}}{2}<\frac{1-r_{1M}-c}{2}$, to signal a high production cost. ■

Proof of Proposition 4

Let $v_1^L(\tilde{c})$ with $\tilde{c} \in \{0, c\}$ denote period-1 user profits. Given that the patent holder has no information about user costs, the license fee for the first period does not depend on these costs. Firstly, if the patent holder allows both types to produce, the fees are set to solve

$$\pi_{1M}^{\text{PH}}(c,\mu)=\max_{\{f_{1M}\geq 0, r_{1M}\geq 0\}} \{f_{1M}+ r_{1M}(\mu q_{1M}(0)+(1-\mu)q_{1M_s}(c))\}, \tag{A7}$$

subject to

[8]In inequalities (A4) and (A5) a fixed fee for period 1 optimally set by the patent holder must be subtracted on both sides. But, since it takes the same value, we omit it. As we will see below in Proposition 5, one important aspect of this omitted fixed fee in (A4) and (A5) is that it can be such that the user's profits can be negative; in this case, the optimal signalling production (and period-1 profits) for the inefficient user will be zero, and (A4) and (A5) must be written as $0 \geq -\frac{c(2-c)}{4}$ and $\frac{1}{4} \geq \frac{c(2-c)}{4}$ respectively. Given our assumption about c it is easy to check that they are verified.

$$v_1^L(c) = (1 - r_{1M} - c - q_{1M_s}(c))q_{1M_s}(c) \geq f_{1M} \tag{A8}$$

and

$$v_1^L(0) = (1 - r_{1M} - q_{1M}(0))q_{1M}(0) \geq f_{1M}, \tag{A9}$$

where $q_{1M}(0) = \frac{1-r_{1M}}{2}$ is taken from Proposition 1, $q_{1M_s}(c) = \frac{1-r_{1M}}{2} - \frac{\sqrt{c(2-c)}}{2}$ from Proposition 3, and (A8) and (A9) are the participation constraints for the inefficient and efficient user respectively. Since $v_1^L(0) > v_1^L(c)$, the problem stated in (A7)–(A9) can be rewritten as the following unconstrained maximization problem

$$\pi_{1M}^{\text{PH}}(c, \mu) = \max_{r_{1M} \geq 0} \{(1 - r_{1M} - c - q_{1M_s}(c))q_{1M_s}(c) + r_{1M}(\mu\ q_{1M}(0) + (1-\mu)q_{1M_s}(c))\}, \tag{A10}$$

with $q_{1M_s}(c)$ and $q_{1M}(0)$ as given above. The solution to (A10) is

$$\{f_{1M}, r_{1M}\} = \begin{cases} \left\{ \dfrac{\left(1-\sqrt{c(2-c)}\right)\left(1-c-\frac{1}{2}\left(1-\sqrt{c(2-c)}\right)\right)}{2}, 0 \right\}, & \text{if } c \leq \dfrac{2(1-\mu)^2}{1+(1-\mu)^2}, \\ \left\{ \dfrac{\left(c-1+\mu\sqrt{c(2-c)}\right)\left(3c-1-(2-\mu)\sqrt{c(2-c)}\right)}{4}, c-(1-\mu)\sqrt{c(2-c)} \right\}, & \text{if } \dfrac{2(1-\mu)^2}{1+(1-\mu)^2} \leq c \leq 1 - \dfrac{\mu}{\sqrt{1+\mu^2}}, \\ \left\{0,\ 1-\sqrt{c(2-c)}\right\}, & \text{if } c \geq 1 - \dfrac{\mu}{\sqrt{1+\mu^2}}. \end{cases} \tag{A11}$$

Period-1 user's net profits are zero if it becomes an inefficient producer and strictly positive if becomes efficient. From (A10) to (A.11), the patent holder's expected payoff in period 1 is

$$\pi_{1M}^{\text{PH}}(c, \mu) = \begin{cases} \dfrac{\left(1-\sqrt{c(2-c)}\right)\left(1-c-\frac{1}{2}\left(1-\sqrt{c(2-c)}\right)\right)}{2}, & \text{if } c \leq \dfrac{2(1-\mu)^2}{1+(1-\mu)^2}, \\ \dfrac{c(2-c)[\mu(\mu-2)-1] + 2c\mu\sqrt{c(2-c)} + 1}{4}, & \text{if } \dfrac{2(1-\mu)^2}{1+(1-\mu)^2} \leq c \leq 1 - \dfrac{\mu}{\sqrt{1+\mu^2}}, \\ \dfrac{\mu\left[\sqrt{c(2-c)} - c(2-c)\right]}{2}, & \text{if } c \geq 1 - \dfrac{\mu}{\sqrt{1+\mu^2}}. \end{cases} \tag{A12}$$

Secondly, the patent holder has the option of setting the fixed fee as equal to the efficient user's profits. Therefore, if the user turns out to be inefficient and does not produce in period 1, the patent holder will not receive any income. The patent holder's revenue in period 1 will only be positive if the user turns out to be efficient, which

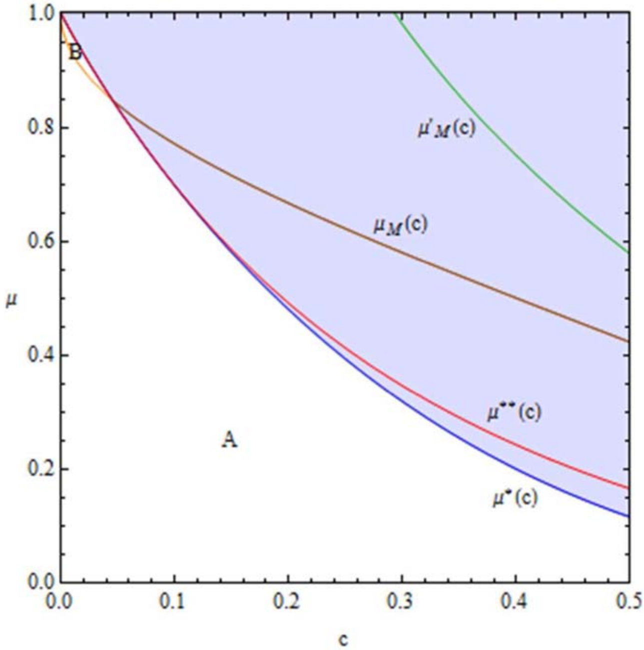

FIG. 2. Period-1 Contracts Under Asymmetric Information When One License is Granted
Shaded area: the best contract for the patent holder is a license such that an inefficient user will not produce in period 1; the subarea between $\mu^*(c)$ and $\mu_M(c)$ is optimal for setting a zero royalty if both types were to produce; in the subarea above $\mu^{**}(c)$ and $\mu_M(c)$ is optimal to set a positive fee if both types were to produce. However, for $\mu \geq \mu_M(c)$ the production of an inefficient type would be zero. Unshaded area: the best contract for the patent holder is to sell a license which allows the user to produce irrespective of his efficiency. (In subarea A the royalty is zero, whereas in subarea B the royalty is positive). [Colour figure can be viewed at wileyonlinelibrary.com]

happens with probability μ. Therefore, instead of (A7)–(A9)—or its unconstrained version (A10)—the patent holder problem is

$$\pi_{1M}^{PH}(\mu) = \max_{r_{1M \geq 0}} \; [\mu((1-r_{1M}-q_{1M}(0))q_{1M}(0)+r_{1M}q_{1M}(0))], \tag{A13}$$

where $q_{1M}(0)$ is the same as before. The solution of (A13) is $r_{1M}=0$ and $f_{1M}=\frac{1}{4}$, resulting in $\pi_{1M}^{PH}(\mu)=\frac{\mu}{4}$. Period-1 net profits of the user are $\pi_1^L(c)=\pi_1^L(0)=0$.

To see which option is better for the patent holder we need to check how $\pi_{1M}^{\mathrm{PH}}(c,\mu)$ in (A12) compares to $\frac{\mu}{4}$. Let $\mu_M(c)$ and $\mu'_M(c)$ denote the values of c and μ that verify with equality the *if* conditions in (A11)–(A12), respectively. Likewise, let $\mu^*(c)$ and $\mu^{**}(c)$ denote the solution to $\pi_{1M}^{\mathrm{PH}}(c,\mu)=\frac{\mu}{4}$, when $\pi_{1M}^{\mathrm{PH}}(c,\mu)$ is given, respectively, by the first and second lines in (A12). The result given by (10) in the main text straightforwardly follows. Figure 2 illustrates the result. ■

Proof of Proposition 5

The proof proceeds in two steps. We first show that the incentive of user i ($i=A,B$) to be perceived as inefficient by the patent holder outweighs the incentive to be understood as efficient by rival j ($j = A, B; j \neq i$). We thus prove the existence of a separating equilibrium.

Step 1. We have two users, $i, j = A, B; i \neq j$, competing *à la* Cournot in the downstream market. Let us denote as $\tilde{c}_i^{b_{PH}} \in \{0, c\}$ the patent holder's beliefs about user i's production cost. For each type of user j, user i's profits according to the patent holder beliefs are

$$v_2^i\left(\tilde{c}_j|\tilde{c}_i^{b_{PH}}\right) = \frac{\left(1-2\tilde{c}_i^{b_{PH}}+\tilde{c}_j-2r_2^i\left(\tilde{c}_j|\tilde{c}_i^{b_{PH}}\right)+r_2^j\left(\tilde{c}_j|\tilde{c}_i^{b_{PH}}\right)\right)^2}{9}, \tag{A14}$$

while the quantity produced is the positive square root of this amount. If we denote by $r_2^i\left(\tilde{c}_j|\tilde{c}_i^{b_{PH}}\right)$ and $r_2^j\left(\tilde{c}_j|\tilde{c}_i^{b_{PH}}\right)$ the period-2 royalties for i and j based on patent holder beliefs, respectively, and by $\pi_2^{PH}\left(\tilde{c}_j|\tilde{c}_i^{b_{PH}}\right)$ the patent holder profits in period 2, fixed fees and royalties chosen by the patent holder for this period are those that solve the following problem

$$\begin{aligned} &\max_{\{r_2^i, r_2^j\}_{i,j=A,B} \geq 0} \pi_2^{PH}\left(\tilde{c}_j|\tilde{c}_i^{b_{PH}}\right) \\ &= \max_{\{r_2^i, r_2^j\}_{i,j=A,B} \geq 0} \sum_{i,j=A,B} \left\{ \frac{\left(1-2\tilde{c}_i^{b_{PH}}+\tilde{c}_j-2r_2^i\left(\tilde{c}_j|\tilde{c}_i^{b_{PH}}\right)+r_2^j\left(\tilde{c}_j|\tilde{c}_i^{b_{PH}}\right)\right)^2}{9} \right. \\ &\left. + \; r_2^i\left(\tilde{c}_j|\tilde{c}_i^{b_{PH}}\right) \frac{1-2\tilde{c}_i^{b_{PH}}+\tilde{c}_j-2r_2^i\left(\tilde{c}_j|\tilde{c}_i^{b_{PH}}\right)+r_2^j\left(\tilde{c}_j|\tilde{c}_i^{b_{PH}}\right)}{3} \right\} . \end{aligned} \tag{A15}$$

The solution to (A15) is

$$\left(f_2^i\left(\tilde{c}_j|\tilde{c}_i^{b_{PH}}\right) = v_2^i\left(\tilde{c}_j|\tilde{c}_i^{b_{PH}}\right), r_2^i\left(\tilde{c}_j|\tilde{c}_i^{b_{PH}}\right)\right) = \begin{cases} \left(0, \dfrac{1-2c}{2}\right), & \text{if } \tilde{c}_i^{b_{PH}} = c \text{ and } \tilde{c}_j = 0, \\ \left(\dfrac{(1-c)^2}{16}, \dfrac{1-c}{4}\right), & \text{if } \tilde{c}_i^{b_{PH}} = c \text{ and } \tilde{c}_j = c, \\ \left(\dfrac{1}{16}, \dfrac{1}{4}\right), & \text{if } \tilde{c}_i^{b_{PH}} = 0 \text{ and } \tilde{c}_j = 0, \\ \left(\dfrac{1}{4}, 0\right), & \text{if } \tilde{c}_i^{b_{PH}} = 0 \text{ and } \tilde{c}_j = c, \end{cases} \tag{A16}$$

which, of course, matches the solution in a duopoly under symmetric information, as given by (3), but with the patent holder's beliefs about user i's cost instead of the true costs.

We shall now denote as $v_2^i\left(\tilde{c}_i, \tilde{c}_j|\tilde{c}_i^{b_{-i}}\right)$, where $\tilde{c}_i, \tilde{c}_j \in \{0, c\}$ are the production costs for users i and j; and $\tilde{c}_i^{b_{-i}} \in \{0, c\}$ denotes beliefs about user i's production costs, both by user j and the patent holder $(-i = \{PH\})$. In addition, let $\tilde{c}_i^{b_{-i}}$ be the perceived state of nature (which depends on how beliefs are revised) by the patent holder and opponent user j. Knowing their own cost and given the beliefs of the patent holder and the opponent user on the state of nature, user i sets quantity so as to maximize profits, i.e.

$$v_2^i\left(\tilde{c}_i, \tilde{c}_j|\tilde{c}_i^{b_{-i}}\right) = \max_{q_i} \left\{\left(1-q_i-q_j\left(\tilde{c}_j|\tilde{c}_i^{b_{-i}}\right)-c_i-r_2^i\left(\tilde{c}_j|\tilde{c}_i^{b_{-i}}\right)\right)q_i\right), \tag{A17}$$

where $q_j\left(\tilde{c}_j|\tilde{c}_i^{b_{-i}}\right)$ is the Cournot equilibrium quantity that user i expects user j to produce

$$q_j = \frac{1-2\left(\tilde{c}_j+r_2^j\left(\tilde{c}_j|\tilde{c}_i^{b_{-i}}\right)\right)+\left(\tilde{c}_i^{b_j}+r_2^i\left(\tilde{c}_j|\tilde{c}_i^{b_{-i}}\right)\right)}{3}. \tag{A18}$$

The solution of (A17) affords

$$v_2^i\left(\tilde{c}_i, \tilde{c}_j|\tilde{c}_i^{b_{-i}}\right) = \frac{\left(2-3\tilde{c}_i+2\tilde{c}_j-\tilde{c}_i^{b_j}-4r_2^i\left(\tilde{c}_j|\tilde{c}_i^{b_{PH}}\right)+2r_2^j\left(\tilde{c}_j|\tilde{c}_i^{b_{PH}}\right)\right)^2}{36}. \tag{A19}$$

In the absence of common knowledge, licensing fees depend on the patent holder's beliefs about users' production costs. The users' period-2 net profits are given by $\pi_2^i\left(\tilde{c}_i, \tilde{c}_j|\tilde{c}_i^{b_{-i}}\right) = v_2^i\left(\tilde{c}_i, \tilde{c}_j|\tilde{c}_i^{b_{-i}}\right) - f_2^i\left(\tilde{c}_j|\tilde{c}_i^{b_{PH}}\right)$. Since both the patent holder and competing user j have the same information about i's marginal cost and since both of them update their beliefs according to Bayes' rule, it follows that their beliefs must match, i.e. $\tilde{c}_i^{b_{PH}} = \tilde{c}_i^{b_j} = \tilde{c}_i^{b_{-i}} \in \{0, c\}$. On one hand, if the patent holder and user j's beliefs are incorrect, taking (A16) into account, the period-2 net profits of each user i are given by

$$\pi_2^i\left(\tilde{c}_i, \tilde{c}_j|\tilde{c}_i^{b_{-i}}\right) = v_2^i\left(\tilde{c}_i, \tilde{c}_j|\tilde{c}_i^{b_{-i}}\right) - f_2^i\left(\tilde{c}_j|\tilde{c}_i^{b_{PH}}\right)$$
$$= \begin{cases} \dfrac{c^2}{4}, & \text{if } \tilde{c}_i=0,\ \tilde{c}_j=0 \text{ and } \tilde{c}_i^{b_{-i}}=c, \\ \dfrac{c}{4}, & \text{if } \tilde{c}_i=0,\ \tilde{c}_j=c \text{ and } \tilde{c}_i^{b_{-i}}=c, \\ \dfrac{c(c-1)}{4}, & \text{if } \tilde{c}_i=c,\ \tilde{c}_j=0 \text{ and } \tilde{c}_i^{b_{-i}}=0, \\ \dfrac{c(c-2)}{4}, & \text{if } \tilde{c}_i=c,\ \tilde{c}_j=c \text{ and } \tilde{c}_i^{b_{-i}}=0. \end{cases} \tag{A20}$$

On the other hand, if the patent holder and user j's beliefs about i's marginal cost are correct, all variables affecting user *i's* profits are common knowledge, $v_2^i\left(\tilde{c}_i, \tilde{c}_j|\tilde{c}_i^{b_{-i}}=\tilde{c}_i\right) = v_2^i(\tilde{c}_i, \tilde{c}_j)$, and—from (A19)—are given by

$$v_2^i(\tilde{c}_i, \tilde{c}_j) = \frac{\left(1-2\tilde{c}_i+\tilde{c}_j-2r_2^i(\tilde{c}_i, \tilde{c}_j)+r_2^j(\tilde{c}_i, \tilde{c}_j)\right)^2}{9}. \tag{A21}$$

Note that if the patent holder's beliefs are correct, then the values chosen for r_2^i and f_2^i are such that $f_2^i(\tilde{c}_i,\tilde{c}_j)=v_2^i\left(\tilde{c}_j|\tilde{c}_i^{b_{PH}}\right)=v_2^i(\tilde{c}_i,\tilde{c}_j)$ with v_2^i given by (A21), so $\pi_2^i\left(\tilde{c}_i,\tilde{c}_j|\tilde{c}_i^{b_{-i}}\right)=\pi_2^i(\tilde{c}_i,\tilde{c}_j)=0$ for all $\tilde{c}_i,\tilde{c}_j\in\{0,c\}$.

In (A20) it can be checked that $\pi_2^i(0,\tilde{c}_j|c)>\pi_2^i(0,\tilde{c}_j)=0$ and $\pi_2^i(c,\tilde{c}_j|c)<\pi_2^i(c,\tilde{c}_j)=0$ for $\tilde{c}_j\in\{0,c\}$. This proves that each user prefers to be perceived as inefficient. Note that, from (A19), user i's profits are decreasing on the basis of the rival's beliefs about the former's costs, meaning that each user i has an horizontal incentive to be perceived as efficient by rivals. However, the fact that user i prefers to be perceived as inefficient would indicate that there are more than off-setting vertical incentives to be perceived as inefficient by the patent holder.

Step 2: Let $i_{\tilde{c}}\in\{A_{\tilde{c}},B_{\tilde{c}}\}$ denote user i with production cost $\tilde{c}\in\{0,c\}$. Also, let $q_{1_s}^{i_{\tilde{c}}}$ and $q_1^{i_{\tilde{c}}}$ denote, respectively, the signalling and non-signalling period 1 output of user $i_{\tilde{c}}$. Finally, assume that there exists $q_{1_s}^{i_c}<q_1^{i_0}$ such that an efficient user sets $q_1^{i_0}$ and an inefficient user sets $q_{1_s}^{i_c}$ in period 1. Now assume that the updated beliefs are

$$\hat{\mu}_{-i}=\begin{cases}0, & \text{if } q_1^i\leq q_{1_s}^{i_c},\\ 1, & \text{otherwise},\end{cases}\tag{A22}$$

where $-i=\{\text{PH},j\}$. The production $q_1^{i_{\tilde{c}}}$ constituting the best response to the expected quantity produced by rival j in period 1, i.e. the best response to $\left(\mu q_1^{j_0}+(1-\mu)q_1^{j_c}\right)$, when firm i's marginal cost is $\tilde{c}\in\{0,c\}$, is given by

$$q_1^{i_0}=\frac{1-r_{1D}-(1-\mu)q_1^{j_c}}{2+\mu},\tag{A23}$$

and

$$q_1^{i_c}=\frac{2(1-c-r_{1D})-\mu c-2(1-\mu)q_1^{j_c}}{2(2+\mu)}.\tag{A24}$$

A separating equilibrium requires that the following incentive compatibility constraints be verified[9]:

- Incentive constraint for the efficient user

$$\begin{aligned}&\mu\left(1-r_{1D}-q_1^{i_0}-q_1^{j_0}\right)q_1^{i_0}+(1-\mu)\left(1-r_{1D}-q_1^{i_0}-q_{1_s}^{j_c}\right)q_1^{i_0}\\&\geq\mu\left[\left(1-r_{1D}-q_{1_s}^{i_c}-q_1^{j_0}\right)q_{1_s}^{i_c}+\frac{c^2}{4}\right]+(1-\mu)\left[\left(1-r_{1D}-q_{1_s}^{i_c}-q_{1_s}^{j_c}\right)q_{1_s}^{i_c}+\frac{c}{4}\right].\end{aligned}\tag{A25}$$

Of all quantities that would lead the user to be identified as efficient, clearly $q_1^{i_0}$ as given by (A23) is the best by definition. Of all quantities that would lead the user to

[9] As in the case of monopoly inequalities (A4) and (A5), here in inequalities (A25) and (A26) there is also a fixed fee in the first period to be subtracted on both sides of the inequalities. However, since the patent holder does not have information on licensee types, it takes the same value on both sides and so we omit it.

be perceived as inefficient the least costly deviation is $q_{1_s}^{i_c}$—to be determined—but (A25) guarantees that the efficient user is not tempted to set $q_{1_s}^{i_c}$.

- Incentive constraint for the inefficient user:

$$\begin{aligned}&\mu\left(1-c-r_{1D}-q_{1_s}^{i_c}-q_1^{j_0}\right)q_{1_s}^{i_c}+(1-\mu)\left(1-c-r_{1D}-q_{1_s}^{i_c}-q_{1_s}^{j_c}\right)q_{1_s}^{i_c}\\&\quad\geq\mu\left[\left(1-c-r_{1D}-q_1^{i_c}-q_1^{j_0}\right)q_1^{i_c}+\frac{c(c-1)}{4}\right]\\&\quad\quad+(1-\mu)\left[\left(1-c-r_{1D}-q_1^{i_c}-q_{1_s}^{j_c}\right)q_1^{i_c}+\frac{c(c-2)}{4}\right].\end{aligned}\tag{A26}$$

Of all quantities that would lead the user to be identified as inefficient, clearly $q_{1_s}^{i_c}$ is the one that causes least costs. Of the quantities that lead the user to be identified as efficient, the optimal deviation is $q_1^{i_c}$ as given by (A24), which the user is not tempted to set under (A26). Solving (A25) and (A26) as equalities and then using (A23) and (A24), we obtain four roots but only the following two simultaneously verify both inequalities, namely

$$q_{1a_s}^{i_c}=\frac{1-r_{1D}}{3}-\frac{c(2+\mu)}{6}-\frac{(2+\mu)\Psi}{6},\tag{A27}$$

where $\Psi=\sqrt{2c-c^2-\mu c}$, and

$$q_{1b_s}^{i_c}=\frac{1-r_{1D}}{3}-\frac{(2+\mu)\Lambda}{6},\tag{A28}$$

where $\Lambda=\sqrt{c-\mu c+\mu c^2}$.

From (A27) and (A28) it follows that $q_{1a_s}^{i_c}<q_{1b_s}^{i_c}$. Further, given that $q_{1b_s}^{i_c}<q_1^{i_c}$, in a separating equilibrium of least cost, the inefficient user chooses the output $q_{1b_s}^{i_c}$ given by (A28). Note here how important it is to have $q_{1b_s}^{i_c}<q_1^{i_c}$, given the updated beliefs. Indeed, if we had $q_{1b_s}^{i_c}>q_1^{i_c}$, the inefficient user would be better off deviating to his best response given the updated beliefs.

Now consider the case where inefficient users do not signal their cost. Using equation (A24) and ignoring inequalities (A27) and (A28) we have

$$q_{1_{ns}}^{i_c}=\frac{1-r_{1D}}{3}-\frac{(2+\mu)c}{6}.\tag{A29}$$

It can easily be verified that the equilibrium output of the inefficient user under signalling, as given by (A28), is lower than the non-signalling equilibrium output characterized by (A29).

Finally, under (A23), the efficient user chooses

$$q_1^{i_0}=\frac{1-r_{1D}}{3}+\frac{(1-\mu)\Psi}{6}.\tag{A30}$$

■

Proof of Proposition 6

Again, the patent holder has no information about the user's type and therefore the license fees cannot depend on this. If the patent holder allows both types of user to produce, f_{1D} and r_{1D} must be such that they solve the problem

$$\pi_{1D}^{\mathrm{PH}}(c,\mu)=\max_{\{f_{1D}\geq 0,\ r_{1D}\geq 0\}}\Big[2f_{1D}+r_{1D}\Big(\mu^2\Big(q_1^{i_0}+q_1^{j_0}\Big) +(1-\mu)^2\Big(q_{1_s}^{i_c}+q_{1_s}^{j_c}\Big)+2\mu(1-\mu)\Big(q_1^{i_0}+q_{1_s}^{j_c}\Big)\Big)\Big] \tag{A31}$$

subject to

$$Ev_1^{i_c}(c,\tilde{c}_j)=\Big(1-c-r_{1D}-q_{1_s}^{i_c}-\mu q_1^{j_0}-(1-\mu)q_{1_s}^{j_c}\Big)q_{1_s}^{i_c}\geq f_{1D} \tag{A32}$$

and

$$Ev_1^{i_0}(0,\tilde{c}_j)=\Big(1-r_{1D}-q_1^{i_0}-\mu q_1^{j_0}-(1-\mu)q_{1_s}^{j_c}\Big)q_1^{i_0}\geq f_{1D}. \tag{A33}$$

In the above expressions we know that $q_{1_s}^{i_c}=q_{1_s}^{j_c}$ and $q_1^{j_0}=q_1^{i_0}$ and that these quantities are given by (A28) and (A30). Since the solution to problem (A31)–(A33) verifies condition (A32) with equality and (A33) with strict inequality, problem (A31)–(A33) can be written as

$$\pi_{1D}^{\mathrm{PH}}(c,\mu)=\max_{r_{1D}\geq 0}\Big\{2\Big(1-c-\mu\ r_{1D}-\mu q_1^{i_0}-(2-\mu)q_{1_s}^{i_c}\Big)q_{1_s}^{i_c}+2\mu\ r_{1D}q_1^{i_0}\Big\}. \tag{A34}$$

It is easy to check that the patent holder's problem (A34) is linear in r_{1D} and, therefore, the solution is

$$r_{1D}=\begin{cases}0, & \text{if } \mu\leq\mu_D''(c),\\ \dfrac{1+3c+2(1-\mu)\Psi-2\big(2-\mu-\mu^2\big)\Lambda}{4}, & \text{if } \mu\geq\mu_D''(c),\end{cases} \tag{A35}$$

where $\mu_D''(c)$ is implicitly defined by the equation

$$c-\frac{2\big(2-\mu-\mu^2\big)\Lambda-2\mu(1-\mu)\Psi-1}{3}=0, \tag{A36}$$

being $\Lambda=\sqrt{c-\mu c+\mu c^2}$, and $\Psi=\sqrt{2c-c^2-\mu c}$.

The fixed fee f_{1D} is given by (A32) with equality. The expected profit of the inefficient user is zero and that of the efficient user is positive. Finally, patent holder revenue is given by (A34) after the substitution of r_{1D} as given by (A35)

$$\pi_{1D}^{\mathrm{PH}}(c,\mu)=\begin{cases}\left(\dfrac{2}{3}-\dfrac{(2+\mu)\Psi}{3}\right)\times \\ \left(1-c-\mu\left(\dfrac{1}{3}+\dfrac{(1-\mu)\Psi}{6}\right)-(2-\mu)\left(\dfrac{1}{3}-\dfrac{(2+\mu)\Lambda}{6}\right)\right), & \text{if } \mu\leq\mu_D''(c),\\ \dfrac{1}{36}[9+2(1-\mu)\mu(2+\mu)(4\mu-3)\Lambda \\ +c^2\big(9-28\mu^2+6\mu^4+4\mu^5\big)-18c+2\mu^3c-10\mu^4c \\ +8\mu^5c+12\mu^2c(\Psi-\Lambda-2) \\ -12\mu c(\Psi+2\Lambda-2)], & \text{if } \mu\geq\mu_D''(c).\end{cases} \tag{A37}$$

On the other hand, if fees established by the patent holder are such that the users only produce if they are efficient, f_{1D} is given by (A33) with equality and r_{1D} must be such that the following problem is solved

$$\pi_{1D}^{\mathrm{PH}}(\mu, c_i, c_j) = \max_{r_{1D}\geq 0}\Big\{2\mu^2\Big(1-r_{1D}-q_1^{i_0}-\mu q_1^{j_0}\Big)q_1^{i_0}+2\mu(1-\mu)\Big(1-r_{1D}-q_1^{i_0}-\ \mu q_1^{j_0}\Big)q_1^{i_0}$$
$$+r_{1D}\Big(\mu^2\Big(q_1^{i_0}+q_1^{j_0}\Big)+2\mu(1-\mu)q_1^{i_0}\Big)\Big\}. \tag{A38}$$

As before, $q_1^{j_0}=q_1^{i_0}$ and is given by (A30). Taking this into account, the problem can be written as

$$\pi_{1D}^{\mathrm{PH}}(\mu, c_i, c_j) = \max_{r_{1D\geq 0}}\ \big[2\mu\big(1-(1+\mu)q_1^{i_0}\big)q_1^{i_0}\big]. \tag{A39}$$

The value of r_{1D} that solves this problem is

$$r_{1D}=\begin{cases} 0, & \text{if } \mu\leq\mu_D'(c),\\ \dfrac{2\mu-1+(1-\mu^2)\Psi}{2(1+\mu)}, & \text{if } \mu\geq\mu_D'(c),\end{cases} \tag{A40}$$

where $\mu_D'(c)$ solves the equation

$$\Psi=\frac{2\mu-1}{\mu^2-1}. \tag{A41}$$

The fixed fee is given by (A33) with equality, after substituting r_{1D} by (A40). Patent holder's expected profits are therefore

$$\pi_{1D}^{\mathrm{PH}}(\mu, c)=\begin{cases} 2\left(\dfrac{1}{3}+\dfrac{1}{6}(1-\mu)\Psi\right)\times & \\ \mu\left(1-(1+\mu)\left(\dfrac{1}{3}+\dfrac{1}{6}(1-\mu)\Psi\right)\right), & \text{if } \mu\leq\mu_D'(c),\\ \dfrac{\mu}{2(1+\mu)}, & \text{if } \mu\geq\mu_D'(c).\end{cases} \tag{A42}$$

To check whether it is better or not for the patent holder to allow inefficient users to produce in period 1, we need to compare patent holder's profits in both situations, as given by (A37) and (A42). First note, from (A36) and (A41), that $\mu_D'(c)$ and $\mu_D''(c)$ cross each other (see also Fig. 3). Taking this into account in comparing (A37) and (A42), it turns out that the patent holder prefers to allow both user types to produce in the first period as long as $\mu\leq\mu_D(c)$ where $\mu_D(c)$ is implicitly defined by the equality between the expressions in the second lines of (A37) and (A42) (see again Fig. 3). The royalty is zero or positive depending on whether μ is smaller or greater than $\mu_D''(c)$. On the contrary, when $\mu\geq\mu_D(c)$, setting the fees such that users only produce if they are efficient is better for the patent holder. Therefore, from (A35) and (A40), it follows that period 1 royalty for the patent holder when two licenses are sold is given by (11) in the main text. The fixed fee in (11) is obtained once the royalty is substituted in the left-hand side of (A32) and (A33) for $\mu\leq\mu_D(c)$ and $\mu\geq\mu_D(c)$, respectively. ■

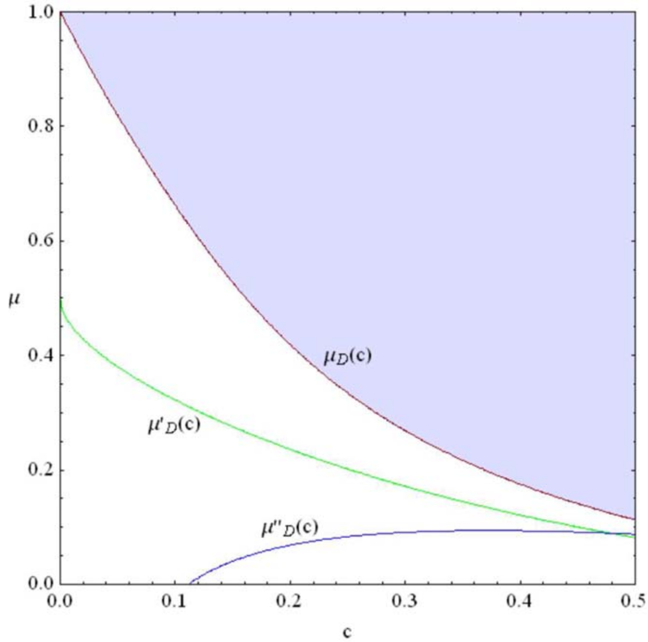

FIG. 3. Period-1 Contracts Under Asymmetric Information with Two Licenses
In the shaded area the best option for the patent holder is to grant a license such that an inefficient firm will not produce in period 1 and the royalty is positive; the line $\mu_D(c)$ indicates where the patent holder is indifferent as to whether both types of user produce or not. In the unshaded area the best option for the patent holder is a license which allows users to produce irrespective of their efficiency, and where the variable fee is positive for $\mu > \mu''_D(c)$ and zero otherwise. In the subarea under $\mu'_D(c)$ it is optimal for the patent holder to set a zero royalty if only efficient users were to produce. [Colour figure can be viewed at wileyonlinelibrary.com]

Proof of Proposition 7

With a single license under asymmetric information, we saw in Proposition 4 that, in order to see if the patent holder prefers to allow the inefficient user to produce or not in period 1, we need to compare the expected profits given by (A12)—all user's types are producing in period 1— with the expected profits of allowing only the efficient user to produce, $\frac{\mu}{4}$. From the comparison it emerges that the patent holder's expected profits in period 1 when selling one license are

$$\pi_{1M}^{PH}(c,\mu)=\begin{cases}\dfrac{\left(1-\sqrt{c(2-c)}\right)\left(1-c-\dfrac{1}{2}\left(1-\sqrt{c(2-c)}\right)\right)}{2}, & \text{if } \mu\le\mu_M(c)\cap\mu\le\mu^*(c),\\[2ex] \dfrac{(\mu(\mu-2)-1)c(2-c)+2c\mu\sqrt{c(2-c)}+1}{4}, & \text{if } \mu_M(c)\le\mu\le\mu^{**}(c),\\[2ex] \dfrac{\mu}{4}, & \text{if } \mu\ge\mu^*(c)\cup\mu\ge\mu^{**}(c),\end{cases} \quad \text{(A43)}$$

where $\mu_M(c)$, $\mu^*(c)$, and $\mu^{**}(c)$ are defined as in Proposition 4. If the showing cost per license amounts to $S^* = \frac{1}{4}c(1-\mu)\mu(2-c)$ as defined in Proposition 2, then, in a separating equilibrium, the patent holder's expected revenue from selling only one license, $E\pi_{MM}^{\mathrm{PH}}$, is

$$E\pi_{MM}^{\mathrm{PH}} = \pi_{1M}^{\mathrm{PH}}(c,\mu) + \mu\ \pi_{2M}^{\mathrm{PH}}(0) + (1-\mu)\pi_{2M}^{\mathrm{PH}}(c) - S^*, \tag{A44}$$

where $\pi_{1M}^{\mathrm{PH}}(c,\mu)$ is given by (A43), while $\pi_{2M}^{\mathrm{PH}}(0)$ and $\pi_{2M}^{\mathrm{PH}}(c)$ are given by (A2), as shown in the proof of Proposition 3.

By issuing two licenses, the patent holder's expected profit amounts to

$$\pi_{1D}^{\mathrm{PH}}(c,\mu) = \begin{cases} \left(\frac{2}{3} - \frac{(2+\mu)\Lambda}{3}\right)\times \\ \left(1 - c - \mu\left(\frac{1}{3} + \frac{(1-\mu)\Psi}{6}\right) - (2-\mu)\left(\frac{1}{3} - \frac{(2+\mu)\Lambda}{6}\right)\right), & \text{if } \mu \le \mu_D''(c), \\ \frac{1}{36}[9 + 2\mu(1-\mu)(2+\mu)(4\mu-3)\Lambda \\ \quad + c^2\left(9 - 28\mu^2 + 6\mu^4 + 4\mu^5\right) - 2c\left(9 + \mu^3 - 5\mu^4 + 4\mu^5\right) \\ \quad + 12\mu^2 c(\Psi - \Lambda - 2) - 12\mu c(\Psi + 2\Lambda - 2)], & \text{if } \mu_D''(c) \le \mu \le \mu_D(c), \\ \frac{\mu}{2(1+\mu)}, & \text{if } \mu \ge \mu_D(c), \end{cases} \tag{A45}$$

which, resulting as it does from the combination of (A37) and (A42), gives rise to the patent holder's profits resulting from his optimal choice of the licensing fees under a duopoly, as studied in Proposition 6 (as before, $\Lambda = \sqrt{c - \mu c + \mu c^2}$ and $\Psi = \sqrt{2c - c^2 - \mu c}$ in (A45)).

Therefore, with two licenses under asymmetric information, the patent holder's expected profits are

$$\begin{aligned} E\pi_{DD}^{\mathrm{PH}} = {} & \pi_{1D}^{\mathrm{PH}}(c,\mu) + \mu^2 \pi_{2D}^{\mathrm{PH}}(0,0) + 2\mu(1-\mu)\left(\pi_{2D}^{\mathrm{PH}}(0,c) + \pi_{2D}^{\mathrm{PH}}(c,0)\right) \\ & + (1-\mu)^2 \pi_{2D}^{\mathrm{PH}}(c,c) - 2S^*. \end{aligned} \tag{A46}$$

In (A46), $\pi_{1D}^{\mathrm{PH}}(c,\mu)$ is given by (A45) and $\pi_{2D}^{\mathrm{PH}}\left(\tilde{c}_i, \tilde{c}_j\right)$ can be computed by using the results of Proposition 5 and substituting expression (A16) into the objective function that the patent holder maximizes, bearing in mind that, in a separating equilibrium with two licenses, period 2 fees coincide with those of a Cournot equilibrium with symmetric information, as given by (A16) when $\tilde{c}_i^{b_{PH}} = \tilde{c}_i$. We thus have that $\pi_{2D}^{\mathrm{PH}}\left(\tilde{c}_i, \tilde{c}_j\right)$ in (A46) is given by

$$\pi_{2D}^{PH}(\tilde{c}_i, \tilde{c}_j) = \begin{cases} \frac{1}{4}, & \text{if } \tilde{c}_i = 0 \text{ or } \tilde{c}_j = 0, \\ \frac{(1-c)^2}{4}, & \text{if } \tilde{c}_i = c \text{ and } \tilde{c}_j = c. \end{cases} \tag{A47}$$

Finally, comparing (A44) with (A46) we obtain the result stated in the main text. ■

References

Antelo, M. (2013). 'Duration and Payment of Licensing Contracts for Users to Reveal What They Know', *Economics of Innovation and New Technology*, Vol. 22, pp. 127–151.

Antelo, M. and Sampayo, A. (2014). 'On the Number of Licenses Under Symmetric Versus Asymmetric Information with Signalling', *MPRA working paper no. 60759.*

Arrow, K. J. (1962). 'Economic Welfare and the Allocation of Resources for Invention', in R. R. Nelson (ed), *The Rate and Direction of Inventive Activity: Economic and Social Factors*, Princeton, NJ, Princeton University Press.

Beggs, A. W. (1992). 'The Licensing of Patents Under Asymmetric Information', *International Journal of Industrial Organization*, Vol. 10, pp. 171–191.

Kamien, M. I. (1992), 'Patent Licensing', in R. J. Aumann and S. Hart (eds), *Handbook of Game Theory* with Economic Applications, Volume 1, Amsterdam: North-Holland.

Schmitz, P. W. (2002). 'On Monopolistic Licensing Strategies under Asymmetric Information', *Journal of Economic Theory*, Vol. 106, pp. 177–189.

Schmitz, P. W. (2007). 'Exclusive versus Non-Exclusive Licensing Strategies and Moral Hazard', *Economic Letters*, Vol. 97, pp. 208–214.