Spatial mode multiplexing/demultiplexing by Gouy phase interferometry

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We present a theoretical study about spatial mode multiplexing/demultiplexing (mux/demux) without theoretical losses by means of interferometry with selective control of the Gouy phase of optical beams, that is, Gouy phase interferometry (GPI). Different Gouy phase values can be obtained by inserting appropriate optical systems at each arm of an interferometer. Thus, spatial mode mux/demux operations, of strategic interest in optical communications with few-mode optical fibers, are implemented by means of constructive interference and regardless the parity and separability of the optical beams. Consequently, unachievable mux/demux by interferometry based on image inversion methods becomes possible with GPI. This kind of operations can also be interesting for optical sensors, optical metrology, image processing and so on.

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In recent years a renewed interest on optical beams has arisen. Optical beams are characterized by a non uniform complex amplitude which provides very special optical properties such as orbital angular momentum transference, non diffracting propagation, Gouy phases, and so on. Such optical beams are highly suitable for different applications as optical trapping, micro-lithography, optical metrology, etc. Motivated both for their optical properties and optical applications a myriad of theoretical solutions of the scalar z-paraxial wave equation, with z indicating the direction of propagation, have been found and analyzed. Examples of investigated paraxial optical beams are those ones with Gaussian amplitude modulated by other function, that is, F-Gaussian beams such as Hermite-Gaussian, Ince-Gaussian, Bessel-Gaussian, Hermite-cosh-Gaussian, Hypergeometric-Gaussian, and so on [1–5]. Moreover, these beams form complete sets of solutions characterized by two integer numbers (optical beam order) which solve the propagation problem of an arbitrary scalar paraxial optical field. These optical beams can be generated in different ways, however it is worthwhile to mention that many of them come from optical modes of few-mode optical fibers (FMF).

In this Letter we present a study on Gouy phase interferometry (GPI), that is, the use of Gouy phases ϕ of optical beams in, for example, a Mach-Zehnder interferometer (MZI). These phases are a consequence of the non uniform amplitude of the optical beams and present a dependence on the propagation distance z, the optical beam order (μ, ν) and other optical parameters p, that is, a selective phase $\phi(z; \mu, \nu, p)$ is obtained. This study is based on the seminal work of Beijersbergen *et.al.* [6] about selective Gouy phases along each spatial direction for mode converter purposes. In our case we consider a selective Gouy phases according to the path followed by the beam in an interferometer. The Gouy phase can be obtained by using either a conventional lenses system or alternatively gradient-index or fiber lenses highly compatible with optical fibers. In consequence, a myriad of Gouy phase shifters can be designed, and thus GPI can be used for implementing spatial mode multiplexing/demultiplexing (mux/demux) in FMFs for optical communications [7] and without any theoretical losses. As will be shown, GPI generalizes interferometric mux/demux based on both reflective [8] and refractive [9] image inversion, and therefore it opens new mux/demux possibilities. Likewise, GPI could also be applied to optical sensors based on FMFs [10]. The method proposed in this work can be applied to different kind of F-Gaussian optical beams, however, without loss of generality, we consider the important set of optical modes called Laguerre-Gaussian (LG) modes (a particular case of LP modes), and therefore a theoretical study will be made with the corresponding LG optical beams.

LG optical beams can come from a FMF (demultiplexing task) or from different mode converters [11] (multiplexing task). Let us consider the following representation of the LG optical beams after propagation along z-direction in free space and with radial mode number $\mu = p \in \mathbb{N}$ and azimuthal mode number $\nu = l \in \mathbb{N}$,

$$\Psi_{lp\binom{c}{s}}(r,\varphi) = F_{lp}(r)\, e^{-\frac{r^2}{w^2(z)}}\, e^{ik_o\frac{r^2}{2R(z)}}\, e^{ik_o z} e^{-i\phi_{lp}(z)} \begin{pmatrix} \cos l\varphi \\ \sin l\varphi \end{pmatrix}, \quad \textbf{(1)}$$

where $F_{lp}(r) = (C_{lp}/w(z))\left(\sqrt{2}r/w(z)\right)^{|l|} L_p^l\left(2r^2/w^2(z)\right)$, with L_p^l the generalized Laguerre polynomials, C_{lp} a normalization constant (not relevant in our analysis), $w(z)$ the half-width of the Gaussian function, $R(z)$ the curvature radius of the wavefronts and $\phi_{lp}(z)$ the Gouy phase and k_o the wavenumber in vacuum. It is interesting to indicate that, as shown recently, these modes present an excellent overlapping with the *LP* modes of a step-index optical fiber when the normalized frequency V is large [12]. Next, we write the expressions of the functions $w(z)$, $R(z)$ and $\phi(z)$ after propagation in an arbitrary Gaussian or *ABCD* optical system and, for reasons which will be evident later, with initial conditions $w(0) = w_o$, $1/R(z) = 0$ [2, 13],

$$w^2(z) = \frac{w_o^2}{z_R^2}\left(z_R^2 A^2(z) + B^2(z)\right), \tag{2}$$

$$\frac{1}{R(z)} = \frac{z_R^2 A(z)C(z) + B(z)D(z)}{z_R^2 A^2(z) + B^2(z)}, \tag{3}$$

$$\phi_{lp}(z) = (2p + l + 1)\arctan\left(\frac{B(z)}{z_R A(z)}\right). \tag{4}$$

with z_R the Rayleigh length. We must stress that functions $w(z)$ and $R(z)$ are identical for many optical beams such as Hermite-Gaussian, Ince-Gaussian, Hypergeometric-Gaussian and so on. However, the Gouy phase usually has a different dependence on the mode numbers, for instance, in the case of Hermite-Gaussian (HG) optical beams, with cartesian mode numbers $\mu, \nu = m, n \in \mathbb{N}$, we have

$$\Upsilon_{mn}(x,y) = H_{mn}(x,y)\, e^{-\frac{x^2+y^2}{w^2(z)}}\, e^{ik_o \frac{x^2+y^2}{2R(z)}}\, e^{ik_o z} e^{-i\phi_{mn}(z)}, \tag{5}$$

$$\phi_{mn}(z) = (m + n + 1)\arctan\left(\frac{B(z)}{z_R A(z)}\right), \tag{6}$$

where $H_{mn}(x,y) = \left(C_{mn}/w(z)\right)H_m(x/\sqrt{2}w(z))H_n(y/\sqrt{2}w(z))$, with $H_{m(n)}$ the Hermite polynomials and C_{mn} a normalization constant. In this case a separable function along x and y direction is obtained. It is important to underline that LG modes can be written as a finite sum of HG modes and viceversa [1].

We start by considering a MZI as shown in Fig. 1. In each one of its arms a symmetric (for sake of simplicity) *ABCD* optical system is placed. For our purposes a symmetric optical system is one that produces the same transformation for both an optical field injected forward as one injected in the backward direction, and, formally, fulfills the condition $A = D$ [14]. In one of the paths (path 1) we have the $A_1B_1C_1D_1$ optical system and in the other path (path 2) is the $A_2B_2C_2D_2$ one. We assume that the length of the optical systems is $L = 2d$ and their centers are located at a distance s from the first Beam-Splitter (BS1) of the MZI. Moreover, we consider a phase shifter θ, implemented by a thin element (for instance, a double prism), which introduces non selective interferometric phases. We also place four lenses (one at each input and output of the MZI) for diverging and converging operations because the optical beams, coming from optical fibers, must have a certain curvature at the input of the *ABCD* optical systems. Next, let us consider two optical beams Ψ_a and Ψ_b incident on the interferometer along horizontal and vertical inputs, respectively. We look for a constructive interference, as shown in Fig. 1, where, for example, we have $\Psi_a + \Psi_b$ at output v and no light at output h. We assume, without loss of generality, that the optical systems have a mirror symmetry with respect to its center where the origin of a reference system is placed. Therefore we can divide the optical systems in two subsystems, that is, $D_{1d}B_{1d}C_{1d}A_{1d}$ and $A_{1d}B_{1d}C_{1d}D_{1d}$, and the same for the second optical subsystem, where subindex d indicates a matrix element of the optical subsystem between $z = 0$ and $z = d$, as indicated in Fig. 1. Ideal constructive interference is obtained if there is a maximum overlapping of the optical beams at the output of the interferometer. At that end, width and curvature of the optical beams have to be match in both paths of the interferometer, for example, at the positions $s + d$ respect to BS1. Since optical beams also present a symmetric behavior inside both optical systems then the conditions $w_1(d) = w_2(d)$, $R_1(d) = R_2(d)$ establish a maximum overlapping, that is,

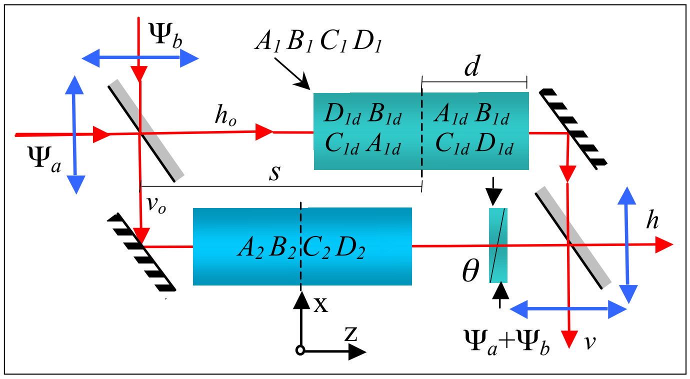

Fig. 1. MZI with *ABCD* systems implementing Gouy phases.

$$\frac{z_{R1}}{z_{R1}^2 A_{1d}^2 + B_{1d}^2} = \frac{z_{R2}}{z_{R2}^2 A_{2d}^2 + B_{2d}^2}, \tag{7}$$

$$\frac{z_{R1}^2 A_{1d}C_{1d} + B_{1d}D_{1d}}{z_{R1}^2 A_{1d}^2 + B_{1d}^2} = \frac{z_{R2}^2 A_{2d}C_{2d} + B_{2d}D_{2d}}{z_{R2}^2 A_{2d}^2 + B_{2d}^2}. \tag{8}$$

In general $z_{R1} \neq z_{R2}$, therefore, different Gouy phases are obtained, and, from a more geometrical point of view, different waist sizes are achieved at the center of each optical system. Under these conditions the optical systems implement Gouy phase shifters. The interferometric Gouy phase $\Phi_{lp} = \phi_{lp1} - \phi_{lp2}$, by defining $n_{pl} = (2p + l + 1)$, is given by

$$\Phi_{lp} = 2n_{pl}\left[\arctan\left(\frac{B_{1d}}{z_{R1}A_{1d}}\right) - \arctan\left(\frac{B_{2d}}{z_{R2}A_{2d}}\right)\right] - \theta. \tag{9}$$

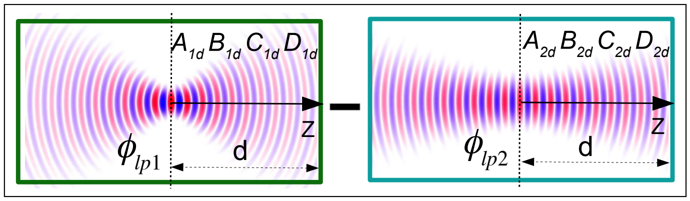

Fig. 2. Sketch of light propagation in two Gouy phase shifters.

Let us consider the particular case of two spherical lenses separated a distance $2d$ and with focal lengths f. The matrix elements of the optical subsystem are $A_{1d} = 1$, $B_{1d} = d$, $C_{1d} = 1/f$ and $D_{1d} = 1 + d/f$. Likewise we assume free space in path 2, that is, $A_{2d} = 1$, $B_{2d} = d$, $C_{2d} = 0$ and $D_{2d} = 1$. Propagation of the optical beams in the Gouy phase shifters, with $z_{R1} \neq z_{R2}$, is schematically shown in Fig. 2. Therefore Eqs. (7) and (8) become

$$\frac{z_{R1}}{z_{R1}^2 + d^2} = \frac{z_{R2}}{z_{R2}^2 + d^2}, \tag{10}$$

$$\frac{z_{R1}^2(1/f) + d + d^2/f}{z_{R1}^2 + d^2} = \frac{d}{z_{R2}^2 + d^2}. \tag{11}$$

These conditions are analogous to those ones imposed by Beijersbergen *et.al.* [6] for a mode converter with two cylindrical lenses in order to obtain beam matching along the two directions of the space at the end of the optical device. In our case beam matching is obtained along the two paths of the MZI by using spherical (or also cylindrical) optical systems. Finally, from Eq. (9) the following expression for the relative Gouy phase between the arms of the MZI (interferometric Gouy phase) is obtained

$$\Phi_{lp} = 2n_{pl}\left[\arctan\left(\frac{d}{z_{R1}}\right) - \arctan\left(\frac{d}{z_{R2}}\right)\right] - \theta. \qquad \textbf{(12)}$$

Eqs. (10) and (11) are easily solved and the following solution is obtained, $z_{R1} = \kappa d$, $z_{R2} = d/\kappa$, $\kappa = \sqrt{1-\gamma}/\sqrt{1+\gamma}$, $\gamma = d/f$. By substituting this solution into Eq. (12) and after a certain calculation the interferometric Gouy phase is given by the expression

$$\Phi_{lp} = (2p + l + 1)\,\Delta\phi - \theta. \qquad \textbf{(13)}$$

with $\Delta\phi = 2\arcsin\gamma$, which characterizes a Gouy phase shifter $G(\Delta\phi)$. Now, by imposing, for sake of expositional convenience, the condition $\Delta\phi = \pi/\sigma, \sigma \in \mathbb{N}^{\star}$, we obtain the following result

$$\gamma = \sin(\pi/2\sigma), \qquad \textbf{(14)}$$

For example, if $\Delta\phi = \pi,\ \pi/2,\ \pi/4$ $(\sigma = 1, 2, 4)$, the corresponding Gouy phase shifters $G(\pi)$, $G(\pi/2)$, $G(\pi/4)$ are implemented if $\gamma = 1$, $\sqrt{2}/2$, $\sin(\pi/8)$. The first case defines a confocal device, that is, $d = f$, which implies $z_{R1} = 0$, $z_{R2} = \infty$. It corresponds to the method of image inversion by refraction [9] or its equivalent of image inversion by reflection [8]. The rest of the cases can not be obtained by image inversion.

Next, it worthwhile to show how we can use the above results to design other symmetric $A_1B_1C_1D_1$ optical systems. The matrix elements of such optical systems can formally be written as follows, $A_{1L} = D_{1L} = \cos\alpha$, $B_{1L} = \sin\alpha/a$, $C_{1L} = -a\sin\alpha$, with $a = (-C_{1L}/B_{1L})^{1/2}$ [14], that is, they are characterized by two independent parameters. Therefore, by taking into account the matrix elements of an optical system formed by two spherical lenses with focal f and separated a distance $L = 2\gamma f$ we obtain the optical parameters

$$a = \frac{2}{f}\frac{(1-\gamma)}{\sqrt{\gamma - \gamma^2}}, \qquad \cos\alpha = 1 - 2\gamma. \qquad \textbf{(15)}$$

As an example, we present results for graded-index lenses since they allow to perform small and compact interferometers [9] highly compatible with optical fibers. For a graded-index lens (or fiber lens) [13] with length l, gradient parameter g_o and base index n_o, the matrix elements are $A_1 = D_1 = \cos g_o l$, $B_1 = \sin g_o l/n_o g_o$, $C_1 = -n_o g_o \sin g_o l$, that is, it is a symmetric optical system and presents a mirror symmetry at $z = l/2$. In this case we have $\alpha = g_o l$ and $a = n_o g_o$. Therefore, from Eq. (15) we obtain that the parameters of the graded-index lens fulfill the relationships $n_o g_o = 2(1-\gamma)/(f\sqrt{\gamma - \gamma^2})$ and $\cos g_o l = 1 - 2\gamma$. For instance, for $\gamma = 1$ we obtain $g_o l = \pi, \forall a = n_o g_o$, which corresponds to a graded index lens of half pitch length which implements a Gouy phase shifter $G(\pi)$. Obviously, a more versatile optical system is obtained by using two or more graded-index lenses separated a variable distance.

On the other hand, we can also consider one-dimensional (cylindrical) optical systems in one arm of the interferometer, that is, optical systems acting only on one direction $\eta = x$ or y (active direction). In this case it is useful, as will be shown, to consider LG optical beams as sum of HG optical beams H_q, $q = m, n$, and accordingly to obtain interferometric Gouy phase of HG beams, which is given, as can be easily verified by taking into account Eq. (6), by the expression

$$\Phi_\eta = (q + \frac{1}{2})\left[2\arctan\left(\frac{B_{1d}}{z_{R1}A_{1d}}\right) - 2\arctan\left(\frac{d}{z_{R2}}\right)\right] - \theta, \quad \textbf{(16)}$$

with $q \equiv m, n$ if $\eta = x, y$, and where η is coincident turn with the active direction of the optical system. The corresponding η-cylindrical Gouy phase shifters will be denoted as $G_\eta(\Delta\phi)$.

Next, we present the application of the previous results to spatial mode mux/demux by using a MZI with a Gouy phase shifter, that is, GPI applied to optical communications. First of all, we recall how a MZI works without any phase shifter. For that, let us consider the modes Ψ_a and Ψ_b coming into the MZI through the horizontal and vertical inputs, respectively, as shown in Fig. 1. We assume, without loss of generality, that the following transformation is obtained after BS1 [15]

$$\Psi_{h_o} = \frac{1}{\sqrt{2}}(\Psi_a - \Psi_b), \qquad \textbf{(17)}$$

$$\Psi_{v_o} = \frac{1}{\sqrt{2}}(\Psi_a + \Psi_b). \qquad \textbf{(18)}$$

By considering that the second Beam Splitter (BS2) implements the same transformation then we obtain, except a global phase, the following results, $\Psi_h = \Psi_b$ and $\Psi_v = \Psi_a$, that is, the modes emerge from BS2 through different outputs, therefore, no mux/demux is obtained. Now, let us consider the LG modes $\Psi_a = \Psi_{00}$ and $\Psi_b = \Psi_{10}$. The corresponding optical beams Ψ_{00} and Ψ_{10} are multiplexed by taking $\Delta\phi = \pi$, that is, a Gouy phase shifter $G(\pi)$. Indeed, according to Eq. (13) the following selective interferometric phases are acquired in the MZI, $\Phi_{00} = \pi$ and $\Phi_{10} = 2\pi$, therefore, $\theta = u\pi, u \in \mathbb{Z}$, has to be chosen. If we consider these selective phases it is easy to prove that $\Psi_h = 0$ and $\Psi_v = \Psi_{00} + \Psi_{10}$, therefore a perfect (without theoretical losses) multiplexing (demultiplexing under backward propagation) is obtained. We must stress that in general the above superposition can be coherent or incoherent, where the incoherent case is usual in spatial mode mux/demux, although for other applications, as optical sensing, a coherent superposition can be needed. Another interesting case corresponds to the input modes $\Psi_a = \Psi_{00}$ and $\Psi_b = \Psi_{03}$. We choose $\Delta\phi = \pi/2$ and therefore $\Phi_{00} = \pi/2$ and $\Phi_{03} = 3\pi + \pi/2$, therefore a perfect multiplexing is again obtained if the common phase $\pi/2$ is compensated with $\theta = (2u+1)\pi/2$. Note that these modes can not be multiplexed by methods based on image inversion [8, 9]. By taking into account these examples it is easy to obtain a first general result. Indeed, let us consider the arbitrary modes $\Psi_a = \Psi_{lp}$ and $\Psi_b = \Psi_{l'p'}$, then the Gouy phases can be written as $\Phi_{lp} = (2p + l + 1)\Delta\phi - \theta$ and $\Phi_{l'p'} = \big(2(p'-p) + (l'-l) + 2p + l + 1\big)\Delta\phi - \theta$, and therefore the common phase can be compensated by $\theta = (2p + l + 1)\Delta\phi - u\pi$. Consequently, the mux/demux is obtained by a Gouy phase shifter characterized by the phase

$$\Delta\phi = \frac{(2v+1)\pi}{2(p'-p) + (l'-l)}, \qquad \textbf{(19)}$$

with $v \in \mathbb{Z}$. For instance, in the case Ψ_{00} and Ψ_{10} we obtain $\Delta\phi = (2u+1)\pi$ (note that $u = 0$ corresponds to the value chosen above), and for Ψ_{00} and Ψ_{03}, $\Delta\phi = (2u+1)\pi/6$ ($u = 1$ corresponds to the value chosen above). Likewise, as commented, we can also obtain mux/demux by using cylindrical optical systems.

It is useful when the parity of the optical beam is different in each spatial direction x and y. For example, let us consider the case $\Psi_a = \Psi_{00}$ and $\Psi_b = \Psi_{10c}$, then with a x-cylindrical Gouy phase shifter $G_x(\pi)$ we obtain, according to Eq. (16), the following Gouy phases $\Phi_{00} = \pi/2$ and $\Phi_{10} = \pi + \pi/2$, therefore, unlike spherical case we have to compensate a phase $\pi/2$ with, for instance, $\theta = \pi/2$.

As a more ambitious example of the GPI possibilites, let us consider the mux/demux of a set of optical modes coming into/from a FMF optical fiber. In particular, the following seven modes: Ψ_{00}, Ψ_{02}, Ψ_{01}, Ψ_{20s}, Ψ_{10u}, Ψ_{10d} and Ψ_{20c}, although, for reasons which will be evident later, we substitute the modes Ψ_{10c} and Ψ_{10s} by the equivalent ones $\Psi_{10u} = (1/\sqrt{2})(\Psi_{10c} + \Psi_{10s})$ and $\Psi_{10d} = (1/\sqrt{2})(\Psi_{10c} - \Psi_{10s})$. Moreover, we will need several concatenated MZIs as inferred from Fig. 3 where a multiplexing protocol, characterized by an increasing sequence of the phase $\Delta\phi$, is shown. First of all, we must stress that modes with $l = 0, p \neq 0$ (radial symmetry) can not be multiplexed by reflective [8] or refractive [9] image inversion systems, however, by using Gouy phase shifters $G(\pi/4)$, $G(\pi/2)$, $G(\pi)$ and $G_{x(y)}(\pi)$ the seven modes can be multiplexed, as shown below. We start by multiplexing the modes Ψ_{00} and Ψ_{02} by using a Gouy phase shifter $G(\pi/4)$. Indeed, the Gouy relative or interferometric phases acquired are: $\Phi_{00} = \pi/4$ and $\Phi_{02} = \pi + \pi/4$, therefore a phase compensation $\theta = 3\pi/4$ can be used, and the optical field $\Psi_{00} + \Psi_{02}$ is obtained (first step in Fig. 3) at one of the the outputs of the interferometer. Next, the optical beams Ψ_{01} and $\Psi_{00} + \Psi_{02}$ can be multiplexed by using a Gouy phase shifter $G(\pi/2)$. It is easily seen that the Gouy relative phases are $\Phi_{01} = \pi + \pi/2$, $\Phi_{00} = \pi/2$ and $\Phi_{02} = 2\pi + \pi/2$, therefore, the total optical field $\Psi_0 = \Psi_{00} + \Psi_{01} + \Psi_{02}$ is obtained at one of the the outputs of the MZI if, for instance, a phase compensation $\theta = \pi/2$ is introduced. Next, we multiplex the optical modes with $l \neq 0$. We start by multiplexing the optical field Ψ_0 with the mode Ψ_{20s} ($l = 2$). By using a Gouy phase shifter $G_x(\pi)$ the mode Ψ_{20s} acquires an interferometric fase $\Phi_{20s} = \pi + \pi/2$, that is, $q = 1$ in Eq. (16), and the optical field acquires an interferometric phase $\pi/2$, although each optical mode acquires a phase $\pi/2\,(\mathrm{mod}\ 2\pi)$, as it can be checked by taking into account the relationships $\Psi_{01} \propto (\mathrm{Y}_{20} + \mathrm{Y}_{02})$ and $\Psi_{02} \propto (\mathrm{Y}_{40} + 2\mathrm{Y}_{22} + \mathrm{Y}_{04})$; therefore if $\theta = \pi/2$ then $\Psi_0 + \Psi_{20s}$ is obtained. Next we multiplex the above optical field with the mode Ψ_{10u} by using a $G(\pi)$, where the mentioned optical field acquires phases π (mod 2π) and the mode a phase $\Phi_{10u} = 2\pi$, therefore we obtain the output $\Psi_0 + \Psi_{20s} + \Psi_{10u}$. By using another $G(\pi)$ we can multiplex the modes Ψ_{10d} and Ψ_{20c}, that is, the relative phases are $\Phi_{10d} = 2\pi$ and $\Phi_{20c} = 3\pi$, and we obtain $\Psi_{10d} + \Psi_{20c}$. Finally, we multiplex the optical fields $\Psi_0 + \Psi_{20s} + \Psi_{10u}$ and $\Psi_{10d} + \Psi_{20c}$. To that end, we consider a rotated reference system $x'y'$, with $x' = (1/\sqrt{2})(x + y)$ and $y' = (1/\sqrt{2})(-x + y)$. In this new reference system the optical fields are: $\Psi_0 + \Psi_{20c} + \Psi_{10c}$ and $\Psi_{10s} + \Psi_{20s}$. Therefore, if we use a Gouy phase shifter $G_{y'}(\pi)$ we obtain a phase $\pi/2$ for Ψ_0, a phase $\pi/2$ for $\Psi_{10c} = \mathrm{Y}_{10}$, and $\pi/2\,(\mathrm{mod}\ 2\pi)$ for $\Psi_{20c} \propto (\mathrm{Y}_{20} - \mathrm{Y}_{02})$. On the other hand we have the phase $\pi + \pi/2$ for $\Psi_{10s} = \mathrm{Y}_{01}$ and $\pi + \pi/2$ for $\Psi_{20s} = \mathrm{Y}_{11}$. Finally, by introducing $\theta = \pi/2$, the seven modes are multiplexed.

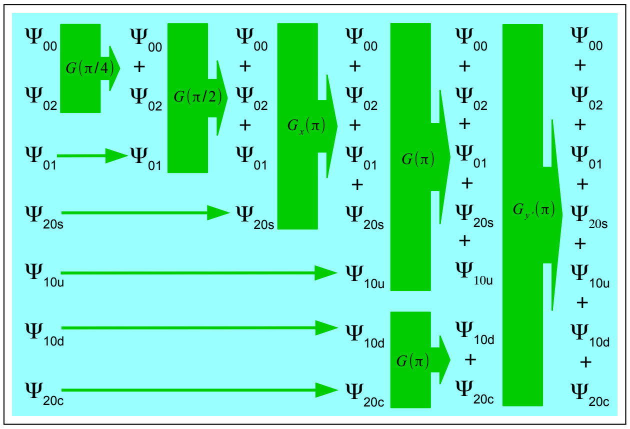

Fig. 3. Protocol of space mode multiplexing/demultiplexing.

In summary, a GPI concept has been presented. It is based on the interferometric Gouy phase acquired by F-Gaussian beams under propagation through refractive optical systems located in the arms, for example, of a MZI. The interferometric Gouy phase depends on the mode number, that is, selective phases are generated and accordingly spatial mode mux/demux in FMF fibers is achieved. It is interesting to note that refractive image inversion corresponds to the particular case of a Gouy phase equal to π. It is interesting underline that Gouy phase shifters can be implemented with graded-index or fiber lenses which are highly compatible with optical fibers and provide small and compact interferometric optical systems. Finally, GPI is a general procedure whose applications in other fields, such as optical sensors, optical metrology, image processing and so on, can be highly promising.

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