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**TESIS DE DOCTORADO**

**ANALYSIS OF TOPICS IN HEALTH MICROECONOMICS  
THROUGH FLEXIBLE REGRESSION MODELS**

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**ESCUELA DE DOCTORADO INTERNACIONAL**

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### ANALYSIS OF TOPICS IN HEALTH MICROECONOMICS THROUGH FLEXIBLE REGRESSION MODELS

D. Francisco Reyes Santías

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Título: Estimating hospital production functions through flexible regression models

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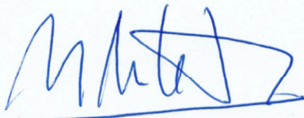
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En Santiago de Compostela, a 7 de mayo de 2019

Fdo: Francisco Reyes Santías



DEDICATORIA

A Fe, la mujer de mi vida

A Rosa, mi madre

A Francisco, mi padre

A Romy, mi hermana





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Finalmente si te encuentras leyendo esto, y de alguna manera no te sientes identificado en ninguno de los párrafos anteriores; gracias a ti también.



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## RESUMEN

*Introducción:* El objeto de discusión es la función de producción de asistencia sanitaria. Del mismo modo, el tiempo de espera para la cirugía electiva es un problema importante en el mundo médico actual.

*Objetivo:* Calcular una nueva función flexible de producción de producción hospitalaria mediante un Modelo Aditivo Generalizado incluyendo interacciones y, compararlo con los modelos clásicos Cobb-Douglas y Translog en la predicción del comportamiento de los factores productivos. Otro tema importante es estudiar cómo el número de camas en el hospital afecta la actividad hospitalaria, la duración de las estancias y, en consecuencia, la lista de espera.

*Material:* Los datos empleados hacen referencia a los hospitales públicos de Galicia para el período 2002-2008, incluyendo el número de camas, el número de facultativos, y el número de altas por DRG así como el número de ingresos y estancias.

*Método:* Se comparan las predicciones de las funciones Cobb-Douglas, Translog, y Modelo Aditivo Generalizado (GAM). Así mismo, reproducir, mediante un análisis de simulación de Monte Carlo, cómo el número de camas de los hospitales (una medida aproximada del capital físico de los hospitales) afecta la actividad de hospitalización, la duración de la estancia y, por consiguiente, el tamaño de las listas de espera de cirugía en los hospitales.

*Resultados:* El modelo GAM es más adecuado que el Cobb-Douglas o el translog para evaluar la función de producción hospitalaria, para los hospitales públicos ubicados en Galicia, para el período de estudio. Parece que no hay diferencias significativas en términos de listas de espera y tasas de ocupación cuando el número de camas de los hospitales de mayor dimensión se incrementa. Las políticas del lado de la oferta también pueden ser decepcionantes en sus efectos sobre los tiempos de espera para los pequeños hospitales rurales. Una mayor capacidad en términos de más camas se asocia con menores tiempos de espera para hospitales medianos.

*Discusión:* Los Modelos Aditivos Generalizados son más flexibles que los modelos paramétricos, proporcionando un mejor ajuste en presencia de relaciones no lineales y

permitiendo, por ello, unos valores de predicción más ajustados. Parece útil analizar la caída en la lista de espera debido a la asignación de camas nuevas, así como el desarrollo de configuraciones de camas más adecuado.

*Conclusiones:* Los resultados de este estudio sugieren que AM es una técnica prometedora para las áreas de investigación y aplicación en economía de la salud. El estudio demuestra la utilidad de las técnicas de simulación para examinar un sistema hospitalario.



## RESUMO

*Introdución:* O obxecto de discusión é a función de produción de asistencia sanitaria. Do mesmo xeito, o tempo de espera para a cirurxía electiva é un problema importante no mundo médico actual.

*Obxectivo:* Calcular unha nova función flexible de produción de produción hospitalaria mediante un Modelo Aditivo Xeneralizado incluíndo interaccións e, comparalo cos modelos clásicos Cobb-Douglas e Translog na predición do comportamento dos factores produtivos. Outro tema importante é estudar como o número de camas no hospital afecta a actividade hospitalaria, a duración das estancias e, en consecuencia, a lista de espera.

*Material:* Os datos empregados fan referencia aos hospitais públicos de Galicia para o período 2002-2008, incluíndo o número de camas, o número de facultativos, e o número de altas por DRG así como o número de ingresos e estancias.

*Método:* Compáranse as predicións das funcións Cobb-Douglas, Translog, e Modelo Aditivo Xeneralizado ( GAM). Así mesmo, reproducir, mediante unha análise de simulación de Monte Carlo, como o número de camas dos hospitais (unha medida aproximada do capital físico dos hospitais) afecta a actividade de hospitalización, a duración da estancia e, por conseguinte, o tamaño das listas de espera de cirurxía nos hospitais.

*Resultados:* O modelo GAM é máis adecuado que o Cobb-Douglas ou o translog para avaliar a función de produción hospitalaria, para os hospitais públicos situados en Galicia, para o período de estudo. Parece que non hai diferenzas significativas en termos de listas de espera e taxas de ocupación cando o número de camas dos hospitais de maior dimensión incrementase. As políticas ao lado da oferta tamén poden ser decepcionantes nos seus efectos sobre os tempos de espera para os pequenos hospitais rurais. Unha maior capacidade en termos de máis camas asóciase con menores tempos de espera para hospitais medianos.

*Discusión:* Os Modelos Aditivos Xeneralizados son máis flexibles que os modelos paramétricos, proporcionando un mellor axuste en presenza de relacións non lineais e

permitindo, por iso, uns valores de predición máis axustados. Parece útil analizar a caída na lista de espera debido á asignación de camas novas, así como o desenvolvemento de configuracións de camas máis adecuado.

*Conclusións:* Os resultados deste estudo suxiren que AM é unha técnica prometedora para as áreas de investigación e aplicación en economía da saúde. O estudo demostra a utilidade das técnicas de simulación para examinar un sistema hospitalario.



## SUMMARY

*Introduction:* The object of discussion is the health care production function. Similarly, waiting time for elective surgery is a major problem in the current medical world.

*Objective:* To calculate a new flexible hospital production function by means of a Generalized Additive Model including interactions and to compare it with the classic models Cobb-Douglas and Translog in the prediction of the behavior of productive factors. Another important issue is to study how the number of beds in the hospital affects the hospital activity, the length of stays and, consequently, the waiting list.

*Material:* The data used refers to the public hospitals of Galicia for the period 2002-2008, including the number of beds, the number of doctors, and the number of registrations by DRG as well as the number of admissions and stays.

*Method:* The predictions of the Cobb-Douglas, Translog, and Generalized Additive Model (GAM) functions are compared. Likewise, to reproduce, by means of a Monte Carlo simulation analysis, how the number of hospital beds (an approximate measure of the physical capital of hospitals) affects the hospitalization activity, the duration of the stay and, therefore, the size of the surgery waiting lists in hospitals.

*Results:* The GAM model is more appropriate than the Cobb-Douglas or the translog to evaluate hospital production functions, for public hospitals located in Galicia and for the study period. It seems that there are no significant differences in terms of waiting lists and occupancy rates when the number of beds in larger hospitals increases. Supply-side policies can also be disappointing in their effects on waiting times for small rural hospitals. Greater capacity in terms of more beds is associated with shorter waiting times for medium size hospitals.

*Discussion:* Generalized Additive Models are more flexible than parametric models, providing a better fit in the presence of non-linear relationships and thus allowing more accurate prediction values. It seems useful to analyze the drop in the waiting list due to the allocation of new beds, as well as the development of a more suitable bed's configuration.

*Conclusions:* The results of this study suggest that AM is a promising technique for the areas of research and application in health economics. The study demonstrates the usefulness of simulation techniques to examine a hospital system.



## RESUMEN EXTENDIDO

### INTRODUCCIÓN:

Los modelos clásicamente usados en Estadística son paramétricos y para ello se ha de suponer que la muestra de observaciones proviene de una familia paramétrica conocida. En estos casos, el problema es estimar los parámetros desconocidos o hallar tests de hipótesis o intervalos de confianza para los mismos. Esta suposición puede ser relativamente fuerte porque el modelo paramétrico supuesto puede no ser el correcto ya que los datos pueden ser tales que no exista una familia paramétrica adecuada que proporcione un buen ajuste. Por otra parte, los métodos estadísticos desarrollados para un modelo paramétrico particular pueden llevar a conclusiones erróneas cuando se aplican a un modelo ligeramente perturbado (falta de robustez de los datos respecto del modelo). Estos problemas llevaron a la tendencia de desarrollar métodos no paramétricos o semiparamétricos para analizar los datos. Un modelo paramétrico razonable produce inferencias precisas mientras que un modelo erróneo posiblemente conducirá a conclusiones equivocadas. Por otro lado, los modelos no paramétricos si bien están asociados con alta estabilidad tienen menor precisión. Recientemente, los modelos no paramétricos han ganado una importante atención en el estudio de fenómenos naturales con comportamiento de complejidad no lineal.

El análisis de estos modelos requiere de técnicas de suavizado multivariadas para la función  $m$  y por lo tanto, encuentra a la hora de aplicarlo el problema conocido como “la maldición de la dimensionalidad” que está asociado al hecho de que cuando estamos estimando considerando un entorno con un número fijo de datos, y tenemos una superficie de gran dimensión, dicho entorno puede ser demasiado grande como para ser llamado local; hecho que produce grandes sesgos. Es decir, se necesita un número exponencialmente mayor de datos para que dichos entornos contengan observaciones de la muestra.

En los últimos años, para resolver este problema, diversos autores han tratado el problema de reducción de la dimensión de las covariables en modelos de regresión no paramétrica. Hastie y Tibshirani (1990) introdujeron los modelos aditivos que

generalizan los modelos lineales, resuelven el problema de “la maldición de la dimensión” y además son de fácil interpretación. Este nuevo planteamiento combina la flexibilidad de los modelos no paramétricos con la simple interpretación del modelo lineal estándar.

La intención de la primera Fase del trabajo de investigación ha sido testar estos modelos comparados con los modelos paramétricos, a las funciones de producción hospitalarias.

El programa estadístico que se ha empleado en las dos fases del trabajo de investigación que se llevado a cabo ha sido el programa de software libre R. El programa R es un entorno de análisis y programación estadístico que forma parte del proyecto de software libre GNU (General Public Licence). R está disponible en la dirección <http://www.r-project.org>. El proyecto R comenzó en 1995 por un grupo de estadísticos de la universidad de Auckland, dirigidos por Ross Ihaka y Robert Gentleman. R, está basado en el lenguaje de programación S, y está diseñado específicamente para la programación de tareas estadísticas en los años 80 por los Laboratorios Bell AT&T. El lenguaje S se considera un lenguaje de programación estadística de alto nivel orientado a objetos. Frente a otros lenguajes de programación, R, es sencillo, intuitivo y eficiente ya que se trata de un lenguaje interpretado (a diferencia de otros como Fortran, C++, Visual Basic, etc.). Como programa de análisis estadístico, R-base permite realizar tareas estadísticas sencillas habituales y además permite extensiones que implementan técnicas estadísticas avanzadas. De este modo se cubre las necesidades de cualquier analista, tanto en el ámbito de la estadística profesional como en el de la investigación estadística. R consta de un sistema base pero la mayoría de las funciones estadísticas vienen agrupadas en distintos packages que se incorporan de forma opcional. Para los métodos de regresión no paramétrica existen funciones disponibles en el package básico stats, no obstante, el uso más adecuado de dichos métodos puede conseguirse a través de funciones incorporadas en varios packages adicionales y actualmente disponibles en la web. Entre estos packages destacan kernSmooth, locpol, np, locfit, loess, sm, lowess, gam y recientemente se ha incluido uno para Smooth Backfitting, el sBF.

El estudio de la primera Fase del Trabajo de Investigación, se describe a continuación:

ANTECEDENTES: El objeto de la Fase I del Plan de Investigación son las funciones de producción de los servicios asistenciales sanitarios. La esencia de las funciones de producción en la economía se refiere a la capacidad productiva y la eficiencia de los factores de producción. Si la capacidad y la eficiencia están relacionadas con varios factores de producción, estaremos evaluando una función de producción de varias variables o, especialmente, podríamos analizar una función relativa a un solo factor de producción, es decir, una función de producción de una variable. El término función de producción se refiere a la relación física entre la organización de los recursos productivos y el resultado en forma de bienes o servicios por unidad de tiempo. Dos modelos son los comunmente utilizados en la estimación de la función de producción hospitalaria (Rosko and Broyles 1988): el modelo Cobb-Douglas y el modelo trascendental logarítmico (modelo translog). Cobb-Douglas ha venido siendo muy popular entre los economistas debido a su sencillez en el cálculo. Sin embargo trabajos teóricos y empíricos han cuestionado frecuentemente la validez del modelo paramétrico Cobb-Douglas como representación de la producción de servicios de asistencia sanitaria (López casanovas, 1988). En comparación con el modelo Cobb-Douglas, el modelo translog presenta la ventaja de añadir a la función los efectos de la interacción entre inputs. El modelo Cobb-Douglas omite estos efectos. Por ello, muchos estudios sobre funciones de producción han empleado la función translog (Rosko and Broyles 1988; McGuire 1987). Sin embargo, en algunas circunstancias, los modelos paramétricos como el Cobb-Douglas o el translog pueden resultar muy restrictivos. Al utilizar estos modelos para la estimación y la predicción, su forma funcional es forzada a seguir una forma lineal paramétrica, que frecuentemente no ajusta correctamente a los datos. La relativa falta de flexibilidad de los modelos paramétricos, ha llevado al desarrollo de técnicas de regresión no paramétricas basadas en la familia de los Modelos Aditivos Generalizados (GAMs; Hastie and Tibshirani, 1990; Wood, 2006). Estas técnicas no imponen una forma paramétrica para los predictores, en su lugar, asumen solo que esos efectos son aditivos y razonablemente suavizados que pueden ser estimados empleando una variedad de métodos suavizados no paramétricos. El texto de Deaton and Muellbauer (1980b) provee varios ejemplos

en microeconomía con estudios de funciones aditivas con unas formulaciones estadísticas y unos resultados para la economía muy deseables. La utilidad de los GAM en aplicaciones prácticas ha sido demostrada en múltiples áreas de investigación, con referencia, entre otras disciplinas, a la biología, la medicina (Hastie and Tibshiani 1990; Wood 2006) y también la economía y las finanzas (Härdle et al 2004);

**OBJETIVOS:** El principal objetivo de este estudio es el cálculo de una nueva función flexible de producción para los hospitales y los servicios clínicos y quirúrgicos hospitalarios. En segundo lugar se examina (a) las principales fortalezas y debilidades de las diferentes formas funcionales empleadas en este estudio, la flexible y las clásicas Cobb-Douglas y Translog; (b) cómo los clusters de hospitales responden a los diferentes modelos de funciones de producción hospitalaria y, finalmente, evaluar la capacidad de predicción de los análisis AM respecto del comportamiento de las funciones clásicas, (C) lo anterior para las funciones de producción de los servicios clínicos (médicos y quirúrgicos);

**METODOLOGÍA:**

**MATERIAL.** Input y Output de la producción hospitalaria. Las variables empleadas en el estudio han sido los inputs, entendidos como capital y trabajo, y los outputs de la producción hospitalaria. En esta investigación se ha utilizado como medida de la producción hospitalaria el número de ingresos estandarizado por la complejidad o case-mix, obteniendo una unidad homogénea de producción llamada UNIDAD DE PRODUCCIÓN HOSPITALARIA (UPH), cuyo cálculo se obtiene multiplicando el número de ingresos por su complejidad obtenida de los pesos de los DRG (López et al, 1999), abordando de este modo la necesidad de tener en cuenta la complejidad de los diferentes hospitales y ajustando en mayor medida el output de la producción de cada hospital. Siguiendo a Ferrier y Valmanis (2004) los inputs de los hospitales: para el input de capital se usa el número de camas para cada hospital y cada año, obteniendo estos datos de las estadísticas hospitalarias oficiales. El input del trabajo se mide como el número de especialistas hospitalarios en la plantilla de cada hospital a 31 de diciembre de cada año. *Datos.* Los datos han sido recogidos y organizados como datos

de panel del sistema de información del Servizo Galego da Saúde completados con las estadísticas de las memorias de los propios hospitales del Servizo Galego da Saúde. Las variables del estudio son: Número de camas total y por servicio, Número de médicos especialistas total y por servicio, Número de ingresos y altas por DRG (total y por servicio), Cluster de inclusión para cada hospital, Número y tipología de los servicios hospitalarios médicos y quirúrgicos por cluster;

**MÉTODOS ESTADÍSTICOS.** Modelos clásicos. La función Cobb-Douglas que fue estimada por Charles W. Cobb y Paul H. Douglas (1928), aunque ya fue anticipada por Knut Wicksell (1901, 1923). Un problema que presenta esta función es la omisión del cambio en la tecnología de producción. Un procedimiento estándar para introducir en la función el cambio tecnológico es incluir el tiempo de la serie. Una alternativa para la ecuación Cobb-Douglas es la función de producción Translog (Christensen, Jorgenson y Lau, 1973). Al igual que en la ecuación Cobb-Douglas, se asume que los efectos del progreso tecnológico son neutros. El nuevo modelo aditivo flexible: Los Modelos Aditivos Generalizados flexibles (GAMs) son útiles como predictores en relaciones funcionales para diferentes grupos de datos sin necesidad de establecer a priori un modelo funcional específico. Estos modelos combinan la habilidad para explorar varias relaciones no paramétricas simultáneamente, a través de la flexibilidad que proporcionan las distribuciones de los Modelos Lineales Generalizados (GLM, McCullagh y Nelder, 1989). En nuestro modelo, la variable de respuesta es continua. En este caso, el Modelo Aditivo Generalizado es referido habitualmente en la literatura estadística como Modelo Aditivo. De este modo, en el presente estudio denotaremos el modelo flexible como AM. Este estudio plantea el modelo tanto para los Cluster de hospitales como para los servicios médicos. *Hospitales.* El modelo flexible considerado ha sido el AM incluyendo la interacción camas-facultativos. Las covariables “Hospital” y “Año” han sido incluidas también en el modelo. Atendiendo a la estimación del modelo (3) penalized thin plate splines (Wood, 2004, 2006<sup>a</sup>, 2006<sup>b</sup>) serán utilizados para representar las funciones suavizadas. La representación del modelo mixto de un penalizado AM ha sido considerado (Wood, 2004). En esta configuración, los parámetros suavizados serán estimados vía REML (Wahba, 1982; Wang, 1998; Lin and Zhang, 1999). *Servicios Médicos.* El modelo flexible a

considerar ha sido una función suavizada desconocida que representa la posible interacción entre el número de camas y el número de facultativos (ambos en escala logarítmica). La covariable “Año” será incluida también en el modelo;

La segunda Fase del Trabajo de Investigación, aborda uno de los problemas asociados a la “producción hospitalaria” como es la Lista de Espera, a través de un estudio de Simulación, con un modelo de cadenas de Markov para generar una simulación de Montecarlo que nos permitió estimar el comportamiento del sistema a través de la aplicación del modelo a la lista de Espera en los hospitales del SERGAS.

El estudio de la segunda Fase del Trabajo de Investigación, se describe a continuación:

ANTECEDENTES: Una lista de espera es una cola de pacientes a los que se les ha indicado un procedimiento asistencial y que por imperativos ajenos a su voluntad, deben esperar a ser atendidos un periodo de tiempo variable (Sampietro & Espallargues 2001 ). Las listas de espera aparecen en diferentes ámbitos (atención primaria y especializada), en distintos niveles asistenciales (ambulatorio y hospitalario) y afectan a distintos tipos de procedimientos terapéuticos (quirúrgicos y no quirúrgicos), diagnósticos y rehabilitadores (Churruca 2000). Pero de forma generalizada siempre se ha hablado de lista de espera haciendo alusión a la lista de espera quirúrgica. Esta ha sido motivo de estudio con mayor frecuencia, alegando mayores problemas de morbi-mortalidad, así como, económicos en relación a la administración (Instituto Nacional de la Salud 1998). Existe poca, pero sólida evidencia para explicar el crecimiento de la demanda de servicios sanitarios. Los cambios en la estructura poblacional, las presiones en el nivel primario de asistencia, las expectativas de los pacientes y la habilidad creciente para tratar a los pacientes son frecuentemente citados como posibles razones del aparente aumento de la demanda asistencial (Nigel & Hensher 1998). Las listas de espera se ven condicionadas por dos verdades incuestionables como son, por un lado, la limitación de los recursos (Manté i Fors 2002) y por el otro, por el crecimiento de la demanda. De las listas de espera se derivan consecuencias de morbimortalidad, sociales, políticas y económicas. En el año 1999, se publicó una revisión sistemática de estudios observacionales, que incluía 87 trabajos publicados en diversos países, con el objetivo de valorar la influencia de la

espera en la supervivencia. Esta revisión ponía de manifiesto que en los pacientes con más de tres meses de demora quirúrgica para el tratamiento del cáncer de mama, la supervivencia se veía reducida en un 10% a los 5 años, en relación con aquellos pacientes con operaciones más tempranas (Richards 1999 ). En cuanto a las consecuencias socio-políticas, un estudio prospectivo publicado en Munich en 1996 con relación al riesgo inherente a las listas de espera demostró que el riesgo de mortalidad de los pacientes en espera de un *by-pass* coronario era de 1.3% por mes y que para minimizar el riesgo de muerte en lista de espera, el *by-pass* debe ser realizado dentro de la primera semana tras el diagnóstico de la angiografía coronaria (Silber y colbs. 1999). La simulación se ha aplicado a las listas de espera en cirugía programada. En el año 2001 se publica un modelo de simulación que optimiza la gestión de los quirófanos programados en el Hospital General de Denia (De la Morena y colbs. 2001). En el año 2002, (Everett 2002) se utilizó un modelo de simulación por ordenador, basado en la construcción de bloques de categorías de pacientes en relación con el tipo de procedimiento que precisaban y al grado de prioridad. Esta última simulación se realizó “día a día” durante un periodo de 1000 días La aplicación de ambos modelos puso de manifiesto el potencial de la simulación en la estrategia y planificación de recursos hospitalarios;

**OBJETIVOS:** Simular los ingresos de pacientes en cada hospital gallego y la duración de sus estadias. Esto nos permitirá analizar cómo se comportan la lista de espera y el índice de ocupación del centro. Además el estudio debía considerar distinto número de camas, para evaluar cómo afecta este factor en la ocupación y en la lista de espera. En concreto, si  $n_i$  es el número de camas del hospital  $i$  se tomaron como valores para el número de camas:  $\{n_i, n_i + 5\%n_i, n_i + 10\%n_i, \dots, n_i + 50\%n_i\}$ .

**MATERIAL Y MÉTODOS:** Los datos de los ingresos que se han obtenido hacen referencia a los ingresos realizados por el hospital durante cada uno de los 365 días del año 2013. Los datos de las estancias muestran el número de enfermos que han tenido una estancia de un día, de dos y así hasta la máxima estancia para un enfermo en el año 2013 que habrá sido de 353 días. La simulación realizada asumirá una distribución de los ingresos de poisson. Se empleó el programa de simulación R. La exposición del trabajo de simulación se hizo explicando los diferentes problemas que el autor tuvo

que abordar para concluir el trabajo y los diferentes pasos que se han dado para resolverlos. Una vez leídos los datos por el programa R, se ha indicado, para los ingresos, qué días corresponden a festivos o no festivos (los días de julio, agosto y diciembre y los sábados, y domingos). Esto se hizo para estimar la distribución de poisson para cada grupo de días. De este modo estimamos el parámetro de poisson que coincide con la media y para ello se ha estimado la media muestral. Esto lo utilizamos en la simulación que hicimos del número de ingresos en un día. Respecto a los datos sobre estancias, en primer lugar se han leído los datos al igual que hemos hecho con los ingresos, identificamos las frecuencias y los datos de los valores de las estancias. A continuación, transformamos las frecuencias que tenemos y las convertimos en frecuencias relativas. Acumulamos las frecuencias, lo que utilizamos después para simular las estancias. Para no partir de un hospital vacío, decidimos plantear una primera simulación, para obtener número de camas ocupadas y el número de estancias simuladas para cada paciente, de modo que la simulación objeto de este trabajo se encuentre con un escenario más realista de camas ocupadas.

En el estudio se simulan datos para los años 2013-2016. Para este período, se han realizado 500 iteraciones en cada una de las cuales se simularon ingresos diarios de pacientes con sus estadías asociadas. Durante lo proceso, se ha construido una lista de espera y el índice de ocupación diario. Para evitar comenzar la simulación con los hospitales “vacíos” (situación poco realista), se comenzó a simular el proceso 1 de agosto de 2006. Las estadías de cada paciente se simularon mediante el estimador no paramétrico tipo kernel de la densidad (con la bandwidth que se calculó anteriormente). Para simular los ingresos, se extrajeron valores de normales con distintos parámetros según el día sea laborable o festivo. Como veíamos anteriormente, el número de camas parece estar relacionado con el patrón de ingresos diarios de los centros. Por eso, hicimos dos estudios diferentes. En una primera aproximación simulamos los ingresos sin tener en cuenta el número de camas que se están considerando (sin elasticidad). Para eso, generamos los ingresos mediante las distribuciones normales que ajustamos anteriormente a los datos. Sin embargo, como se vino anteriormente, el número de camas de un centro parece afectar a los ingresos diarios de pacientes. Para tener en cuenta este efecto, adaptamos el procedimiento de

simulación de ingresos mediante lo ajuste lineal que obtuvimos: la distribución normal que se usará para generar los datos tendrá por media  $c_1 + c_2 * n^{\circ}$  de camas y por desviación típica  $c_3 + c_4 * n^{\circ}$  de camas, donde los coeficientes  $c_j$  son los obtenidos nos ajustes lineales que se realizaron anteriormente (con elasticidad).

Para ello, creamos un vector con los días de la semana, de modo que cada uno de los días de la simulación tenga asignado un día de semana: le pedimos que repitiera la secuencia de los días 4,5,6,7,1,2,3 para los días de la simulación. Hicimos lo mismo para los meses. A continuación, construimos una matriz con el mismo número de filas que el número de camas y el mismo número de columnas que el número de días. La rellenamos con valores que significan que la cama esta vacía. La celda nos dice, si está en ese valor, que la cama está vacía y si está con un número, este representa el número de días que la cama aún estará ocupada. Inicializamos otro vector, que nos da el número de ingresos que tendremos cada día y el número de personas que cada día están esperando que una cama quede vacía y puedan ingresar. Para eso, el primer día tiene un 0 que representa que nadie está esperando el primer día de la simulación inicial. El bucle funciona día a día y nos dice que si el mes es 7, 8 o 12, es decir meses que consideramos como festivos (por tener una tasa más baja de ingresos que el resto de los meses del año) o bien si el día de la semana es 6 o 7 (sábado o domingo, por los mismos motivos) entonces el valor que tomamos es el parámetro de los días festivos, en caso contrario empleamos el landa de los días laborales. A continuación generamos el número de ingresos y generamos un valor de una distribución de poisson con parámetro lambda y después le sumamos la lista de espera que tengamos para ese día. Guardamos los ingresos nuevos sin tener en cuenta la lista de espera. A continuación, miramos qué camas libres había. Para ello seleccionamos la columna correspondiente al día en el que estamos, vemos si la cama está vacía y también qué camas exactamente están vacías; con lo que sabemos el número de camas libres. Ahora observamos si el número de ingresos es mayor que el número de camas libres. En la lista de espera del día siguiente, guardamos el número de personas en lista de espera y, por otro lado, guardamos el número de persona que vamos a ingresar realmente. En otro caso, la lista de espera será 0. Si tenemos ingresos, con un nuevo bucle le

asignamos a los pacientes las camas libres y simulamos sus estancias. Seleccionamos cada ingreso y generamos un uniforme. El intervalo  $[0, 1]$  lo dividimos en segmentos de longitudes igual a las frecuencias relativas que habíamos obtenido para las estancias. Observamos cada uniforme a que segmento del intervalo corresponde y le asignamos al individuo  $j$  la estancia correspondiente. Para ver si la estancia obtenida supera el número de días hasta el término de la simulación, en el caso de que lo supere calculamos el número de días de las estancias generadas que caen dentro del período que estamos simulando, por eso escogemos el mínimo entre la estancia que generamos y el número de días que quedan hasta acabar el período. Además, en la fila correspondiente a una cama libre que tengamos, la ocupamos con la estancia que generamos y rellenamos las casillas desde el día de hoy, el día del ingreso hasta el final de la estancia, con un número que nos indica el número de días que falta para que le den el alta al paciente. A continuación se inicia la simulación objetivo. Guardamos el valor que teníamos en esa cama el último día de la simulación inicial. Si lo que teníamos en esa cama el último día de la simulación inicial es un valor distinto de 0, es decir, si esa cama estaba ocupada, rellenamos la matriz actual con esa estancia y los días siguientes con los días que faltaban para su alta. No miramos la diferencia entre el valor de la estancia y el final del período, porque no tenemos ningún caso de paciente con estancias de más de 1000 días. En la lista de espera tenemos como primer valor el último valor de la simulación inicial. Introducimos el cálculo del índice de ocupación (porcentaje de camas ocupadas en un año) anual y mensual, para cada uno de los cuatro años. Por ejemplo, en el cuarto año de cada repetición, desde la columna 1 hasta la 365 identificamos si la cama está ocupada o no (1, 0). Sumamos estos valores y nos da el total del número de camas ocupadas en el año y dividimos esto por el número de camas (395) multiplicado por el número de días del año. Este es el Índice de Ocupación Anual. Para el Índice de Ocupación Mensual, hacemos lo mismo, pero en lugar de seleccionar toda la matriz, solo seleccionamos las columnas correspondientes a cada mes  $x$  de la matriz. Sumamos los valores de las columnas cuyo mes es el  $x$  y nos da el número de camas ocupadas en el mes, esto se divide por el número total de camas multiplicado por el número de días que tiene el mes y obtenemos el porcentaje de camas ocupadas por mes. Calculamos también, para todo el período, el porcentaje de días que tienen una ocupación del 100% de las camas, desde el primer día hasta el

último día, observando si las camas están todas ellas ocupadas o no (1, 0), y calculamos la media de estos días para todo el período. Esto es el porcentaje de días con una ocupación de camas del 100%”.





# AN ANALYSIS OF TOPICS IN THE MICROECONOMICS OF HEALTH USING FLEXIBLE REGRESSION MODELS

## 1. EXTENDED ENGLISH SUMMARY

The models classically used in statistics are parametric, for which it is assumed that the sample of observations comes from a known parametric family. In these cases, the problem is to estimate the unknown parameters and to find hypothesis tests or confidence intervals for them. This assumption may be relatively strong because the assumed parametric model may not be correct since the data may be such that there is no suitable parametric family that provides a good fit. On the other hand, the statistical methods developed for a particular parametric model can lead to erroneous conclusions when applied to a slightly disturbed model (lack of robustness of the data with respect to the model). These problems led to the development of nonparametric or semiparametric methods to analyze the data. A reasonable parametric model produces precise inferences while an erroneous model may lead to false conclusions. On the other hand, non-parametric models, although associated with high stability, have less precision. Recently, nonparametric models have attracted significant attention in the study of natural phenomena with behaviour of non-linear complexity.

The analysis of these models requires multivariate smoothing techniques for the  $m$  function and therefore, when applying this smoothing, the problem known as "the curse of dimensionality" is associated with the fact that where an estimation relates to an environment with a fixed amount of data, and with a large area, this environment may be too large to be considered local; a fact that produces large biases. That is, an exponentially larger amount of data is needed for these environments to contain the observations of the sample.

In recent years, to solve this problem, several authors have addressed the problem of reducing the dimension of covariates in non-parametric regression models. Hastie and

Tibshirani (1990)<sup>1</sup> introduced additive models that generalize the linear models, solving the problem of "the curse of dimensionality" and they are also easy to interpret. This new approach combines the flexibility of non-parametric models with the simple interpretation of the standard linear model.

The intention of the first Phase of this research work has been to test these models compared with the parametric models, for the functions of hospital production. The statistical program used in the two phases of the research work that has been carried out is the free software program R. The R program is an environment of statistical analysis and programming that is part of the GNU free software project (General Public License). R is available at <http://www.r-project.org>. Project R was established in 1995 by a group of statisticians from the University of Auckland, led by Ross Ihaka and Robert Gentleman. R is based on the programming language S, and was designed specifically for the programming of statistical tasks in the 1980s by Bell Laboratories AT & T. The S language is considered a high-level object-oriented statistical programming language. In contrast to other programming languages, R is simple, intuitive and efficient since it is an interpreted language (unlike others such as Fortran, C ++, Visual Basic, etc.). As a statistical analysis program, R-base allows to carry out common simple statistical tasks and also offers extensions that implement advanced statistical techniques. In this way, the needs of any analyst are covered, both in the field of professional statistics and in the field of statistical research. R consists of a base system, but most of the statistical functions are grouped in different packages that are incorporated as an option. For non-parametric regression methods there are functions available in the stats basic package, however, the most appropriate use of these methods can be achieved through functions incorporated in several additional packages and currently available on the web. These packages include kernSmooth, locpol, np, locfit, loess, sm, lowess, gam and recently one has been included for Smooth Backfitting, the sBF.

The study of the first Phase of the Research Work is described below:

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<sup>1</sup> Hastie, T.J.; Tibshirani, R.J. (1990) Generalized Additive Models, Chapman and Hall, London

BACKGROUND: The purpose of Phase I of the Research Plan is to analyse the production functions of health care services. The essence of the functions of production in the economy refers to the productive capacity and the efficiency of the factors of production. If capacity and efficiency are related to several factors of production, economists will be evaluating a production function of several variables or they could analyze a function relative to a single production factor, that is, a production function of a variable. The term production function refers to the physical relationship between the organization of productive resources and the result in the form of goods or services per unit of time. Two models are commonly used in the estimation of the hospital production function (Rosko and Broyles 1988)<sup>2</sup>: the Cobb-Douglas model and the logarithmic transcendental model (translog model). Cobb-Douglas has been very popular among economists because of its simplicity in calculation. However, theoretical and empirical studies have frequently questioned the validity of the Cobb-Douglas parametric model as a representation of the production of health care services (López Casanovas 1988). In comparison with the Cobb-Douglas model, the translog model has the advantage of adding to the function the effects of the interaction between inputs. The Cobb-Douglas model omits these effects. For this reason, many studies on production functions have used the translog function (Rosko and Broyles 1988<sup>3</sup>, McGuire 1987<sup>4</sup>). However, in some circumstances, parametric models such as Cobb-Douglas or translog can be very restrictive. When using these models for estimation and prediction, their functional form is forced to follow a parametric linear form, which frequently does not adjust correctly to the data. The relative lack of flexibility of the parametric models has led to the development of nonparametric regression techniques based on the family of Generalized Additive Models (GAMs, Hastie and Tibshirani 1990<sup>5</sup>, Wood 2006<sup>6</sup>). These techniques do not

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<sup>2</sup> Rosko, M.D.; Broyles, R.W. (1988) *The Economics of Health Care: A Reference Handbook*, Greenwood Press, Inc., New York, Westport, CT, 1988

<sup>3</sup> Rosko, M.D.; Broyles, R.W. (1988) *The Economics of Health Care: A Reference Handbook*, Greenwood Press, Inc., New York, Westport, CT, 1988

<sup>4</sup> McGuire, A. (1987) The measurement of hospital efficiency, *Social Science and Medicine* 24, 719–724

<sup>5</sup> Hastie, T.J.; Tibshirani, R.J. (1990) *Generalized Additive Models*, Chapman and Hall, London

<sup>6</sup> Wood, S.N. (2006) *Generalized Additive Models: An Introduction with R*, Chapman and Hall/CRC Press

impose a parametric form for the predictors, instead, they assume only that these effects are additive and reasonably smooth and can be estimated using a variety of nonparametric smoothing methods. A paper by Deaton and Muellbauer (1980b)<sup>7</sup> provides several examples in microeconomics with studies of additive functions with statistical formulations and very desirable results for the economy. The usefulness of GAMs in practical applications has been demonstrated in multiple research areas, with reference, among other disciplines, to biology, medicine (Hastie and Tibshiani 1990<sup>8</sup>, Wood 2006<sup>9</sup>) and also economics and finance (Härdle et al 2004<sup>10</sup>).

**OBJECTIVES:** The main objective of this study is the calculation of a new flexible production function for hospitals and hospital clinical and surgical services. Secondly, the main strengths and weaknesses of the different functional forms used in this study, the flexible and the classic Cobb-Douglas and Translog, are examined (a); (b) how the hospital clusters respond to the different models of hospital production functions and, finally, assess the prediction capacity of the AM analysis with respect to the behaviour of the classical functions, (c) the above for the production functions of clinical services (medical and surgical).

#### METHODOLOGY:

**MATERIAL.** The input and output of hospital production: The variables used in the study were the inputs, understood as capital and labour, and the outputs of hospital production. In this investigation, the number of patients admitted standardized by the complexity or case-mix has been used as a measure of hospital production, obtaining a homogeneous unit of production called the HOSPITAL PRODUCTION UNIT (HPU), whose calculation is obtained by multiplying the number of inpatients by its complexity obtained from the weights of the DRG (Diagnostic related Groups) (López

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<sup>7</sup> Deaton, A.S.; Muellbauer, J. (1980b), An almost Ideal Demand System, *American Economic Review*, 70, 312-326

<sup>8</sup> Hastie, T.J.; Tibshirani, R.J. (1990) *Generalized Additive Models*, Chapman and Hall, London

<sup>9</sup> Wood, S.N. (2006) *Generalized Additive Models: An Introduction with R*, Chapman and Hall/CRC Press

<sup>10</sup> Härdle, W., Müller, M.; Sperlich, S.; Werwatz, A. (2004). *Nonparametric and semiparametric models*. Berlin: Springer-Verlag

et al 1999<sup>11</sup> ), thus addressing the need to take into account the complexity of the different hospitals and adjusting to a greater extent the production output of each hospital. Following Ferrier and Valmanis (2004)<sup>12</sup>, regarding the inputs of hospitals: for the input of capital the number of beds for each hospital and each year is used, obtaining this data from official hospital statistics. The input of labour is measured as the number of hospital specialists among the staff of each hospital at 31 December of each year.

DATA. The data has been collected and organized as panel data by the information system of the Servizo Galego da Saúde, complete with the statistics for the hospitals of the Servizo Galego da Saúde. The variables of the study are: Total number of beds and per service, Total number of medical specialists and per service, Number of admissions and discharges according to the DRG (total and per service), Cluster of inclusion for each hospital, and Number and typology of medical and surgical hospital services per cluster;

STATISTICAL METHODS. Classic models: The Cobb-Douglas function was estimated by Charles W. Cobb and Paul H. Douglas (1928)<sup>13</sup>, although it was already anticipated by Knut Wicksell (1901<sup>14</sup>, 1923<sup>15</sup>). A problem with this function is the omission of the change in production technology. A standard procedure for introducing technological change into the function is to include the time of the series. An alternative to the Cobb-Douglas equation is the production function Translog (Christensen, Jorgenson and Lau 1973<sup>16</sup>). As in the Cobb-Douglas equation, it is

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<sup>11</sup> López Rois, F.J.; Mateo, R.; Gómez, J.R.; Ramón, C.; Pereiras, M. (1999) Methodological criteria for drawing up a contract-programme or singular sector based agreement of specialized care using HPUs, Secretara Xeral SERGAS, Consellera de Sanidade e Servicios Sociais, Xunta de Galicia, Santiago de Compostela

<sup>12</sup> Ferrier, G.; Valmanis, V. (2004) Do mergers improve hospital productivity? *Journal of the Operational Research Society* 55, 1071–1080

<sup>13</sup> Cobb, C. W., and P. H. Douglas. 1928. A theory of production. *American Economic Review* 18:139–65

<sup>14</sup> Wicksell, K. (1901). *Lectures on Political Economy*. Vol. 1, Translated by E. Classen, London: George Routledge & Sons

<sup>15</sup> Wicksell, K. (1923). *Realkapital und Kapitalzins*, Review of *Realkapital und Kapitalzins*, by Gustaf Akerman, English translation by E. Classen as appendix 2 of Wicksell (1901) 1934, 258-99

<sup>16</sup> Christensen, L.R.; Jorgenson, D.W.; Lau, L.J. (1973) Transcendental logarithmic production *Frontiers, The Review of Economics and Statistics* 55, 28–45

assumed that the effects of technological progress are neutral. The new flexible additive model: Flexible Generalized Additive Models (GAMs) are useful as predictors in functional relationships for different groups of data without the need to establish a specific functional model a priori. These models combine the ability to explore several nonparametric relationships simultaneously, through the flexibility provided by the distributions of Generalized Linear Models (GLM, McCullagh and Nelder 1989<sup>17</sup>). In our model, the response variable is continuous. In this case, the Generalized Additive Model is usually referred to in the statistical literature as an Additive Model. Thus, in the present study we will denote the flexible model as AM. This study proposes the model for both hospital Clusters and medical services.

Hospitals: The flexible model considered has been the AM including the beds-facultative interaction. The covariates "Hospital" and "Year" have also been included in the model. Based on the estimation of the model penalized thin plate splines (Wood 2004<sup>18</sup>, 2006a<sup>19</sup>, 2006b<sup>20</sup>) will be used to represent smoothed functions. The representation of the mixed model of a penalized AM has been considered (Wood 2004<sup>21</sup>). In this configuration, the smoothed parameters will be estimated via REML (Wahba 1985<sup>22</sup>, Wang 1998<sup>23</sup>, Lin and Zhang 1999<sup>24</sup>).

Medical services: The flexible model to be considered has been an unknown smoothed function that represents the possible interaction between the number of beds and the

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<sup>17</sup> McCullagh, P.; Nelder, JA. (1989) Generalized Linear Models. London: Chapman & Hall

<sup>18</sup> Wood, S.N. (2004) Stable and efficient multiple smoothing parameter estimation for generalized additive models. *Journal of the American Statistical Association*. 99:673- 686

<sup>19</sup> Wood, S.N. (2006a) Low rank scale invariant tensor product smooths for generalized additive mixed models. *Biometrics*. 62(4):1025-1036

<sup>20</sup> Wood S.N. (2006b) Generalized Additive Models: An Introduction with R. Chapman and Hall/CRC Press

<sup>21</sup> Wood, S.N. (2004) Stable and efficient multiple smoothing parameter estimation for generalized additive models. *Journal of the American Statistical Association*. 99:673- 686

<sup>22</sup> Wahba, G. (1985) A Comparison of GCV and GML for choosing the smoothing parameter in the generalized spline smoothing problem. *Ann. Statist.* 13:1378-1402

<sup>23</sup> Wang, Y. (1998) Mixed effects smoothing spline analysis of variance *J.R. Statist. Soc. B.* 60, 159-174

<sup>24</sup> Lin, X. and Zhang, D. 1999. Inference in generalized additive mixed models using smoothing splines. *J.R. Statist. Soc. B.* 61, 381-400

number of physicians (both in logarithmic scale). The covariate "Year" will also be included in the model.

The second phase of the research work addresses one of the problems associated with "hospital production" namely the waiting list, through a simulation study, with a Markov chain model to generate a Monte Carlo simulation that allowed to estimate the behaviour of the system through the application of the model to the waiting list in the SERGAS hospitals.

The study of the second Phase of the Research Work is described below:

**BACKGROUND:** A waiting list consists of the patients who have been indicated for an assistance procedure and who, due to imperatives beyond their control, must wait for a variable period of time (Sampietro and Espallargues 2001<sup>25</sup>). Waiting lists appear in different areas (primary and specialized care), at different levels of care (ambulatory and hospital) and affect different types of therapeutic procedures (surgical and non-surgical), diagnostics and rehabilitation (Churruca 2000<sup>26</sup>). But in a generalized way waiting lists have often been referred to in alluding to the surgical waiting list.

This has been the most frequent subject of study, implying greater problems of morbidity and mortality, as well as economic problems in relation to the administration (Instituto Nacional de la Salud 1998<sup>27</sup>). There is little, but solid, evidence to explain the growth in the demand for health services. Changes in the population structure, pressures at the primary care level, patient expectations and the increasing ability to treat patients are frequently cited as possible reasons for the apparent increase in the demand for care (Nigel and Hensher 1998<sup>28</sup>). Waiting lists are conditioned by two unquestionable truths, on the one hand, the limitation of resources

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<sup>25</sup> Sampietro, L.; Espallargues, M. (2001) 'Nuevas fórmulas para dar solución a las listas de espera', *El Médico*, 15 June of 2000: 60-64. <http://www.diariomedico.com/sanidad/listas/debate.html>

<sup>26</sup> Churruca, S. (2000) 'Los líderes en gestión de listas de espera advierten: un suceso no puede condicionar la estrategia', *Diario Médico*, 15/06/2000. Available at <http://www.diariomedico.com/sanidad/listas/debate.html>

<sup>27</sup> INSALUD (1998). Guía para la gestión de la lista de espera quirúrgica. Madrid : Instituto Nacional de la Salud, Área de Estudios, Documentación y Coordinación Normativa

<sup>28</sup> [Edwards](#), N.; Hensher, M. (1998). Managing demand for secondary care services: the changing context. *BMJ*. Jul 11; 317(7151): 135–138

(Manté and Fors 2002<sup>29</sup>) and, on the other, by the growth in demand. Morbidity, social, political and economic consequences derive from the waiting lists. In 1999, a systematic review of observational studies was published, which included 87 papers published in various countries, with the aim of assessing the influence of waiting on patient survival. This review showed that in patients with more than three months of surgical delay for the treatment of breast cancer, survival was reduced by 10% at 5 years, in relation to those patients with earlier operations (Richards 1999<sup>30</sup>). Regarding the socio-political consequences, a prospective study published in Munich in 1996 regarding the risk inherent in waiting lists showed that the mortality risk of patients waiting for a coronary bypass was 1.3% per month and that to minimize the risk of death on the waiting list, the by-pass should be performed within the first week after the diagnosis by coronary angiography (Silber et al 1999<sup>31</sup>).

Simulation has been widely applied to waiting lists in scheduled surgery. In 2001, a simulation model was published that optimizes the management of the operating theatres at the General Hospital of Denia (De la Morena et al 2001<sup>32</sup>). In 2002, a computer simulation model was used (Everett 2002<sup>33</sup>), based on the construction of blocks of patient categories in relation to the type of procedure they required and the degree of priority. This last simulation was carried out "day by day" over a period of 1,000 days. The application of both models showed the potential of simulation in the strategy and planning of hospital resources.

**OBJECTIVES:** To simulate the admission of patients in each Galician hospital and the duration of their stays. This will allow us to analyze how the waiting list behaves and

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<sup>29</sup> Manté i Fors, C. (2002). "Las listas de espera. Un problema de todos". El periódico de Cataluña. Cataluña 20 de mayo

<sup>30</sup> Richards, M.A. (1999) "The influence on survival of delay in the presentation and treatment of symptomatic breast cancer". *British Journal Cancer*, vol. 79, pp. 858-64

<sup>31</sup> Sigmund, S.; Mühlhng, H.; Dörr, R.; Zindler, G.; Preuss, A.; Stümpfl, A. (1996) 'Waiting times and death on the waiting list for coronary artery bypass operation. Experiences in Munich with over 1,000 patients', *Herz* 21(6): 389-396

<sup>32</sup> De la Morena, E., Martínez, A., García, P., Vicente Gimeno, J.& Toledo, J. (2001) "Infratilización de quirófano de cirugía programada en el Hospital General de Denia: simulación artesanal". *Revista Española de Sistemas*. Diciembre Vol. 2, núm. 1, pp. 35-45

<sup>33</sup> Everett J.E. (2002) "A Decision Support Simulation Model for the Management of an Elective Surgery Waiting System". *Health Care Management Science* vol5, pp. 89-95

the occupation index of the centre. In addition, the study should consider different numbers of beds, to evaluate how this factor affects occupation and the waiting list. Specifically, regarding the number of hospital beds, the following were taken as values for the number of beds:  $\{n_i, n_i + 5\% n_i, n_i + 10\% n_i, \dots, n_i + 50\% n_i\}$

**MATERIAL AND METHODS:** The admissions data obtained refer to the hospital's admissions during each of the 365 days of 2013. The data on the patients' stays show the number of patients who had a stay of one day, two days and so on until the maximum stay for a patient in 2013 that was 353 days. The simulation performed will assume a Poisson distribution of admissions. The simulation program R was used. The simulation work is presented explaining the different problems that the author had to face in order to complete the work and the different steps that were taken to solve them. Once the data was read by the R program, it was indicated, for the patient admissions, which days correspond to holidays or non-holidays (the days of July, August and December and Saturdays and Sundays). This was done to estimate the Poisson distribution for each group of days. In this way, we estimate the Poisson parameter that coincides with the mean and for this the sample mean was estimated. We used this in the simulation on the number of admissions in one day. Regarding the data on patient stays, first we read the data as we did with the admissions, then we identified the frequencies and the data on the values of the stays. Next, we transformed the frequencies converting them into relative frequencies. By accumulating the frequencies, we were able to simulate the stays. In order not to permit an empty hospital, we decided to run an initial simulation to obtain the number of beds occupied and the number of simulated stays for each patient, so that the simulation object of this work met a more realistic scenario of occupied beds.

In the study, data are simulated for the years 2013-2016. For this period, 500 iterations were performed, in each of which the daily admission of patients with their associated stays was simulated. During the process, a waiting list and the daily occupation index were created. To avoid starting the simulation with "empty" hospitals (unrealistic situation), the process began to simulate on 1 August 2006. The stays of each patient were simulated using the non-parametric kernel density estimator (with the bandwidth that was calculated previously). To simulate the admissions, standard values with

different parameters were extracted according to whether it was a working day or holiday. As we saw earlier, the number of beds seems to be related to the centre's daily admission pattern. This is why we conducted two different studies. In a first approximation we simulate the admissions without taking into account the number of beds that are being considered (without elasticity). For this, we generate the admissions through the normal distributions that we previously adjusted to the data. However, as previously mentioned, the number of beds in a centre seems to affect the daily admissions of patients. To take this effect into account, we adapted the admissions simulation procedure by means of the linear adjustment we obtained: the normal distribution that will be used to generate the data will have mean  $c_1 + c_2 * n^{\circ}$  of Beds

and standard deviation  $c_3 + c_4 * n^{\circ}$  of Beds, where the coefficients are obtained from linear adjustments that were performed previously (with elasticity).

To do this, we created a vector with the days of the week, so that each day of the simulation is assigned a week day: we repeated the sequence of days 4,5,6,7,1,2,3 for the days of the simulation. We did the same for the months. Next, we created an array with the same number of rows as the number of beds and the same number of columns as the number of days. We filled it with 0 values that mean that the bed is empty. The cell tells us, if it has that 0 value, that the bed is empty and if it contains a number, it represents the number of days that the bed will still be occupied. We initialized another vector, which gives us the admission number we will have each day and the number of people each day waiting for a bed to be empty so they can enter. Therefore, the first day has the value 0 that represents that nobody is waiting for the first day of the initial simulation. The loop works day by day and tells us that if the month is 7, 8 or 12, that is to say months that we consider as holidays (because there is a lower admission rate than in the remaining months of the year) or if the day of the week is 6 or 7 (Saturday or Sunday, for the same reasons), then the value we take is the parameter for the holidays, otherwise we use the lambda for the working days. Next we generate the number of admissions and generate a value of a Poisson distribution with lambda parameter and then we add the waiting list we have for that day. We consider the new admissions without taking into account the waiting list. Next, we

looked at what free beds there were. For this we select the column corresponding to the day we are in, we see if the bed is empty and also which beds precisely are empty; with what we know about the number of free beds. Now we see if the admission number is greater than the number of free beds.

On the waiting list the next day, we keep the number of people on the waiting list and, on the other hand, we keep the number of people we are going to actually be admitted to hospital. In another case, the waiting list will be 0. If we have admissions, with a new loop we assign patients free beds and simulate their stays. We select each admission and generate a report. The interval  $[0, 1]$  is divided into segments of lengths equal to the relative frequencies we obtained for the stays. We observe each report to which each segment of the interval corresponds and we assign to the individual  $j$  the corresponding stay. To see if the stay obtained exceeds the number of days until the end of the simulation, in the event that it exceeds the number of days of the generated stays that fall within the period we are simulating, we choose the minimum between the stay that we generate and the number of days remaining until the end of the period. In addition, in the row corresponding to a free bed that we have, we occupy it with the bed we generate and fill in the boxes from the current day, the day of admission until the end of the stay, with a number that tells us the number of days remaining until the discharge of the patient. The objective simulation is then started. We kept the value we had for that bed on the last day of the initial simulation. If what we had for that bed on the last day of the initial simulation is a value other than 0, that is, if that bed was occupied, we filled in the current matrix with that stay and the following days with the days remaining until discharge. We do not look at the difference between the value of the stay and the end of the period, because we have no patient case with a stay of more than 1,000 days. For the waiting list we have as the first value the last value of the initial simulation. We now introduce the calculation of the occupation index (percentage of beds occupied in a year), annual and monthly, for each of the four years. For example, in the fourth year of each repetition, from column 1 to 365 we identify whether the bed is occupied or not (1, 0). We add these values to give us the total number of beds occupied in the year and we divide this by the number of beds (395) multiplied by the number of days of the year. This is the Annual Occupancy Index. For the Monthly Occupancy Index, we do the same, but instead of selecting the

entire matrix, we only select the columns corresponding to each month  $x$  of the matrix. We add the values of the columns relating to month  $x$  to obtain the number of beds occupied in the month, this is divided by the total number of beds multiplied by the number of days in the month and we obtain the percentage of beds occupied per month. We also calculate, for the whole period, the percentage of days that have 100% occupation of the beds, from the first day to the last day, observing whether the beds are all occupied or not (1, 0), and we calculate the average of these days for the entire period. This is the percentage of days with 100% bed occupancy.

#### DISCUSSION:

Guillem López Casasnovas (1988)<sup>34</sup> has suggested that the objective that all planning pursues is to achieve an efficient use of resources. In this sense, a relevant question is whether or not the combination of factors used in terms of their minimum cost combination is optimal. The intention is, therefore, to know if we are in a situation of economic efficiency, using the inputs so that their marginal productivities have a ratio identical to that of their relative prices. An approximation to the answer to this question could be given in terms of the results derived from the estimation of the production function.

As indicated by Salvador Peiró (2000)<sup>35</sup>, the strategies for dealing with waiting lists can be classified according to the reduction of the list (or waiting time) or its management. Among the first, structural increases in resources include the opening / expansion of new hospitals or services, but also of specific care modalities (surgery without admission, day hospital, home hospitalization, minor surgery in primary care, etc.), as well as the organizational, technological and other reforms that have in common a stable increase in productive capacity, either by increasing resources, by increasing productivity or by both factors together. In general, there is some agreement that an increase in the productive capacity tends to shorten the average wait, but the

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<sup>34</sup> López Casasnovas, G. (1988) "La combinación de los factores productivos en el hospital: una aproximación a la función de producción". *Investigaciones Económicas (Segunda época)* vol. XII, nº 2, pags 305-327

<sup>35</sup> Peiró, S. (2000) "Algunos elementos para el análisis de las listas de espera". *Gestión Clínica y Sanitaria*, Invierno, vol. 2, nº 4, pags 126-131

number of people on the waiting list increases<sup>3637</sup> and this theory is corroborated by most of the studies<sup>383940</sup> and the evolution of empirical experiences.

The temporary increases in availability, either by extending the days or hours of the operating theatre, the injection of specific funds to the public hospitals and referrals to the private sector are the most commonly used formulas in Spain for temporarily increasing resources, usually to act on specific waiting lists.

In principle, it is expected that they act as structural increases, although subject to their own incentives according to the schemes used. For example, a study in Spain showed that afternoon sessions with payments for intervention did not reduce productivity the next day (requirement to establish the contract), but the type of interventions selected tended to maximize payments to doctors and was practically inverse to the clinical priority of interventions<sup>41</sup>.

Article: Applying a simulation model in order to manage waiting lists for hospital inpatient activity in an EU region.

Kroneman and Siegers (2004)<sup>42</sup>, in their study on the effect of reducing hospital beds in ten European countries, found that admission rates seem to be sensitive to bed provision, with a positive elasticity of 1.44; countries with the highest bed supply show higher admission rates. The same has been found in the present study where admission rates seem to be sensitive to the supply of beds.

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<sup>36</sup> Pope C (1999). Cutting queues or cutting corners: waiting lists and the 1990 NHS reforms. *BMJ* 1992; 305: 577-9; Glazer A, Rothenberg LS. Increased capacity may exacerbate rationing problems: with applications to medical care. *J Health Economics*; 18: 671-80

<sup>37</sup> Goldacre MJ, Lee A, Don B. (1987) Waiting list statistics I: Relation between admissions from waiting list and length of waiting list. *BMJ*; 295: 1105-1108

<sup>38</sup> Newton JN, Henderson J, Goldacre MJ. (1995). Waiting list dynamics and the impact of earmarked funding. *BMJ*; 311: 783-5

<sup>39</sup> Dowling B. (1997). Effect of fundholding on waiting times: database study. *BMJ* Aug 2; 315 (7103): 290-2

<sup>40</sup> Hamblin R, Harrison A, Boyle S. (1998). Waiting lists. The wrong target. *Health Serv J* Apr 2; 108 (5598): 28-31

<sup>41</sup> Márquez S, Portella E. (1994). Evaluación de un programa de reducción de la lista de espera quirúrgica basado en el pago por acto. *Med Clin (Barc)*; 103: 169-73

<sup>42</sup> Kroneman, M.; Siegers, J.J. (2004) The effect of hospital bed reduction on the use of beds: a comparative study of 10 European countries, *Social Science & Medicine* 59, 1731–1740

It seems that there are no significant differences in terms of waiting lists and occupancy rates when the number of beds in teaching hospitals are increased. In this sense, Zeraati et al. (2005)<sup>43</sup> suggested that an increase in the supply of hospital beds tends to generate additional demand, either in the form of more inpatients or patients treated for longer periods of time or a combination of both. This is known as Roemer's Law, after a study by Roemer (1961)<sup>44</sup> that reported on a natural experiment in which a sudden increase in hospital beds in a country, without changes in other factors, led to a sharp increase in usage rates.

Supply policies can also be disappointing in their effects on waiting times for small rural hospitals. The same results have been found by Siciliani and Hurst (2003)<sup>45</sup> who show that a common experience is to take measures aimed at reducing waiting times by increasing activity, only to discover that after a short period the demand has increased and waiting times have returned to levels similar to those before the introduction of measures, as demand responds positively to reductions in waiting times. The same conclusions have been demonstrated by Donald et al. (2003)<sup>46</sup> who found that, although staffing and other capacity constraints may make it difficult to reduce waiting times, the increase in capacity will not necessarily achieve sustained reductions in wait times because changes in work practices and accurate measurement of capacity is also needed to ensure that changes in other parts of the health system do not reduce the effect of additional capacity.

On the other hand, our study shows that greater capacity in terms of more beds is associated with lower waiting times for medium sized hospitals. Likewise, Siciliani and Hurst (2003)<sup>47</sup> found that, on the supply side, the pronounced and prolonged

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<sup>43</sup> Zeraati, H.; Zayeri, F.; Babae, G.; Khanafshar, N.; Ramezanzadeh, F. (2005) Required hospital beds estimation: a simulation study, *Journal of Applied Sciences* 5, 1189–1191

<sup>44</sup> Roemer, M.I. (1961) Bed supply and hospital utilization: a national experiment, *Hospitals: JAHA* 35, 988–993

<sup>45</sup> Siciliani, L.; Hurst, J. (2003) Explaining waiting times variations for elective surgery across OECD countries, *OECD Health Working Papers* 7

<sup>46</sup> Donald, J.; Huby, C.; Maxwell, D. (2003). The outpatient waiting times problem (and the solutions), waiting for elective admission, *Review of national findings, Audit Commission, London*

<sup>47</sup> Siciliani, L.; Hurst, J. (2003). Explaining waiting times variations for elective surgery across OECD countries, *OECD Health Working Papers* 7

reductions in the long waiting times for coronary revascularization surgery in Denmark were due to significant increases in activity, supported by increases in capacity. The same results were found by Martin and Smith (1999)<sup>48</sup> and Lindsay and Feigenbaum (1984)<sup>49</sup>, which show that econometric evidence suggests that greater capacity, in terms of more beds and doctors, is associated with shorter waiting times.

Although simulation offers the opportunity to investigate the effect of many different alternatives in situations where real experiments are impossible, or too expensive, slow or risky, according to Álvarez and Centeno (1999)<sup>50</sup>, the main reason for the reluctance of the health care industry to accept simulation was the reluctance of the administration to reduce a complex process in the field of medical care to a model representation.

Article: Estimating hospital production functions through flexible regression models

The decision to measure hospital production by AM (Additive Model) was an attempt to improve the flexibility for the functional form. The proposed model is certainly a simplified version of the complete specification of the econometric model (some other variables, in fact, may affect the phenomenon analyzed), but, also at this preliminary stage, the results obtained are really close to the desirable hypotheses.

A selected set of simple production indicators has been analyzed. These indicators were compared in different types of hospitals. This comparative analysis provides important information about the different variations between hospitals.

While medium and small primary care hospitals are almost homogeneous in terms of bed productivity, large hospitals represent a more complex trend in bed productivity. Among the types of hospitals, the AM shows great variability in the consultants' productivity. The interpretation of these results is surely an interesting instrument for decision makers in order to analyze the productive conditions of each hospital and the

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<sup>48</sup> Martin, S.; Smith, P.C. (1999). Rationing by waiting lists: an empirical investigation, *Journal of Public Economics* 71 141–164

<sup>49</sup> Lindsay, C.M.; Feigenbaum, B. (1984). Rationing by waiting lists, *American Economic Review* 74, 404–417

<sup>50</sup> Álvarez, A.; Centeno, M. (1999). Enhancing simulation models for emergency rooms using VBA, *Winter Simulation Conference*, 1685–1693

health sector as a whole. In addition, AMs can also be applied to verify the performance of classic models.

The results of this study suggest that AM is a promising technique for areas of research and application in health economics.

In addition, the results allow us to characterize the areas in which our approach can be effective, such as those related to demand, costs and utility functions in health care.

Article: Comparing Some Production Functions for Inpatient Health Services in Selected Public Hospitals in Spain

Although empirical studies of hospital efficiency have often included an estimation of the production functions using Cobb-Douglas or translog models, they have neglected both the possibility of using more flexible models such as AM, and the comparison of the various possible models. In this document, we have tried to fill these two gaps. Our findings suggest that, although for some inpatient health services in selected public hospitals in Spain, a Cobb-Douglas or translog model is adequate (the translog model is best if there is significant synergy or anti-synergy between the capital and labour inputs, the Cobb-Douglas model otherwise), this is not always the case. Specifically, we show in this study that the most flexible AM is always superior to any of the others, but although its superiority was only marginal for the gynaecology and obstetrics service, it was very clear for the traumatology and orthopaedic surgery service, with intermediate status for the other two services considered.

One reason for this trend may be revealed by examining the average HPU per patient in each service in one of the participating hospitals (the Juan Canalejo Hospital), which contributes one third of the total number of HPU in the study, ranging from 0.67 in the gynaecology and obstetrics service up to 1.51 in internal medicine, 1.54 in general and digestive surgery, and 2.34 in orthopaedic surgery<sup>51</sup> (Juan Canalejo University Hospital Complex 2002). As the average resources needed per patient correlate well with the average duration of hospitalization, this finding suggests that the root cause of the observed trend is oversimplification in the assumption that

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<sup>51</sup> Complejo Hospitalario Universitario Juan Canalejo. (2002). *Memoria anual 2001*. Santiago de Compostela, Spain: Xunta de Galicia

production depends only on FTC (Full-Time Consultant) (the input of work) and the number of beds (capital inflow). When the average product (i.e., hospital production units) grows due to the increasing influence of operating costs not captured by labour, the observed relationship between the total hospital production units on one side and the FTCs and beds on the other becomes complex, and this complexity can be captured by AMs but not by Cobb-Douglas or translog models. In particular, in the case of orthopaedic surgery in this study, it seems likely that the larger services were more productive due to the referral of patients requiring longer hospitalization periods.

It is worth noting that while the production of inpatient health services depends on both work and capital inputs in two of the services examined (general and digestive surgery, as well as traumatology and orthopaedic surgery), in the other two it depends fundamentally on the number of beds.

Under the reasonable assumption that there was no shortage of demand for any of these services during the study period, this indicates that the number of medical personnel was not a limiting factor for the productivity of the internal medicine service or of the gynaecology and obstetrics service. In other words, while an increase in medical personnel would have been necessary to take full advantage of any increase in the number of beds in both general and digestive surgery and in trauma and orthopaedic surgery services, it would not have been necessary to increase personnel in the internal medicine service or gynaecology and obstetrics service, in which an increase in medical personnel would in fact have had little effect unless accompanied by an increase in the number of beds. The capacity of the analysis of the production function to highlight this type of relationship between inputs has been emphasized by Jensen and Morrissey (1986)<sup>52</sup> and by Thurston and Libby (2002)<sup>53</sup> and Morikawa (2010)<sup>54</sup>, among others, while Hellinger (1975)<sup>55</sup> pointed out that incorrectly analysing

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<sup>52</sup> Jensen, G. A., and M. A. Morrissey. (1986). The role of physicians in hospital production. *Review of Economics and Statistics* 63:432–42

<sup>53</sup> Thurston, N., and A. Libby. (2002). A production function for physician services revisited. *The Review of Economics and Statistics* 84:184–91

<sup>54</sup> Morikawa, M. (2010). *Economies of scale and hospital productivity: An empirical analysis of medical area level panel data*. RIETI Discussion Paper Series 10-E-050

<sup>55</sup> Hellinger, F. (1975). Specification of a hospital production function. *Applied Economics* 7:149–60

the specific production functions leads to incorrect estimates of economies of scale and substitution relations between inputs.

Finally, our results suggest several considerations of a more general nature. Flexible AMs promise interesting advantages for research in other areas of health economics in addition to the particular application considered in this study, especially as models of demand, cost, and utility functions. Even when the data handled is susceptible to more traditional techniques, AMs can, as in the present study, provide useful verification of the validity of these less sophisticated methods. More empirical and simulation studies should be conducted to explore in greater detail the relative merits of traditional and data-based econometric methods.

Article: Bed capacity and surgical waiting lists: a simulation analysis

The expansion of the physical capacity of the hospital (building new surgical units, for example) is a long-term policy that may require time to be implemented. The increase in the work force in the health sector can be even slower, since doctors and specialists need several years training before they become active. Although personnel can be hired from abroad, these personnel may face assimilation difficulties, so that such a policy can also take time. This means that different ways of increasing supply will generally involve different costs and will require different time scales. In the short term, the purchase of additional activity from public facilities at low marginal cost may be possible if there is additional capacity. However, if public facilities are already operating near their maximum capacity, the purchase of additional activity in the short term will only be possible at high marginal costs.

In the medium and long term, it may be cheaper to expand activity by expanding public capacity. For example, Denmark adjusted its public capacity to respond to the increased demand for coronary revascularization procedures more quickly than England in the 1990s. Consequently, waiting times for revascularization were reduced in Denmark, while they increased abruptly in England (Siciliani and Hurst 2003)<sup>56</sup>.

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<sup>56</sup> Siciliani, L.; Hurst, J. (2003), Explaining waiting times variations for elective surgery across OECD countries, OECD Health Working Papers 7

It is argued that, in principle, waiting times can be reduced through supply policies, if the volume of surgery is not considered adequate, or by policies on the demand side, if the volume of surgery is considered adequate. Policies on the supply side include increasing production capacity by increasing the number of beds and specialists, or by using the capacity available in other (private) hospitals. They also include increasing productivity by financing additional activities, encouraging day surgery and linking the remuneration system of doctors and hospitals with their performance (Hurst and Siciliani, 2006)<sup>57</sup>.

However, it is common to take measures to reduce waiting times by increasing activity, and then find that, after a short period, demand has increased and waiting times have reverted to levels similar to those before the implementation of the measures. Such responses can be difficult to overcome, since demand responds positively to reductions in waiting times. This is the result presented by the present study. Through the simulation analysis, we observed that if the demand pattern is not modified in relation to the increase in the number of beds, a (slight) increase in the number of beds would significantly help to reduce the surgical waiting lists. Particularly, a 10 percent increase in the number of available beds would reduce surgical waiting lists of variable size from 62 percent to 5 percent for the period studied. In this context, this increase in production capacity would be highly effective. However, if the demand responds positively to the increase in supply, then each increase in the number of beds (regardless of size) would have little impact on the reduction of the waiting list.

The flow (supply) of elective surgery depends on the surgical capacity, both public and private, and the productivity with which the capacity is used. The econometric evidence (cross-sectional and national) suggests that greater capacity – in terms of a greater number of beds and doctors – is associated with lower waiting times.

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<sup>57</sup> Hurst, J.; Siciliani, L. (2006) Tackling excessive waiting times for elective surgery: a comparison of policies in twelve OECD countries, OECD Health Working Paper, No. 6

Martin and Smith (1999)<sup>58</sup> provide evidence on the impact of capacity through an English database of Hospital Episode Statistics in 1991/92. These authors showed that the waiting time is negatively associated with the number of beds available. In particular, they found that the elasticity is equal to -0.242. Similarly, Lindsay and Feigenbaum (1984)<sup>59</sup> found that waiting times are negatively associated with the number of doctors and beds available.

In addition, Álvarez and Centeno (1999)<sup>60</sup> used a simulation analysis for the Washington Adventist Hospital to evaluate the impact of an expansion in the number of beds in the Emergency Room. It was found that an expansion in the number of beds resulted in a reduction of 0.6 hours of average stay. Kirtland et al. (1995)<sup>61</sup> used simulation to improve performance by reducing the patient's time in the system and determining adequate levels of staffing. Eleven alternatives were studied and resulted in a reduction of 38 minutes on average.

However, larger increases in capacity may have a different impact on waiting times depending on the level of excess demand and the initial waiting time. It is likely that countries with low supply and high initial waiting times have an elastic demand for variations in the waiting time. For this reason, the effect of even greater increases in the capacity of waiting times can be quite modest (Hurst and Siciliani 2006)<sup>62</sup>.

In general, supply-side policies may succeed in their goal of increasing the elective surgery rate, but they may be disappointing in their effects on waiting times. This is because an increase in supply may continue instead of generating an increase in demand, or may be overcome by further increases in demand. In addition, any

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<sup>58</sup> Martin, S.; Smith, P.C. (1999) 'Rationing by waiting lists: an empirical investigation', *Journal of Public Economics* 71(1): 141-64

<sup>59</sup> Lindsay, C.M.; Feigenbaum, B. (1984) 'Rationing by waiting lists', *American Economic Review* 74(3): 404-417

<sup>60</sup> Alvarez, A.M.; Centeno, M. (1999) Enhancing simulation models for emergency rooms using VBA, Winter Simulation Conference, 1685-1693

<sup>61</sup> Kirtland, A.; Poisker, K.; Stamp, L.; Wolfe, P. (1995), Simulating an ED 'is as much fun as?', Proceedings of Winter Simulation Conference, C. Alexopoulos, K. Kang, W.R. Lilegdon and D. Goldsman (eds.), Institute of Electrical and Electronics Engineers, Washington, 1039-1042

<sup>62</sup> Hurst, J.; Siciliani, L. (2006) Tackling excessive waiting times for elective surgery: a comparison of policies in twelve OECD countries, OECD Health Working Paper, No. 6

reduction in waiting times may encourage an increase in enrolment rates due to reduced clinical thresholds (Hurst and Siciliani 2006)<sup>63</sup>.

In addition, many commenters have suggested that an increase in the supply of hospital beds tends to generate additional demand either in the form of more patients admitted or patients treated for longer periods of time, or a combination of both (Zeraati et al 2005)<sup>64</sup>. Previously, Shain and Roemer (1959)<sup>65</sup> found very close correlations between the availability of short-term general hospital beds per 1,000 inhabitants and the use rates measured by hospital days per 1,000 inhabitants.

Later, Roemer (1961)<sup>66</sup> also reported on a natural experiment in which a sudden increase in hospital beds in a country – without changes in other factors – led to an abrupt increase in utilization rates.

Using data from 10 hospitals in European countries, Kroneman and Siegers (2004)<sup>67</sup> [67] found that hospital activity increased by 1.44 percent for each 1 percent increase in the number of hospital beds. Taking into account the elasticity of the activity of beds–inpatients, the results regarding the impact of beds on waiting times offered by our study differ widely from those obtained without considering the impact of the capacity of beds in the hospital activity. An increase in bed capacity does not lead to significant differences in waiting lists and occupancy rates, even for huge bed capacity increases. Our simulation model reveals the small effect that supply policies can have on waiting lists when the demand for medical care adapts to the new supply conditions.

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<sup>63</sup> Hurst, J.; Siciliani, L. (2006) Tackling excessive waiting times for elective surgery: a comparison of policies in twelve OECD countries, OECD Health Working Paper, No. 6

<sup>64</sup> Zeraati, H.; Zayeri, F.; Babae, G.; Khanafshar, N.; Ramezanzadeh, F. (2005) 'Required hospital beds estimation: A simulation study', *Journal of Applied Sciences* 5(7): 1189-1191

<sup>65</sup> Shain, M.; Roemer, M.I. (1959) 'Hospital costs relate to the supply of beds', *Modern Hospital* 92(4): 71-73

<sup>66</sup> Roemer, M.I. (1961) 'Bed supply and hospital utilization: A natural experiment', *Hospitals* 35, 988-993

<sup>67</sup> Kroneman, M.; J. Siegers, J.J. (2004) 'The effect of hospital bed reduction on the use of beds: A comparative study of 10 European countries', *Social Science & Medicine* 59(8): 1731-1740

## CONCLUSIONS

1. The study shows the usefulness of simulation techniques to examine a hospital system and, in particular, daily hospital activity and the length of stay. In addition, it has been very useful to analyze the reduction in the waiting list, due to the grouping in the allocation of new beds; as well as the development of bed configurations more directly suitable for grouping.
2. The results in this study suggest that AM is a promising technique for the areas of research and application in health economics. In addition, the results allow us to characterize the domains in which our approach can be effective, such as those related to demand, costs and useful functions in health care.
3. The flexibility of additive models offers interesting advantages for research in other areas of health economics. Even when the data handled are susceptible to more traditional techniques, AMs can, as in the present study, provide useful verification of the validity of less sophisticated methods, such as Cobb-Douglas and the Translogarithmic models.
4. On the other hand, if hospital admission patterns are affected by the number of beds, each increase in the number of beds, including notable increases, lead to no significant differences in hospital occupation rates or hospitalization rates and waiting lists.



## 2. CALCULATION OF THE PRODUCTION FUNCTION OF HOSPITALS AND SURGICAL AND CLINICAL HOSPITAL SERVICES THROUGH FLEXIBLE REGRESSION MODELS

### 2.1. INTRODUCTION.

The objects of our discussion are the production functions for health care services. The essence of functions of production in the economy refers to the productive capacity and efficiency of factors of production. If capacity and efficiency are related to several factors of production, we will be evaluating a production function with several variables or, in particular, we may analyse a function relative to a single production factor, that is, a production function with one variable.

The analysis of production functions has been used by economists since the 1930s to study efficiency, and it is one of the econometric methods most commonly used by health economists (Eastaugh, 1992)<sup>68</sup>. In relation to these production models, the concept is defined first, then the model is specified, then the input and output are measured and, finally, the function is used for the measurement of hospital efficiency.

The analytical expression of these functions is the quantitative expression of the success of the management in achieving the economic objectives of a hospital. For this, it is necessary to provide a design of the form and dimensions of the analytical expression. In this context, it is interesting to examine the behaviour of the function for the relevant measures of the result, such as product, profit or benefit, and the relevant elements of the investment, such as the consumption of capital goods. Since the product is the first element of the result, the production function expresses the form and character of the interdependence between the product and the factors of production.

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<sup>68</sup> Eastaugh, S. (1992) *Health economics: Efficiency, Quality, and Equity*. Westport Conn. Auburn House Pub. Co

The term production function refers to the physical relationship between the organization of productive resources and the result, in the form of goods or services per unit of time.

Two models are commonly used in the estimation of the hospital production function (Rosko & Broyles, 1988)<sup>69</sup>: the Cobb–Douglas model and the logarithmic transcendental model (translog model).

Cobb–Douglas has been very popular among economists because its calculation is simple. However, theoretical and empirical studies have frequently questioned the validity of the Cobb–Douglas parametric model as a representation of the production of health care services (López Casanovas, 1988)<sup>70</sup>.

In comparison with the Cobb–Douglas model, the translog model has the advantage of adding the effects of the interaction between inputs to the function. The Cobb–Douglas model omits these effects. Therefore, many studies on production functions have used the translog function (McGuire, 1987<sup>71</sup>; Rosko & Broyles, 1988<sup>72</sup>).

However, in some circumstances, parametric models such as Cobb–Douglas or translog can be very restrictive. When using these models for estimation and prediction, their functional form is forced to follow a parametric linear form, which frequently does not fit the data correctly.

The relative lack of flexibility of parametric models has led to the development of nonparametric regression techniques based on the family of generalized additive models (GAMs: Hastie & Tibshirani, 1990<sup>73</sup>; Wood, 2006<sup>74</sup>). These techniques do not impose a parametric form for the predictors; instead, they assume only that those

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<sup>69</sup> Rosko M D, Broyles R W. (1988) *The Economics of Health Care: A Reference Handbook*. New York. Westport, CT: Greenwood Press, Inc

<sup>70</sup> Lopez Casanovas G, Wagstaff A. (1988) La combinación de los factores productivos en el hospital: una aproximación a la función de producción. *Investigaciones Económicas (Segunda época)*. Vol. 12, nº 2: 305-327

<sup>71</sup> McGuire, A. (1987) The Measurement of Hospital Efficiency. *Social Science and Medicine* 24(9): 719–24

<sup>72</sup> Rosko M D, Broyles R W. (1988) *The Economics of Health Care: A Reference Handbook*. New York. Westport, CT: Greenwood Press, Inc

<sup>73</sup> Hastie TJ, Tibshirani RJ. (1990) *Generalized Additive Models*. London: Chapman and Hall

<sup>74</sup> Wood S.N. (2006b) *Generalized Additive Models: An Introduction with R*. Chapman and Hall/CRC Press

effects are additive and reasonably smoothed so that they can be estimated using a variety of nonparametric smoothing methods.

The text by Deaton and Muellbauer (1980)<sup>75</sup> provides several examples in microeconomics of studies of additive functions with statistical formulations and highly desirable results for the economy.

The utility of GAM in practical applications has been demonstrated in multiple research areas, with reference, among other disciplines, to biology, medicine (Hastie & Tibshirani, 1990<sup>76</sup>; Wood, 2006<sup>77</sup>), and also economics and finance (Härdle et al., 2004<sup>78</sup>). However, despite the potential advantages of using GAMs in practice many models have, until now, not been sufficiently explored in health economics problems.

### 2.2. OBJECTIVE

The main objective of this study is the calculation of a new flexible production function for hospitals and hospital clinical and surgical services. Secondly, the main strengths and weaknesses of the different functional forms used in this study, the flexible form and the classic Cobb–Douglas and translog forms, are examined; the study then considers how hospital clusters respond to the different models of hospital production functions and, finally, assesses the predictive capacity of additive model (AM) analyses regarding the behaviour of the classic functions, (C) the above for the production functions of clinical services (medical and surgical).

The new GAM model and the classic Cobb–Douglas and translog models are applied to the database of public hospitals of the Galician health service for the 2002-2008 time series.

This study is structured as follows: Section 2 presents information about the Galician public hospital sector, Section 3 summarizes the database under analysis, Section 4

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<sup>75</sup> Deaton A, Muellbauer J. (1980) *Economics and Consumer Behaviour*. Cambridge, Cambridge University Press

<sup>76</sup> Hastie TJ, Tibshirani RJ. (1990) *Generalized Additive Models*. London: Chapman and Hall

<sup>77</sup> Wood S.N. (2006b) *Generalized Additive Models: An Introduction with R*. Chapman and Hall/CRC Press

<sup>78</sup> Härdle W., Müller M., Sperlich S., Werwatz A. (2004) *Nonparametric and Semiparametric Models*. Berlin: Springer

briefly describes the Cobb–Douglas and translog production functions and introduces the flexible GAM model, Section 5 calculates the results for the database with these three models, and Section 6 performs a comparison between the three models, evaluating whether the specification of the Cobb–Douglas and translog functions fits the hospital production in the period of study. Finally, relevant results are presented regarding the ability of the GAM models to predict the behaviour and adjustment of the classic models, as well as the adjustment of the GAM models themselves for the production functions.

### **2.3. THE GALICIAN HOSPITAL SYSTEM.**

The Spanish national health system was created in 1987, as a development of the Spanish Constitution of 1978. The new model thus established is characterized by its universal coverage, equity criteria and tax financing. In fact, the Spanish Constitution guarantees all citizens the right to health, with the provision of health care services by the health institutions of the public system.

At the beginning of the 1990s, Galicia joined the process of the decentralization of health care resources, which gives the Autonomous Community of Galicia and its government control over health care resources through the creation of its own health service within the national health system in Spain. The public health service in Galicia consists of ten hospitals or hospital complexes, managed by the autonomous organization service Galego da Saúde (SERGAS).

Some data on the size and characteristics of the hospital sector show that the number of beds in SERGAS hospitals is 7,446, and that there are 3,917 physicians. In 2008, the number of admissions to Galician hospitals was 248,371, with 2,233,894 overnight stays, 1,431,011 first visits and 1,105,083 emergencies.

## 2.4. MATERIAL

### 2.4.1.- *Inputs and outputs of hospital production.*

The variables used in the study are the inputs, understood as capital and labour, and the outputs of hospital production.

Hospitals are multiproduct production centres, with a variety of patients being treated with a variety of inputs. There is no consensus on the most accurate measure of the output of hospital production Q, so that researchers have used different indicators to measure it, including the number of discharges, and the number of admissions or overnight stays. However, these measures fail to capture in a convenient way the health care provided by hospitals to patients.

In this investigation, the number of income, standardized by complexity or case-mix, is used as a measure of hospital production, to give a homogeneous unit of production called UNIT OF HOSPITAL PRODUCTION (UPH). The UPH is obtained by multiplying the number of income by its complexity obtained from the weights of the DRG (López et al., 1999<sup>79</sup>), thus addressing the need to take into account the complexity of the different hospitals and adjusting to a large extent the production output of each hospital.

Following Ferrier and Valmanis (2004)<sup>80</sup>, the inputs of the hospitals can be measured as follows: for the capital input, the number of beds is used for each hospital and each year, with these data being obtained from the official hospital statistics. The work input is measured as the number of hospital specialists on the staff of each hospital on December 31 of each year.

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<sup>79</sup> López Rois F J, Mateo R, Gómez J R, Ramón C, Pereiras M. (1996) *Methodological criteria for drawing up a contract-programme or singular sector-based agreement of specialized care using HPUs*. Secretaria Xeral SERGAS. Consellería de Saniadde e Servicios Sociais. Xunta de Galicia. Santiago de Compostela

<sup>80</sup> Ferrier, G.; Valmanis, V. (2004) Do mergers improve hospital productivity?. *Journal of the Operational Research Society*, 55: 1,071-1,080

#### *2.4.2.- Data.*

The data have been collected and organized as panel data from the SERGAS information system, using statistics from the SERGAS hospitals. The data for the hospitals collected for the period 2002-2008 are:

- Total number of beds and services
- Number of medical specialists (total and per service)
- Number of admissions and registrations by DRG (total and per service)
- Cluster in which each hospital is included
- Number and typology of medical and surgical hospital services per cluster

In Galicia, the hospitals have been classified into three clusters by Reyes (2009)<sup>81</sup>. This classification indicates the number of specialties a hospital has, as a reflection of the type of services it provides. For example, Cluster 2 hospitals only provide internal medicine services, general surgery and some basic specialties, while Cluster 3 hospitals provide a considerable range of specialized services. By contrast, Cluster 1 hospitals provide specialized services with high technology medical equipment and highly qualified personnel. In this context, the hospitals with the fewest specialties treat the simplest cases, since, compared with the hospitals in Clusters 1 and 2, they are less well equipped with high technology medical equipment such as computerized tomography or magnetic resonance scanners.

Table 1 shows how the different SERGAS hospitals belong to the different Clusters, while Table 2 contains some descriptive data on the different clusters.

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<sup>81</sup> Reyes F. (2009) Adopción, difusión y utilización de la Alta Tecnología Médica en Galicia. Tomografía Computerizada y Resonancia Magnética. Universidade de A Coruña, Servizo de Publicacions. A Coruña

Table 1. Hospital distribution according to Cluster.

CLUSTER	HOSPITAL
CLUSTER 1	C.H. UNIVERSITARIO JUAN CANALEJO C.H. UNIVERSITARIO DE SANTIAGO C.H. UNIVERSITARIO DE VIGO
CLUSTER 2	C.H. ARQUITECTO MARCIDE-NOVOA SANTOS H. DA COSTA H. COMARCAL DE MONFORTE H. COMARCAL DE VALDEORRAS
CLUSTER 3	C.H. XERAL-CALDE C.H. DE OURENSE C.H. DE PONTEVEDRA

Table 2. Some characteristics of the public hospital sector of Galicia

	UPHs	HOSPITAL BED SUPPLY	CONSULTANTS
CLUSTER 1	>60,000	>1,000	>500
CLUSTER 2	<20,000	<450	<200
CLUSTER 3	34,000-50,000	550-800	250-500

## 2.5. STATISTICAL METHODS.

### 2.5.1.- *Classic models.*

The Cobb–Douglas function was originally estimated by Charles W. Cobb and Paul H. Douglas (1928)<sup>82</sup>, although it had already been considered by Knut Wicksell (1900)<sup>83</sup> and, according to some authors, by J.H. von Thünen (1863)<sup>84</sup>. It has the following form:

$$Q = \alpha L^{\beta_1} K^{\beta_2}$$

<sup>82</sup> Cobb, C W, Douglas, P H. (1928). A Theory of Production. *The American Economic Review*, vol.18, Supplement, March 1928, pp. 139-165

<sup>83</sup> Wicksell, K. (1900) *Om gränsproduktiviteten såsom grundval för den nationalekonomiska fördelningen.* *Ekon Tidskrift*

<sup>84</sup> Thünen, J. H. von. (1863) *The Isolated State.* London. Pergamon

where  $Q$ ,  $L$  and  $K$  represent the output, the work factor and the capital factor, respectively, and  $\alpha$ ,  $\beta_1$  and  $\beta_2$  are constants.

A problem with this function is the omission of the change in production technology. The need to take this technological change into account was identified by Handsaker and Douglas (1937)<sup>85</sup> and Williams (1945)<sup>86</sup>. A standard procedure for introducing technological change into the function is to include the time of the series. This allows the function to capture changes in the technology, although it is assumed that this is exogenous to the specification of the function.

where,  $\alpha(T) = \alpha e^{\phi T}$  are constants.  $\phi$  is a measure of the proportion of the change in the output per period of time, keeping the levels of the inputs constant. This implies that technological change is exogenous.

The above equation is usually estimated as:

$$\ln Q = \phi T + \ln(\alpha) + \beta_1 \ln(L) + \beta_2 \ln(K) + \varepsilon,$$

where  $\varepsilon$  is an error term that follows a normal distribution. The log-linear specification assumes that the estimates of  $\beta_1$  and  $\beta_2$  are elasticities.

An alternative to the Cobb–Douglas equation is the translog production function (Christensen, Jorgenson, & Lau, 1973<sup>87</sup>). As in the Cobb–Douglas equation, if we assume that the effects of technological progress are neutral, the form of the translog production function is simplified as follows:

$$\ln Q = \beta_0 + \beta_1 \ln(L) + \beta_2 \ln(K) + \beta_3 \ln(L) \ln(K) + \beta_4 T + \varepsilon,$$

where  $Q$  represents the aggregate output,  $T$  is time,  $K$  is fixed capital and  $L$  is work.  $\beta$  stands for the parameters of the function.

<sup>85</sup> Handsaker M L, Douglas P H. (1937) The Theory of Marginal Productivity Tested by Data for Manufacturing in Victoria, *Quarterly Journal of Economics*, 52, 1-36

<sup>86</sup> Williams J. (1945) Professor Douglas' Production Function. *Economic Record*, 25, 55-64

<sup>87</sup> Christensen L R, Jorgenson D W, Lau L J. (1973) Transcendental Logarithmic Production Frontiers. *The Review of Economics and Statistics*, Vol. 55, No. 1 Feb., pp. 28-45

The Cobb–Douglas and translog models that have been used in this study are:

- For the Cobb–Douglas model:

$$\ln UPHs = \ln(\alpha) + \beta_1 \ln(FTEs) + \beta_2 \ln(Beds) + Hospital + Year + \varepsilon, \quad (1)$$

- For the translog model:

$$\ln UPHs = \ln(\alpha) + \beta_1 \ln(FTEs) + \beta_2 \ln(Beds) + \beta_3 \ln(FTEs) \ln(Beds) + Hospital + Year + \varepsilon, \quad (2)$$

### 2.5.2.- *The new flexible additive model.*

Flexible generalized additive models (GAMs) are useful as predictors in functional relationships for different groups of data, and avoid the need to establish a specific functional model a priori. These models combine the ability to explore several nonparametric relationships simultaneously, through the flexibility provided by the distributions of generalized linear models (GLM: McCullagh & Nelder, 1989<sup>88</sup>).

In our model, the response variable is continuous. In this case, the generalized additive model is usually referred to in the statistical literature as an additive model. Thus, in the present study we will denote the flexible model by AM. This study proposes the model for both hospital clusters and medical services.

#### 2.5.2.1.- Hospitals.

The flexible model considered is AM including the beds–facultative interaction:

$$\ln UPHs = f_1(\ln Beds) + f_2(\ln FTEs) + f_3(\ln Beds, \ln FTEs) + Hospital + Year + \varepsilon, \quad (3)$$

where  $f_1$  and  $f_2$  are unknown smoothed functions of the number of beds (on a logarithmic scale) and the number of practitioners (on a logarithmic scale), respectively,  $f_3$  is an unknown smoothed function representing the possible interaction between the number of beds and the number of physicians (both on a logarithmic scale), and  $\varepsilon$  is the error term, which follows a normal distribution with zero mean. Since the database contains observations for the different medical services

<sup>88</sup> McCullagh P, Nelder JA. (1989) *Generalized Linear Models*. London: Chapman & Hall

for each hospital for the period 2002-2008, the covariates "Hospital" and "Year" have also been included in the model.

Based on the estimation of the model (3), penalized thin plate splines (Wood, 2004<sup>89</sup>, 2006a<sup>90</sup>, 2006b<sup>91</sup>) are used to represent the smoothed functions  $f_1$ ,  $f_2$  and  $f_3$  and the representation of the mixed model of a penalized AM is considered (Wood, 2004<sup>92</sup>). In this configuration, the smoothed parameters are estimated via REML (Lin & Zhang, 1999<sup>93</sup>; Wahba, 1985<sup>94</sup>; Wang, 1998<sup>95</sup>).

#### 2.5.2.2.- Medical Services.

The flexible model considered is:

$$\ln UPHs = f_1(\ln Beds) + f_2(\ln FTEs) + f_3(\ln Beds, \ln FTEs) + Year + \varepsilon, \quad (4)$$

where  $f_1$  and  $f_2$  are unknown smoothed functions of the number of beds (on a logarithmic scale) and the number of practitioners (on a logarithmic scale), respectively,  $f_3$  is an unknown smoothed function representing the possible interaction between the number of beds and the number of physicians (both on a logarithmic scale) and  $\varepsilon$  is the error term, which follows a normal distribution with zero mean. Since the database contains observations for the period 2002-2008, the covariate "Year" has also been included in the model. For the estimation of the model (4), the same procedure has been followed as for the model (3).

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<sup>89</sup> Wood, S.N. (2004) Stable and efficient multiple smoothing parameter estimation for generalized additive models. *Journal of the American Statistical Association*. 99:673- 686

<sup>90</sup> Wood, S.N. (2006a) Low rank scale invariant tensor product smooths for generalized additive mixed models. *Biometrics*. 62(4):1025-1036

<sup>91</sup> Wood S.N. (2006b) *Generalized Additive Models: An Introduction with R*. Chapman and Hall/CRC Press

<sup>92</sup> Wood, S.N. (2004) Stable and efficient multiple smoothing parameter estimation for generalized additive models. *Journal of the American Statistical Association*. 99:673- 686

<sup>93</sup> Lin, X. and Zhang, D. (1999) Inference in generalized additive mixed models using smoothing splines. *J.R. Statist. Soc. B*. 61, 381-400

<sup>94</sup> Wahba, G. (1985) A Comparison of GCV and GML for choosing the smoothing parameter in the generalized spline smoothing problem. *Ann. Statist.* 13:1378-1402

<sup>95</sup> Wang, Y. (1998) Mixed effects smoothing spline analysis of variance *J.R. Statist. Soc. B*. 60, 159-174

## 2.6. RESULTS FOR THE CALCULATION OF THE PRODUCTION FUNCTION OF HOSPITALS THROUGH FLEXIBLE REGRESSION MODELS

In this section we present the estimated results for each model, for the whole of SERGAS, for each cluster of hospitals and for selected medical specialties. We assess which specification of the production function (Cobb–Douglas, translog or the flexible additive model) is better adjusted for the public hospital sector of Galicia, in the period 2002-2008.

The models are evaluated based on the AIC (Akaike Information Criterion: Akaike, 1974<sup>96</sup>) and on the economic interpretation of the changes in output due to changes in the input factors. The value of the corrected  $R^2$  is also presented for each model.

### 2.6.1.- Hospital clusters.

#### 2.6.1.1.- The global model.

First, the models are estimated for SERGAS globally.

According to Table 3, in all three models the variables Beds (Beds), Optional (FTEs) and Hospital are all significant ( $p < 0.001$  in all cases). In addition, the interaction between beds and facultative is also significant for the translog models ( $p < 0.001$ ) and flexible AM ( $p < 0.001$ ). However, the variable Year (Year) as an approximation for changes in production technology is not significant (CD  $p = 0.476$ , translog  $p = 0.477$ , AM  $p = 0.690$ ). This result indicates that the change in production technology is neutral in relation to output.

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<sup>96</sup> Akaike, H. (1974) A new look at the statistical model identification. *IEEE Transactions on Automatic Control* 19 (6): 716–723. doi:10.1109/TAC.1974.1100705. MR0423716

Table 3. Results of the Cobb-Douglas (CD), translog and Flexible AM models for the global SERGAS.

Model	Variables	Coefficients	gl	P-value	R <sup>2</sup> ( x100% )	AIC
CD	Beds	0.917	1.00	<0.001	78.12	2610.09 2
	Consultants	0.014	1.00	0.6839		
	Year	-0.011	1.00	0.476		
	Hospital		9.00	<0.001		
Trans Log	Beds	1.030	1.00	<0.001	78.34	2599.27 3
	Facultativos	0.147	1.00	0.004		
	Beds-Consultants	-0.053	1.00	<0.001		
	Year	-0.011	1.00	0.477		
AM	Hospital		9.00	<0.001	82.90	2355.80 1
	s(Beds)	-	5.44	<0.001		
	s(Consultant)	-	5.32	<0.001		
	s(Beds,Cons)	-	15.01	<0.001		
	Year	-0.006	1.00	0.6897		

AIC (Akaike Information Criteria)

gl (degrees of freedom)

Regarding the goodness of fit for the models, it is observed that, from the values of R<sup>2</sup> and the AIC, the flexible model presents a better fit than the classic models.

Figure 1 shows the increase in productivity as a result of the increase in inputs, based on the flexible model. The figure shows three lines. The upper line represents the change in productivity according to variations in the input. The lower right line represents the variations in the capital factor (Beds) while the lower left line shows the variations in the work factor (FTEs).

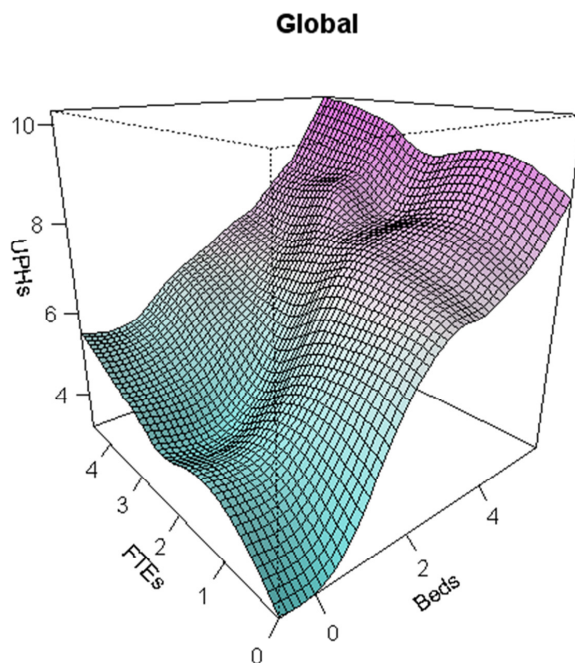


Figure 1. Increase in overall SERGAS productivity according to the flexible model. Variables (UPHs, Optional, Beds) are expressed on a logarithmic scale.

According to the upper line, Figure 1 shows a rapid increase in productivity. This increase is determined by the trend of the capital factor variable while the contribution of the work factor to the increase in productivity is quite modest.

#### 2.6.1.2.- Estimates of the models for the hospital clusters.

The results for Cluster 1 are presented in Table 4. The first noteworthy aspect for the estimated variables is that, unlike the results of the previous section, no statistical significance was found for the Beds variable with either the flexible model or the classic models. However, the flexible model presents statistical significance for the facultative variable (FTEs), while the classic models do not. In addition, the flexible model is the only one to capture statistical significance for the interaction between beds and physicians. None of the models finds statistical significance for the change in production technology or for the Hospital variable.

Table 4. Results of the Cobb-Douglas (CD), translog and Flexible AM models for Cluster 1.

Model	Variables	Coefficients	gl	P-value	R <sup>2</sup> ( x100% )	AIC
CD	Beds	0.816	1.00	<0.001	63.41	1073.084
	Consultants	0.018	1.00	0.706		
	Year	-0.011	1.00	0.654		
	Hospital		2.00	0.849		
Trans Log	Beds	0.684	1.00	<0.001	63.54	1072.302
	Facultativos	-0.128	1.00	0.205		
	Beds-Consultants	0.051	1.00	0.098		
	Year	-0.010	1.00	0.683		
AM	Hospital		2.00	0.848	78.50	875.337
	s(Beds)	-	4.38	0.193		
	s(Consultant)	-	5.42	0.018		
	s(Beds,Cons)	-	15.16	0.041		
	Year	0.015	1.00	0.453		
	Hospital		2.00	0.970		

AIC (Akaike Information Criteria)

gl (degrees of freedom)

Looking at the AIC and R<sup>2</sup> values, we can observe greater explanatory power for the flexible model than the classic models.

The evolution of productivity in the output, based on the behaviour of the two production factors evaluated in this study, is shown in Figure 2. The changes in productivity show that, while the facultative variable (FTEs) gives continuous increases followed by a decrease, the capital variable (Beds) gives sustained increases throughout its curve. On the other hand, the product seems to have diseconomies of scale.

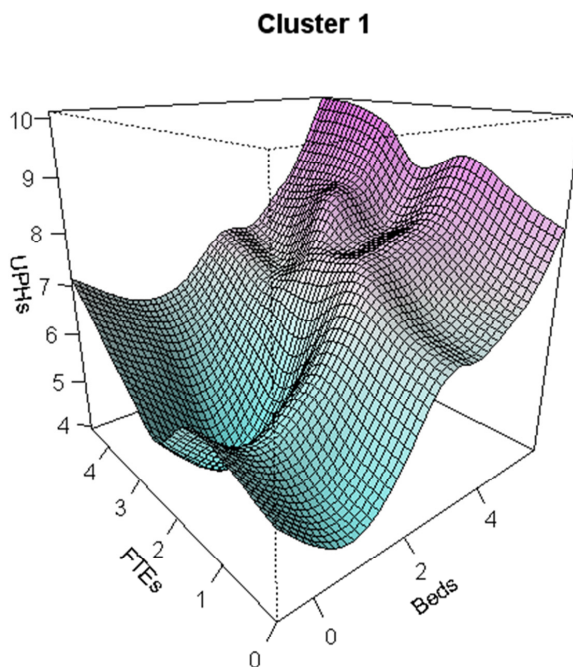


Figure 2. Increase in productivity for Cluster 1 according to the flexible model. Variables (UPHs, Optional, Beds) are expressed on a logarithmic scale

Table 5 contains the results for Cluster 2. It is interesting to note that the variable factor of capital, represented by the number of beds (Beds), is statistically significant for all three models, while the work production factor (FTEs) is not significant for the AM model (0.638). In addition, the effect of the interaction between production factors is also captured by the translog and AM models. The AM model, unlike the classic models, even shows itself capable of capturing the effects that changes in production technology, identified through the time series, produce in the output. The significant statistical effect in the three models of the hospital variable shows some variability in the size of the hospitals belonging to Cluster 2.

Table 5. Results of the Cobb-Douglas (CD), translog and Flexible AM Cluster 2 models.

Model	Variables	Coefficients	gl	P-value	R <sup>2</sup> ( x100% )	AIC
<b>CD</b>	Beds	1.021	1.00	<0.001	77.68	735.695
	Consultants	0.087	1.00	0.343		
	Year	-0.047	1.00	0.158		
	Hospital		3.00	<0.001		
<b>Trans Log</b>	Beds	1.167	1.00	<0.001	78.32	728.013
	Facultativos	0.262	1.00	0.015		
	Beds-Consultants	-0.107	1.00	0.002		
	Year	-0.049	1.00	0.140		
<b>AM</b>	Hospital		3.00	<0.001	85.30	627.366
	s(Beds)	-	5.66	<0.0010.		
	s(Consultant)	-	3.38	638		
	s(Beds,Cons)	-	10.411.	<0.001		
	Year	-0.063	00	0.022		

AIC (Akaike Information Criteria)

gl (degrees of freedom)

As in the previous results, both the AIC and the R<sup>2</sup> show that the AM model is a better fit than the classic models.

Figure 3 shows an almost linear increase in productivity for increases in the capital factor (Beds), while increases in the labour factor (FTEs) have a lower contribution to the increase in productivity.

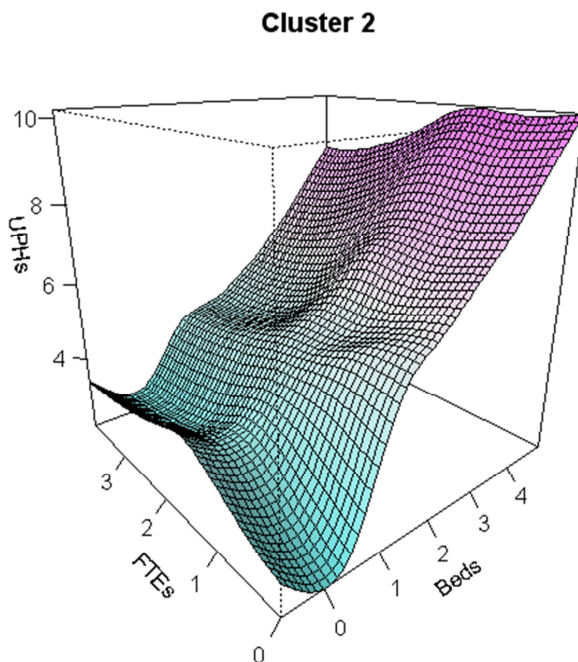


Figure 3. Increase in productivity for Cluster 1 according to the flexible model. Variables (UPHs, Optional, Beds) are expressed on a logarithmic scale

The estimated results for Cluster 3 are presented in Table 6. Statistically significant values are shown for the variable Beds (Beds) in the three models, while the optional variable (FTEs), Hospital and change in technology present statistical significance only in the AM model. Likewise, only the AM model shows statistical significance for the interaction between work and capital.

Table 6. Results of the Cobb-Douglas (CD), translog and Flexible AM Cluster 3 models.

Model	Variables	Coefficients	gl	P-value	R <sup>2</sup> ( x100% )	AIC
<b>CD</b>	Beds	0.930	1.00	<0.001	78.01	775.441
	Consultants	-0.030	1.00	0.595		
	Year	0.023	1.00	0.359		
	Hospital		2.00	0.417		
<b>Trans Log</b>	Beds	0.904	1.00	<0.001	77.96	777.311
	Facultativos	-0.057	1.00	0.545		
	Beds-Consultants	-	1.00	0.772		
	Year	0.011	1.00	0.363		
	Hospital	0.023	2.00	0.418		
<b>AM</b>	s(Beds)	-	4.20	<0.001	86.50	619.218
	s(Consultant)	-	6.10	0.037		
	s(Beds,Cons)	-	16.41	0.022		
	Year	0.021	1.00	0.293		
	Hospital		2.00	0.814		

AIC (Akaike Information Criteria)

gl (degrees of freedom)

The goodness of the fit of the models, measured by the R<sup>2</sup> and the AIC, shows advantages for the flexible AM model over the classic models.

Figure 4 presents the gains in productivity arising from additional resources in the capital factor, reflected by a linear relationship between output and the capital factor. However, the trend for the labour variable is represented by a quadratic curve that shows gains and reductions in production as a consequence of increases in that factor.

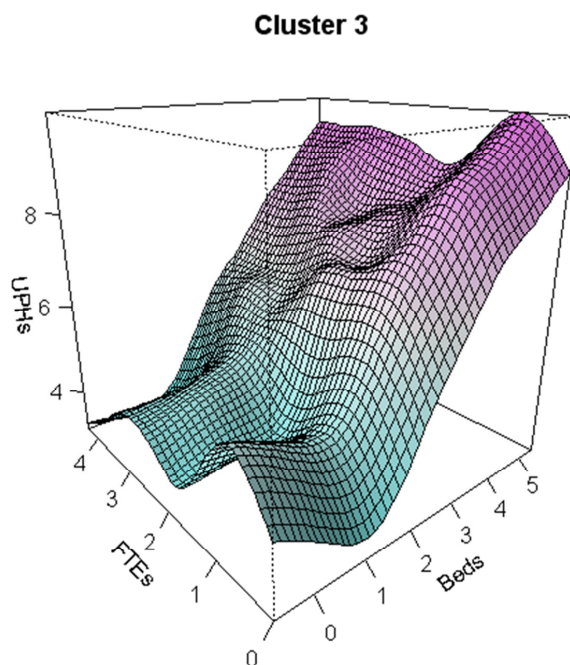


Figure 4. Increase in productivity for Cluster 1 according to the flexible model. Variables (UPHs, Optional, Beds) are expressed on a logarithmic scale

## 2.7.- RESULTS FOR THE CALCULATION OF THE PRODUCTION FUNCTION OF SURGICAL AND CLINICAL HOSPITAL SERVICES THROUGH FLEXIBLE REGRESSION MODELS

Table 7 shows the results for the specialty of General Surgery for the three models. For the classic models, both factors, capital (Beds) and labour (FTEs), are statistically significant although the variables of technological change and the interaction between the two productive factors are not significant. On the other hand, the flexible AM model shows statistical significance for the work variable and its interaction with capital, but the capital factor and technological change variables are not significant.

Table 7. Results of the Cobb-Douglas (CD), TransLog and Flexible AM General Surgery models.

Model	Variables	Coefficients	gl	P-value	R <sup>2</sup> ( x100% )	AIC
CD	Beds	0.458	1.00	<0.001	96.32	-47.184
	FETs	0.483	1.00	<0.001		
	Year	-0.004	1.00	0.789		
Trans Log	Beds	0.645	1.00	<0.001	96.30	-46.109
	FETs	0.656	1.00	<0.001		
	Beds-FTs	-0.054	1.00	0.364		
	Year	-0.002	1.00	0.789		
AM	s(Beds)	-	1.00	0.330	98.30	-46.246
	s(FETS)	-	1.00	<0.001		
	s(Beds,F)	-	1.00	<0.001		
	Year	-0.009	12.8	0.441		
			8	1.00		

AIC (Akaike Information Criterion)

gl (degrees of freedom)

The results for the Cobb–Douglas model show a better fit than the TransLog model and the AM model, as observed from the AIC values, while the R<sup>2</sup> seems to prefer the flexible AM model. This is an excellent example of the ability of the flexible AM model as an explorer of the goodness of fit of other models. In this case, we are shown that we can trust the calculations and results offered by the Cobb–Douglas model.

Figure 5 presents poor gains in output as a consequence of a continuous and sustained increase in the labour factor, while the capital factor shows small increases in the product as a consequence of continued increases in the factor. However, this behaviour does not extend to other surgical specialties such as traumatology, as shown in Figure 6, where productivity losses are observed as a consequence of increases in the labour factor until reaching a turning point, after which the production begins to increase. It can be said that the study of the production function for a specialty such as traumatology particularly benefits from a flexible model.

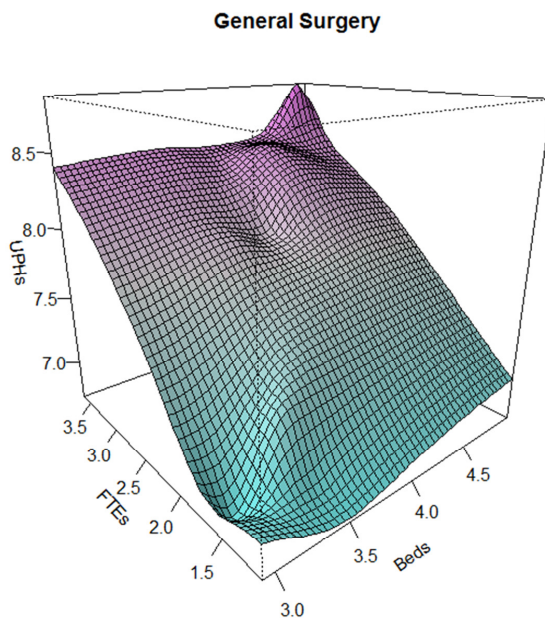


Figure 5. Increase in productivity of General Surgery according to the Flexible model. Variables (UPHs, Optional, Beds) are expressed in logarithmic scale.

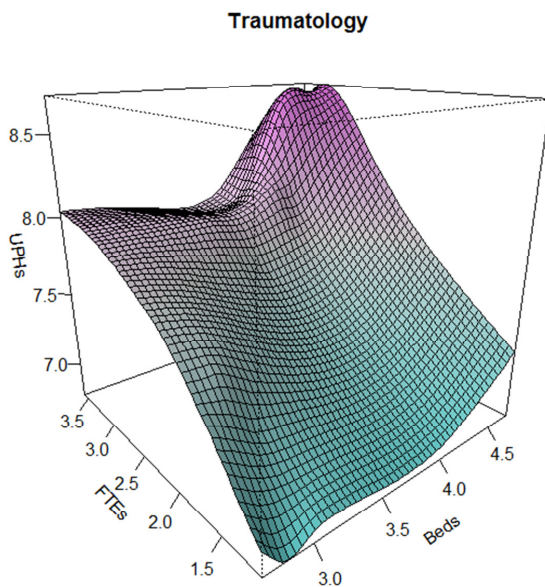


Figure 6. Increase in productivity of Traumatology according to the Flexible model. Variables (UPHs, Optional, Beds) are expressed in logarithmic scale.

Considering Table 8, the results for the specialty of internal medicine show statistically significant values for the variable of capital in the three models, but do not present significant values for the variables of work or technological change in any of the three models. In the same way, the variable of interaction between the two productive factors is significant in the flexible AM model but not in the TransLog model.

Table 8. Results of the Cobb–Douglas (CD), TransLog and Flexible AM Internal Medicine models.

Model	Variables	Coefficients	gl	P-value	R <sup>2</sup> ( x100% )	AIC
CD	Beds	0.868	1.00	<0.001	95.88	-17.789
	FETs	0.126	1.00	0.109		
	Year	0.013	1.00	0.510		
Trans Log	Beds	0.831	1.00	<0.001	95.80	-15.939
	FETs	0.056	1.00	0.784		
	Beds-FTs	0.016	1.00	0.715		
	Year	0.012	1.00	0.530		
AM	s(Beds)	-	1.00	<0.001	97.90	-13.113
	s(FETS)	-	1.00	0.262		
	s(Beds,F)	-	12.1	<0.001		
	Year	0.028	6	0.091		
			1.00			

AIC (Akaike Information Criterion)

gl (degrees of freedom)

Following the AIC values, the Cobb–Douglas model seems to present a better fit than the flexible or TransLog models, although for the values of the R<sup>2</sup> the flexible model AM would present the best fit.

The evolution in productivity for the specialty of internal medicine is presented in Figure 7, which shows gains in production as a linear relationship between the product and the variable capital (Beds) with little influence of the increases in the variable work (FTEs). This behaviour coincides with that of other medical specialties such as Gynaecology, as can be deduced from Figure 8.

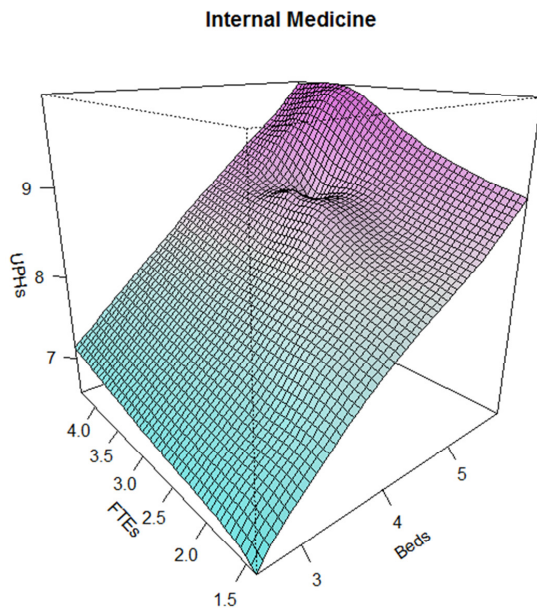


Figure 7. Increase in productivity of Internal Medicine according to the Flexible model. Variables (UPHs, Optional, Beds) are expressed in logarithmic scale

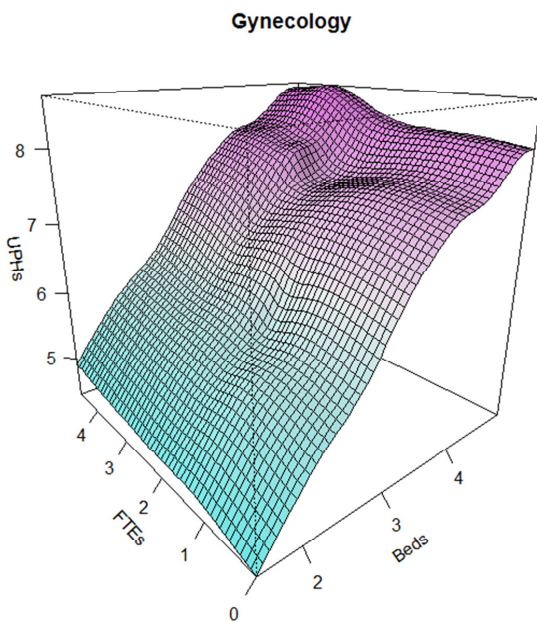


Figure 8. Increase in productivity of Gynaecology according to the Flexible model. Variables (UPHs, Optional, Beds) are expressed in logarithmic scale



### 3. INPATIENT WAITING LISTS: HOW MANY HOSPITAL BEDS ARE ENOUGH? A SIMULATION ANALYSIS

#### 3.1. INTRODUCTION

A waiting list for healthcare is a queue of patients who have been given a care procedure but, due to reasons beyond their control, must wait to be served within a variable time period (Sampietro and Espallargues, 2000)<sup>97</sup>. Surgical waiting lists are often studied due to their major problems of morbidity and mortality, as well as their greater economic impact (Richards, 1999<sup>98</sup>; Silber et al., 1996<sup>99</sup>).

This paper is focused on analyzing the situation in a university hospital. Specifically, the impact of bed capacity—a scarce and expensive input in healthcare—on daily inpatient activity, patients' length of stay and, consequently, waiting lists are analyzed in a teaching hospital. The high cost of academic health centers and other teaching hospitals is largely attributed to the unique missions pursued by these institutions—including graduate medical education, biomedical research, and the maintenance of standby capacity for highly specialized patients (Commonwealth Fund, 1997<sup>100</sup>).

Experimentation with the real system would cause a lot of trouble both for patients and staff. For that reason, we are obliged to perform a simulation approach, which represents the system and can be manipulated with no daily healthcare practice

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<sup>97</sup> Espallargues M, Gall P, Pons Jm, Sampietro-Colom L. (2000) Situació i abordatge de les llistes d'espera a Europa. Informe Tècnic. *Agència d'avaluació de Tecnologia i Recerca Mèdica (AATRM)*. Servei Català de la Salut. Departament de Sanitat i Seguretat Social. Generalitat de Catalunya. Novembre

<sup>98</sup> Richards MA, Smith P, Ramirez AJ, Fentiman IS, Rubens RD. (1999) The influence on survival of delay in the presentation and treatment of symptomatic breast cancer. *Br J Cancer*;79 (5-6:858–864

<sup>99</sup> Silber Sigmund, H. Mühlring, R. Dörr, G. Zindler, A. Preuss and A. Stümpfl (1996) 'Waiting times and death on the waiting list for coronary artery bypass operation. Experiences in Munich with over 1,000 patients', *Herz* 21(6): 389-396.

<sup>100</sup> Commonwealth Fund (1997), *Leveling the Playing Field: Financing the Missions of Academic Health Centers*, Report of the Commonwealth Fund Task Force on Academic Health Centers New York: Commonwealth Fund, May

disruptions. Indeed, one of the often-mentioned reasons for using simulation as a tool is the experimentation with non-existing systems (Law and Kelton, 1991<sup>101</sup>). Once validated, the simulated model can yield accurate estimates of the behaviour of the real system and help to understand and clarify complex dynamic processes (Yamaguchi et al., 1994<sup>102</sup>). Finally, simulating a process such as admission to elective surgery can also help to identify bottle-neck and congestion points. Besides, the simulation model can be useful to monitor hospital system performance and assess the relative effectiveness of alternative policies aimed at coping with historical or statistically-generated patient load.

To examine the pattern of waiting lists size in programmed surgery and reproduce the behaviour of daily inpatient activity, length of stay and, consequently, waiting lists, a known distribution was fit to each variable. Such distribution then allows us to generate new values for daily inpatient activity and patient length of stay by means of the Monte-Carlo method. Finally, after the generation of new observations of inpatient activity and length of stay, the corresponding simulated waiting list and daily percentage of occupied beds (occupancy rate) can also be created.

In this context, waiting-list variations with increased number of hospital beds (a rough proxy for capital input) can also be studied by examining the impact of the number of beds on inpatient activity, length of stay and, consequently, waiting lists. With this purpose, the simulation process was replicated for various increased percentages in the number of available beds in two alternative scenarios. First, increased number of beds is assumed to lead to no modification of the inpatient activity pattern (no bed effect). Then, a change in the number of beds was assumed to modify the behavior of inpatient activity in the amount given by the inpatient activity-

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<sup>101</sup> Law, Averill M. and W. David Kelton (1991) *Simulation Modelling and Analysis*, McGraw-Hill, New York, 2nd ed.

<sup>102</sup> Yamaguchi Naoito, Tomohide Tamura, Tomotaka Sobue, Suminori Akiba, Megu Ohtaki, Yoshinobu Baba, Shigeto Mizuno and Shaw Watanabe (1994) 'Evaluation of Cancer prevention strategies by computerized simulation model: Methodological issues', *Environmental Health Perspectives* 102, Suppl. 8, 64-71

beds elasticity contributed by Kroneman and Siegers (2004)<sup>103</sup> in a comparative study for ten European countries. All computational programming was performed with statistical free software R.

### 3.2. OBJECTIVE

The objective of this work was to reproduce, through a simulation study, admissions to Galician hospitals and the length of their stays, in order to analyse how the number of beds in a hospital affects waiting list length. Data on admissions and stays in Galician hospitals during the year 2007 were used in the simulation. The hospitals considered are those shown in Table 1. Note that the hospitals are ordered according to the number of beds they had in operation during 2007.

Table 1. Galician hospitals considered, with the abbreviations by which they are identified in this work and the number of beds in operation in 2007

Hospital	Abrev.	Camas <sup>104</sup>
C.H. UNIVERSITARIO DE A CORUÑA	( CHUAC )	1494
C.H. UNIVERSITARIO DE VIGO	( CHUVI )	1224
C.H. UNIVERSITARIO DE SANTIAGO	( CHUS )	1100
C.H. DE OURENSE	( CHOU )	809
C.H. XERAL-CALDE	( CAL )	725
C.H. DE PONTEVEDRA	( CHOP )	598
C.H. ARQUITECTO MARCIDE-NOVOA SANTOS	( MAR )	411
H. DA COSTA	( COS )	140
H. COMARCAL DE MONFORTE	( MNF )	133
H. COMARCAL DE VALDEORRAS	( VAL )	103
H. DO SALNES	( SAL )	86
H. DO BARBANZA	( BBZ )	85
H. DE VERIN	( VER )	79
H. VIRXE DA XUNQUEIRA	( XUN )	74

<sup>103</sup> Kroneman, M.; Siegers, J.J. (2004) 'The effect of hospital bed reduction on the use of beds: A comparative study of 10 European countries', *Social Science & Medicine* 59(8): 1731-1740

<sup>104</sup> Los datos fueron obtenidos de la Memoria 2007 Sistema Público de Saúde de Galicia (<http://www.sergas.es/Publicaciones/DetallePublicacion.aspx?IdPaxina=40008&IDCatalogo=1732> ).

### 3.3. MATERIAL AND METHODS

The admissions data obtained refer to the hospital's admissions during each of the 365 days of 2013. The data on the patients' stays show the number of patients who had a stay of one day, two days and so on until the maximum stay for a patient in 2013 that was 353 days. The simulation performed will assume a Poisson distribution of admissions. The simulation program R was used. The simulation work is presented explaining the different problems that the author had to face in order to complete the work and the different steps that were taken to solve them. Once the data was read by the R program, it was indicated, for the patient admissions, which days correspond to holidays or non-holidays (the days of July, August and December and Saturdays and Sundays). This was done to estimate the Poisson distribution for each group of days. In this way, we estimate the Poisson parameter that coincides with the mean and for this the sample mean was estimated. We used this in the simulation on the number of admissions in one day. Regarding the data on patient stays, first we read the data as we did with the admissions, then we identified the frequencies and the data on the values of the stays. Next, we transformed the frequencies converting them into relative frequencies. By accumulating the frequencies, we were able to simulate the stays. In order not to permit an empty hospital, we decided to run an initial simulation to obtain the number of beds occupied and the number of simulated stays for each patient, so that the simulation object of this work met a more realistic scenario of occupied beds.

In the study, data are simulated for the years 2013-2016. For this period, 500 iterations were performed, in each of which the daily admission of patients with their associated stays was simulated. During the process, a waiting list and the daily occupation index were created. To avoid starting the simulation with "empty" hospitals (unrealistic situation), the process began to simulate on 1 August 2006. The stays of each patient were simulated using the non-parametric kernel density estimator (with the bandwidth that was calculated previously). To simulate the admissions, standard values with different parameters were extracted according to whether it was a working day or holiday. As we saw earlier, the number of beds seems to be related to the centre's daily admission pattern. This is why we conducted two different studies. In a first approximation we simulate the admissions without taking into account the number of beds that are being considered (without elasticity). For this, we generate the

admissions through the normal distributions that we previously adjusted to the data. However, as previously mentioned, the number of beds in a centre seems to affect the daily admissions of patients. To take this effect into account, we adapted the admissions simulation procedure by means of the linear adjustment we obtained: the normal distribution that will be used to generate the data will have mean  $c_1 + c_2 * n^\circ$  of Beds and standard deviation  $c_3 + c_4 * n^\circ$  of Beds, where the coefficients are obtained from linear adjustments that were performed previously (with elasticity).

To do this, we created a vector with the days of the week, so that each day of the simulation is assigned a week day: we repeated the sequence of days 4,5,6,7,1,2,3 for the days of the simulation. We did the same for the months. Next, we created an array with the same number of rows as the number of beds and the same number of columns as the number of days. We filled it with 0 values that mean that the bed is empty. The cell tells us, if it has that 0 value, that the bed is empty and if it contains a number, it represents the number of days that the bed will still be occupied. We initialized another vector, which gives us the admission number we will have each day and the number of people each day waiting for a bed to be empty so they can enter. Therefore, the first day has the value 0 that represents that nobody is waiting for the first day of the initial simulation. The loop works day by day and tells us that if the month is 7, 8 or 12, that is to say months that we consider as holidays (because there is a lower admission rate than in the remaining months of the year) or if the day of the week is 6 or 7 (Saturday or Sunday, for the same reasons), then the value we take is the parameter for the holidays, otherwise we use the lambda for the working days. Next we generate the number of admissions and generate a value of a Poisson distribution with lambda parameter and then we add the waiting list we have for that day. We consider the new admissions without taking into account the waiting list. Next, we looked at what free beds there were. For this we select the column corresponding to the day we are in, we see if the bed is empty and also which beds precisely are empty; with what we know about the number of free beds. Now we see if the admission number is greater than the number of free beds.

On the waiting list the next day, we keep the number of people on the waiting list and, on the other hand, we keep the number of people we are going to actually be admitted to hospital. In another case, the waiting list will be 0. If we have admissions, with a

new loop we assign patients free beds and simulate their stays. We select each admission and generate a report. The interval  $[0, 1]$  is divided into segments of lengths equal to the relative frequencies we obtained for the stays. We observe each report to which each segment of the interval corresponds and we assign to the individual  $j$  the corresponding stay. To see if the stay obtained exceeds the number of days until the end of the simulation, in the event that it exceeds the number of days of the generated stays that fall within the period we are simulating, we choose the minimum between the stay that we generate and the number of days remaining until the end of the period. In addition, in the row corresponding to a free bed that we have, we occupy it with the bed we generate and fill in the boxes from the current day, the day of admission until the end of the stay, with a number that tells us the number of days remaining until the discharge of the patient. The objective simulation is then started. We kept the value we had for that bed on the last day of the initial simulation. If what we had for that bed on the last day of the initial simulation is a value other than 0, that is, if that bed was occupied, we filled in the current matrix with that stay and the following days with the days remaining until discharge. We do not look at the difference between the value of the stay and the end of the period, because we have no patient case with a stay of more than 1,000 days. For the waiting list we have as the first value the last value of the initial simulation. We now introduce the calculation of the occupation index (percentage of beds occupied in a year), annual and monthly, for each of the four years. For example, in the fourth year of each repetition, from column 1 to 365 we identify whether the bed is occupied or not (1, 0). We add these values to give us the total number of beds occupied in the year and we divide this by the number of beds (395) multiplied by the number of days of the year. This is the Annual Occupancy Index. For the Monthly Occupancy Index, we do the same, but instead of selecting the entire matrix, we only select the columns corresponding to each month  $x$  of the matrix. We add the values of the columns relating to month  $x$  to obtain the number of beds occupied in the month, this is divided by the total number of beds multiplied by the number of days in the month and we obtain the percentage of beds occupied per month. We also calculate, for the whole period, the percentage of days that have 100% occupation of the beds, from the first day to the last day, observing whether the beds are all occupied or not (1, 0), and we calculate the average of these days for the entire period. This is the percentage of days with 100% bed occupancy.

### 3.4. RESULTS

#### 3.4.1. Length of Stay

##### 3.4.1.1. Descriptive analysis

We carried out a descriptive study of patients' stays in these Galician hospitals in 2007. Table 2 shows some descriptive measures of interest for our variable: the quartiles and the average length of stay in the various hospitals. We can see that when average stay is considered, the largest hospitals seem to have longer patient stays than the smaller hospitals, but the difference in median stay is less pronounced. This may be because longer stays are concentrated in hospitals that treat more serious and / or complex pathologies such as CHUAC, CHUV or CHUS. In this sense longer stays can be seen as outliers. This can be seen from the box plot in Figure 1.

Table 2 Average and quartiles of the stays in the different hospitals. Hospital Min. 1st Quartile  
Medium Med. 3rd Quartile Max.

Hospital	Mín.	1º Cuartil	Mediana	Media	3º Cuartil	Máx.
CHUAC	1	2	5	9.699	11	365
CHUVI	1	2	6	10.060	11	365
CHUS	1	3	6	9.983	12	365
CHOU	1	3	7	9.581	12	365
CAL	1	3	6	8.949	11	365
CHOP	1	3	6	8.836	11	365
MAR	1	3	6	8.544	11	166
COS	1	2	5	7.040	9	197
MNF	1	3	6	7.810	10	280
VAL	1	3	5	8.003	10	118
SAL	1	2	4	6.284	8	71
BBZ	1	2	5	7.577	10	182
VER	1	3	5	6.613	9	63
XUN	1	3	5	6.950	9	66

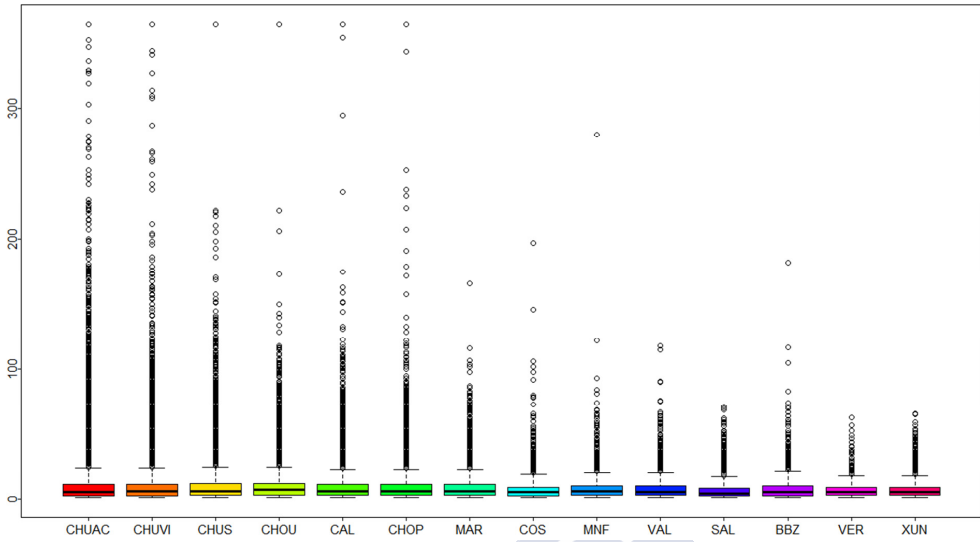


Figure 1. Box plots of the duration of patient stays in the different hospitals.



### 3. INPATIENT WAITING LISTS: HOW MANY HOSPITAL BEDS ARE ENOUGH? A SIMULATION ANALYSIS

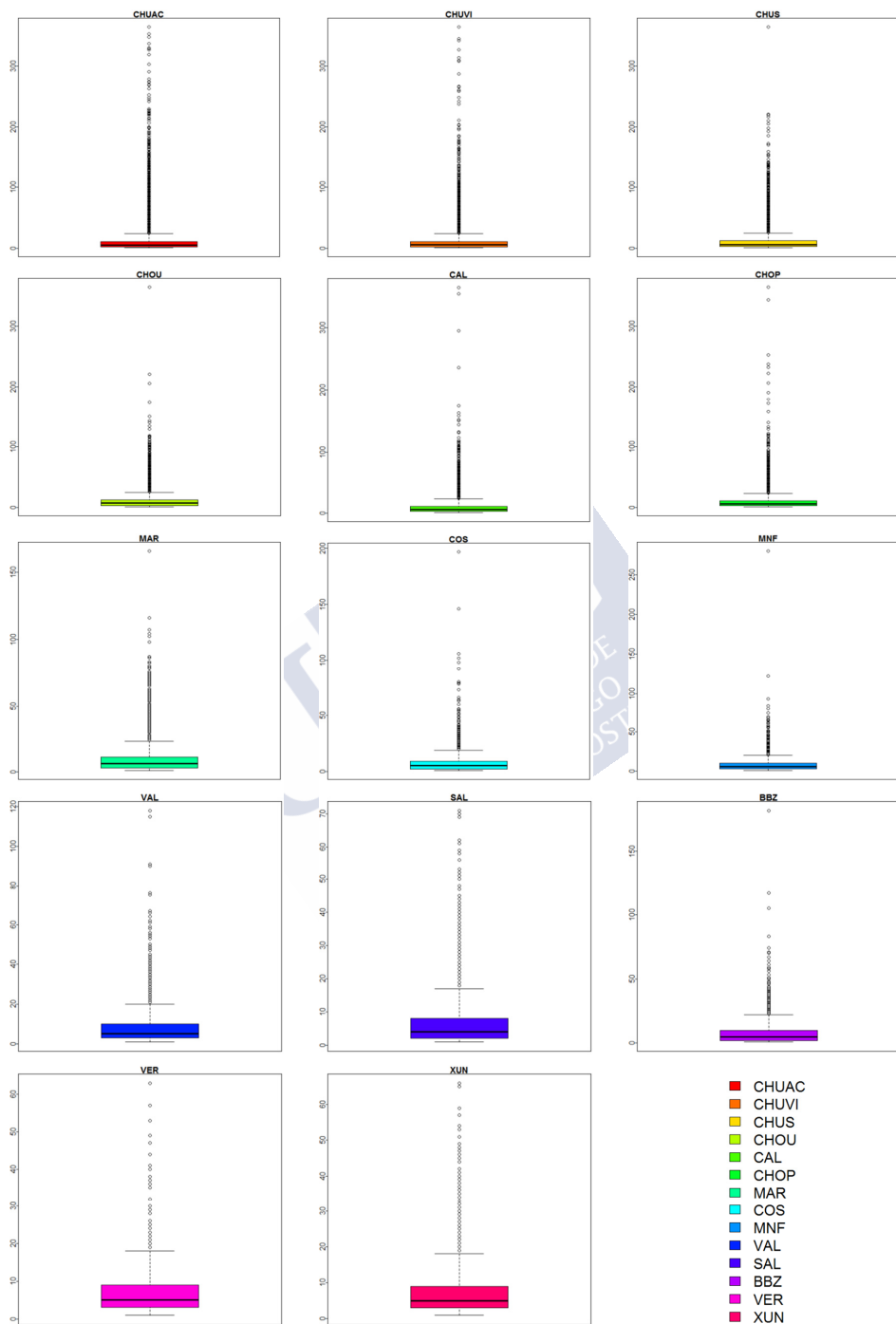


Figure 2. Box plots of the length of patient stays in each hospital.

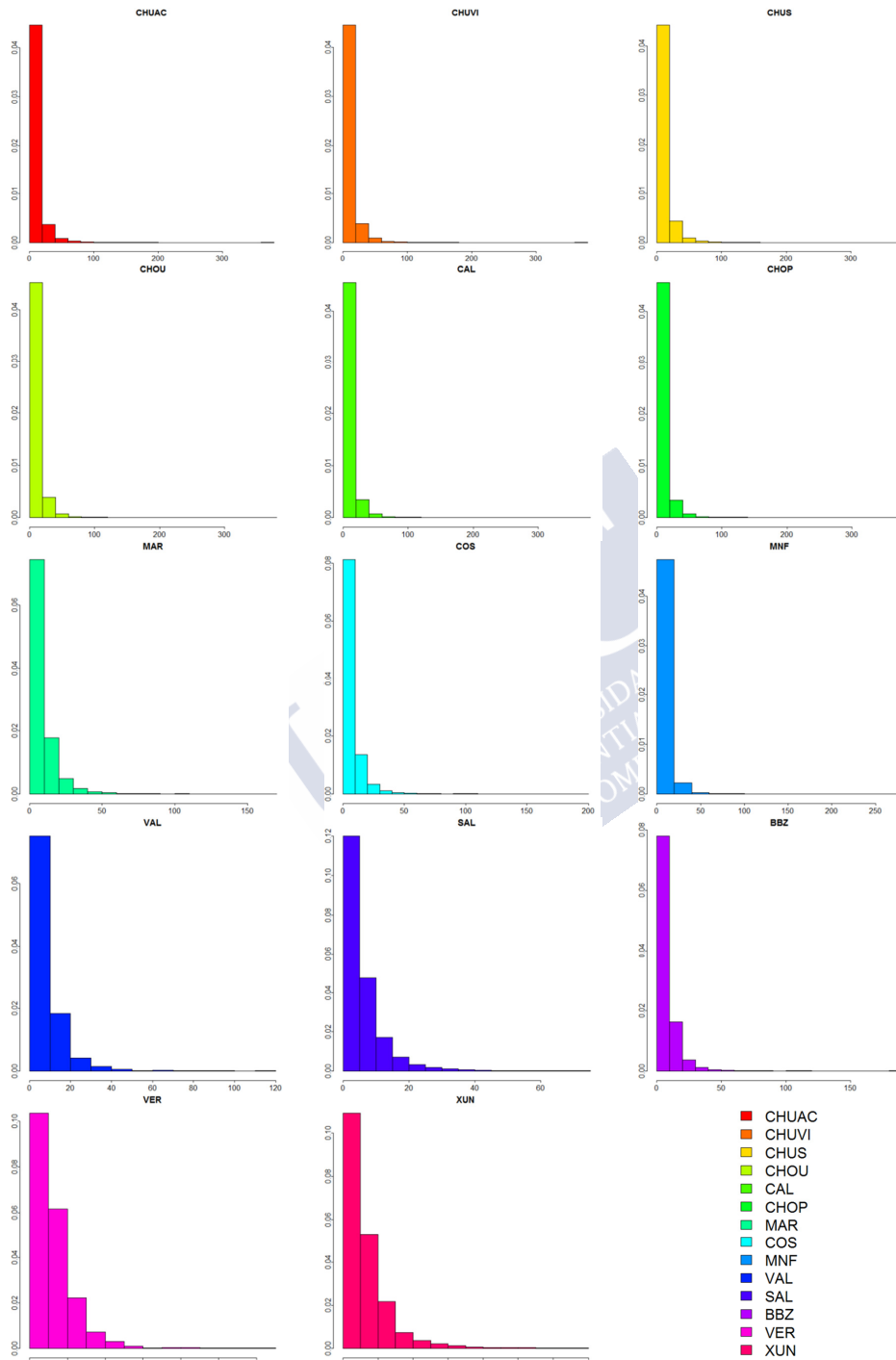


Figure 3. Histograms of the length of patient stays in each hospital.

Figures 2 and 3 show the box plot and histograms for each hospital centre.

### 3.4.1.2. Clusters

The Kolmogorov-Smirnov test was applied to each pair of hospitals to test the null hypothesis that patient stays followed the same distribution in both. From the results presented in Table 3 we can see that this was accepted only in the VER-XUN case.

Table 3. *P*-values for pairwise Kolmogorov-Smirnov tests.

Hospital	CHUAC	CHUVI	CHUS	CHOU	CAL	CHOP	MAR	COS	MNF	VAL	SAL	BBZ	VER	XUN
CHUAC	1	0	0	0	0	0.000	0.000	0	0.000	0.000	0	0.000	0.000	0.000
CHUVI	-	1	0	0	0	0.000	0.000	0	0.000	0.000	0	0.000	0.000	0.000
CHUS	-	-	1	0	0	0.000	0.000	0	0.000	0.000	0	0.000	0.000	0.000
CHOU	-	-	-	1	0	0.000	0.000	0	0.000	0.000	0	0.000	0.000	0.000
CAL	-	-	-	-	1	0.000	0.000	0	0.000	0.000	0	0.000	0.000	0.000
CHOP	-	-	-	-	-	1.000	0.000	0	0.000	0.004	0	0.000	0.000	0.000
MAR	-	-	-	-	-	-	1.000	0	0.000	0.004	0	0.000	0.000	0.000
COS	-	-	-	-	-	-	-	1	0.000	0.000	0	0.000	0.000	0.000
MNF	-	-	-	-	-	-	-	-	1.000	0.009	0	0.000	0.000	0.000
VAL	-	-	-	-	-	-	-	-	-	1.000	0	0.002	0.000	0.000
SAL	-	-	-	-	-	-	-	-	-	-	1	0.000	0.000	0.000
BBZ	-	-	-	-	-	-	-	-	-	-	-	1.000	0.000	0.015
VER	-	-	-	-	-	-	-	-	-	-	-	-	1.000	0.191
XUN	-	-	-	-	-	-	-	-	-	-	-	-	-	1.000

### 3.4.2. Relationship between stay and the number of beds

We calculated the average length of stay and variability in length of stay for all hospitals and we know the number of beds they had in operation during 2007. We then made a linear adjustment to allow us to determine whether there was a linear relationship between the average length of stay or variance in length of stay and the number of beds in a hospital. As we noted above, long stays can be considered atypical within our samples. Hence we used both the average and standard deviation of the stays as well as the median stay and mad of the stays (median and mad are stronger measurements).

3.4.2.1. Average of stays and beds

Table 4 shows the data after the adjustment has been made. The estimated model would be

$$\text{Average of Stays} \sim 7.092 + 0.0023 * \text{number of beds}$$

obtaining a good fit (). In this model both the intercept and the slope are significantly different from zero ( $p <$  usual level of significance). An  $F$ -test also showed that the slope is different from zero ( $p <$  .05).

To confirm the validity of the adjusted linear model it is necessary to verify that we are in the required hypotheses, that is, that the obtained residuals are normal, homoscedastic and independent. Applying the Shapiro-Wilks test to the residuals indicated that they were normally distributed (Table 5). To assess homoscedasticity we plotted the residuals against the number of beds (Figure 4), which showed that the residuals may not have been homoscedastic. Finally, we verified the independence of the residuals using the Durbin-Watson test (Table 5), which indicated an absence of autocorrelation. In summary, these tests showed that the residuals were independent and normally distributed, but their homoscedasticity was doubtful.

Table 4. Linear fit for the mean / median (left / right) of stay length and the number of beds.

Coefficients:					Coefficients:				
	Estimate	Std. Error	t value	Pr(>  t )		Estimate	Std. Error	t value	Pr(>  t )
(Intercept)	7.0961563	0.2261904	31.372	6.93e-13 ***	(Intercept)	5.1500416	0.2751295	18.719	3.02e-10 ***
n_cam	0.0023486	0.0003265	7.194	1.10e-05 ***	n_cam	0.0006939	0.0003971	1.747	0.106
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Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1					Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1				
Residual standard error: 0.5803 on 12 degrees of freedom					Residual standard error: 0.7058 on 12 degrees of freedom				
Multiple R-squared: 0.8118, Adjusted R-squared: 0.7961					Multiple R-squared: 0.2028, Adjusted R-squared: 0.1364				
F-statistic: 51.76 on 1 and 12 DF, p-value: 1.095e-05					F-statistic: 3.053 on 1 and 12 DF, p-value: 0.1061				

Average Test of the stays Average of the stays

Table 5. *P*-values for the contrasts assessing the normality and independence of the residuals.

Test	Media de las estancias	Mediana de las estancias
Shapiro-Wilk	0.0945	0.4133
Durbin-Watson	0.4204	0.0638

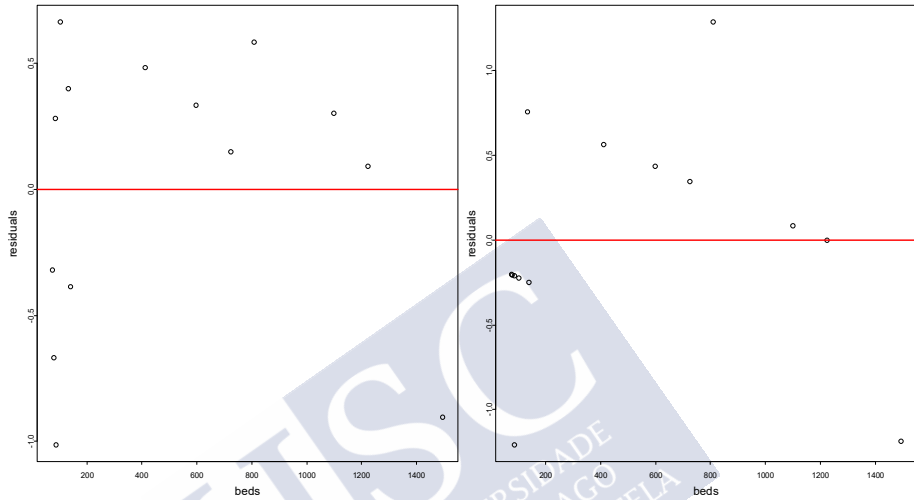


Figure 4. Graphs of the residuals of the mean / median of the stays (left / right) versus the number of beds.

### 3.4.2.2. Medium of the stay length and number of beds

The linear adjustment for the median stay length is given in Table 4. The estimated model comes from

$$\text{Median absolute deviation (MAD)} \sim 5.1500 + 0.0007 * \text{number of beds}$$

for which we have a bad adjustment (). We can assume that the intercept is not zero, but the pendiente is not significantly null for. The latter can also be seen from the *F*-test. We checked the validity of the model by applying the Shapiro-Wilks test and the Durbin-Watson test to the residuals to assess their normality and independence (Table 5); we plotted the residuals against the number of beds to assess homoscedasticity (Figure 4). As in the previous case these analyses indicated that the residuals were normal and not autocorrelated, but it is not clear that they were homoscedastic.

3.4.2.3. Standard deviation of stay length and number of beds

When making the axuste the output contained in Table 6 is obtained. Therefore, the estimated model would be

$$\text{Standard Deviation} = 6.7978 + 0.0072 * \text{number of beds}$$

obtaining an acceptable adjustment (). In this model both the intercept and the pendent were significantly non-zero ( $p <$  the usual level of significance). An  $F$ -test also confirmed that the slope was different from zero ( $p < .05$ ).

Table 6. Linear adjustment for the sd / mad (left / right) of the stays and or number of beds.

Coefficients:					Coefficients:				
	Estimate	Std. Error	t value	Pr(> t )		Estimate	Std. Error	t value	Pr(> t )
(Intercept)	6.7977903	0.6232622	10.907	1.39e-07 ***	(Intercept)	4.5195742	0.2377048	19.013	2.52e-10 ***
n_cam	0.0071620	0.0008995	7.962	3.95e-06 ***	n_cam	0.0009075	0.0003431	2.645	0.0214 *
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Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1					Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1				
Residual standard error: 1.599 on 12 degrees of freedom					Residual standard error: 0.6098 on 12 degrees of freedom				
Multiple R-squared: 0.8408, Adjusted R-squared: 0.8276					Multiple R-squared: 0.3684, Adjusted R-squared: 0.3157				
F-statistic: 63.39 on 1 and 12 DF, p-value: 3.948e-06					F-statistic: 6.998 on 1 and 12 DF, p-value: 0.02136				

Test Sd of the stays Mad of the stays

Table 7. P-values for tests of the normality and independence of the residuals.

Test	Sd de las estancias	Mad de las estancias
Shapiro-Wilk	0.1535	0.0653
Durbin-Watson	0.7033	0.0132

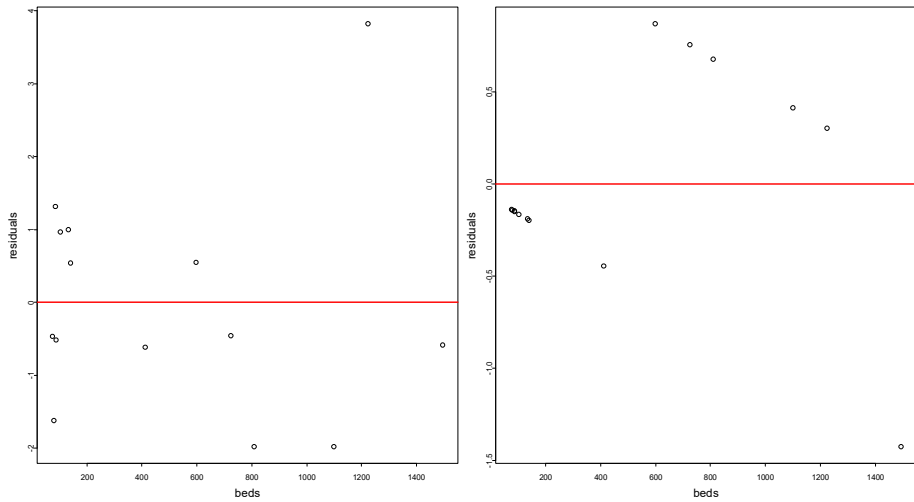


Figure 5. Graphs of the residuals of the sd / mad of the stays (left / right) versus the number of beds.

To verify the validity of the adjusted linear model it is necessary to verify that we are in the required hypotheses, that is, that the obtained residuals are normal, homoscedastic and independent. Applying the Shapiro-Wilks test to the residuals indicated the residuals were normally distributed (Table 7). We checked homoscedasticity by plotting the residuals against the number of beds (Figure 5), which suggested they may not have been homoscedastic. Finally, we verified the independence of the residuals using the Durbin-Watson test (Table 7). In summary, we concluded that the residuals were normally distributed and independent ( $\alpha = 0.05$ ), but there were doubts about homoscedasticity.

#### 3.4.2.4. Mad of the stays and beds

The linear adjustment for the mad of the stays is recorded in Table 6. The estimated model has been

$$\text{Mad of Stays} \sim 4.5196 + 0.0009 * \text{number of beds}$$

for lime we have a bad adjustment (). We can conclude that the intercept is significantly different from zero, the slope is not significantly different from zero for . The latter can also be seen with the  $F$ -test. We assessed the validity of the model by

checking the normality and independence of the residuals with the Shapiro-Wilks test and the Durbin-Watson test respectively (Table 7), and we plotted the residuals against the number of beds to check homoscedasticity (Figure 5). The results indicated that the residuals were normally distributed but autocorrelated and it was not clear that they were homoscedastic.

### 3.4.2.5. Graphical representation and conclusions

Figure 6 shows average and median stay length versus the number of beds in each hospital, along with the lines adjusted in the previous sections. Figure 7 compares the standard deviation and the mad of stay length with the number of beds and shows the corresponding linear adjustments. In conclusion, if we consider median stay length (respectively, in the mad) we can assume that the slope of the line is null for  $\alpha = 0.01$ .

This led us to conclude that perhaps the number of beds does not influence the pattern of patients' stays, when we consider the longer stays as associated data (associated with the centres that treat the most complicated pathologies and that have the highest number of beds). Beds).

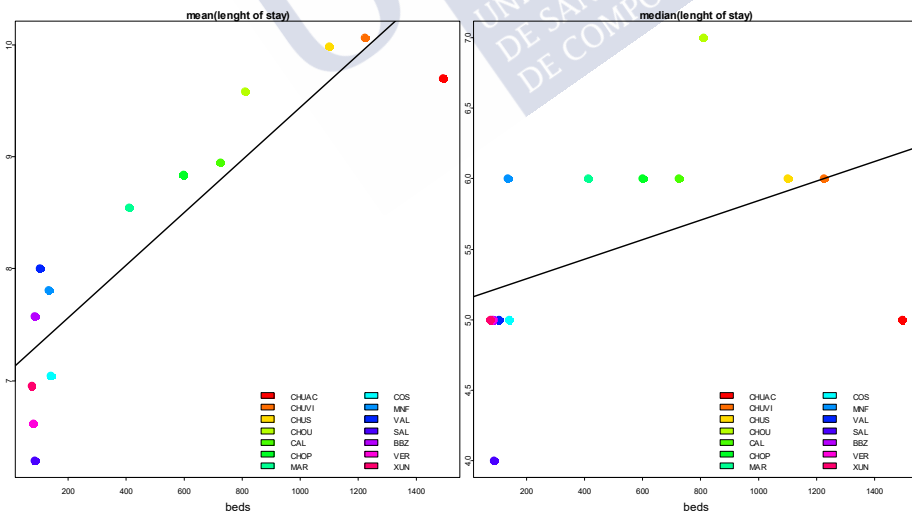


Figure 6. Mean and median stay length versus the number of beds and linear adjustments.

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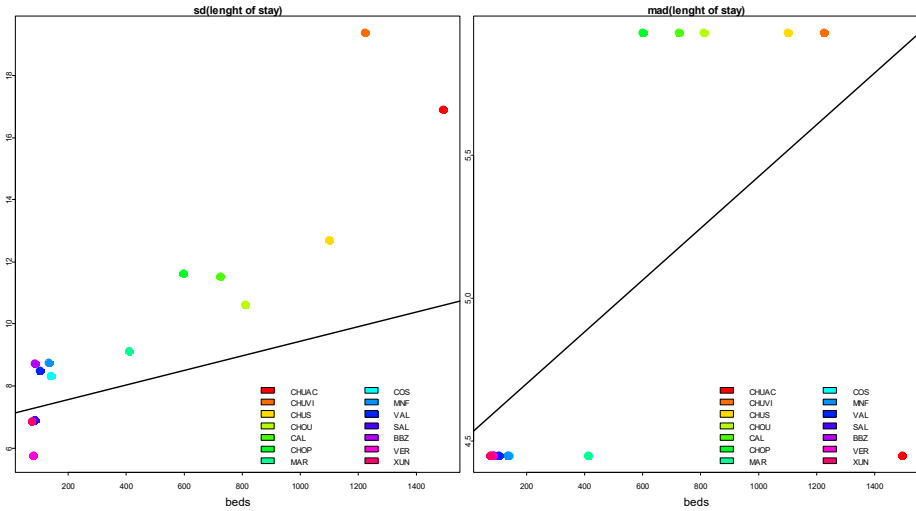


Figure 7. Sd and mad of stay length versus the number of beds and linear adjustments.

#### 3.4.3. Simulation of new values

Figure 3 contains histograms of stay length for each hospital. Our goal was to simulate new values for the variable stay using the observations from 2007.

First we tried to fit a known probability model to the data, namely a Poisson distribution. Recall that if, then the variable takes values in, the probability mass function is given by

$$P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

and, for the adjustment of this distribution we only need to estimate the parameter. We also applied the Kolmogorov-Smirnov test to assess whether the data followed this distribution.

We also used a data generation method based on a non-parametric adjustment. For this, we used a kernel estimator in the density function. Let us suppose that it is a random sample obtained from a univariate, continuous density function. The kernel density estimator is expressed as

$$\hat{f}_h(x) = \frac{1}{nh} \sum_{i=1}^n K\left(\frac{x - X_i}{h}\right)$$

where  $K$  is a kernel function  $\int K(x)dx = 1$  that verifies (usually, a symmetric unimodal density function), and  $h$  is a positive number called bandwidth (‘window’). If we assume that stay length is a continuous random variable we can obtain the kernel estimator for its density, taking as the density function of a normal distribution with a mean = 0 and variance = 1. In this case, it will be necessary to estimate the value of labandwidth for different hospitals.

### 3.4.3.1. Parametric adjustment: poisson distribution

Table 8 shows the estimated values for the parameters of the Poisson distribution. The  $p$ -values associated with the Kolmogorov-Smirnov statistic were also calculated. The results showed that the data did not follow a Poisson distribution in any of the hospitals.

Table 8. Estimated values of the parameter of the Poisson distribution and  $p$ -value of the associated Kolmogorov-Smirnov test.

Hospital	CHUAC	CHUVI	CHUS	CHOU	CAL	CHOP	MAR	COS	MNF	VAL	SAL	BBZ	VER	XUN
$\hat{\lambda}$	9.6988	10.0619	9.9827	9.5808	8.9489	8.8357	8.5443	7.0398	7.8097	8.0033	6.2835	7.5772	6.6130	6.9498
$p$ -valor	0	0	0	0	0	0	0	0	0	0	0	0	0	0

We wish to emphasise that we also attempted to fit a normal distribution to the original data and to Box-Cox transformed (to correct the asymmetry) data. In none of the cases was accepted the hypothesis of normality to apply the Kolmogorov-Smirnov test.

### 3.4.3.2. Noparametric adjustment: kernel estimator of density

The values selected for the smoothing parameter used in the non-parametric adjustment appear in Table 9.

Table 9. Bandwidth estimation for the kernel estimator of the density function for  $X$  for each hospital.

Hospital	CHUAC	CHUVI	CHUS	CHOU	CAL	CHOP	MAR	COS	MNF	VAL	SAL	BBZ	VER	XUN
$\hat{h}$	0.7151	0.7321	0.7444	0.7829	0.7136	0.7314	0.7931	0.8380	0.8686	0.8980	0.7522	1.0401	0.8262	0.8080

### 3.4.3.3. Comparison of poisson and non-parametric estimators

Figure 8 is a histogram of stay length across all hospitals. The solid black line represents the density function of a Poisson parameter (Table 8). The dashed line corresponds to the kernel estimator with the value of the labandwidth collected in Table 9. As can be seen, the non-parametric estimator reproduces the pattern of the longer stays better, since the probability of obtaining such high values using the estimated Poisson distribution is practically zero.

Finally, we performed a small test. We simulated one data set that followed the distribution and another based on the estimated non-parametric density function. The simulated data based on the non-parametric density function were generated as follows:

- Be  $x_1, x_2, \dots, x_n$  the stays of a hospital during the year 2007. We randomly take one of these data  $x_{i_0}$ .
- We generated a value of a standard normal  $z$ .
- We simulated stay length as  $x^* = x_{i_0} + \hat{h} * z$ , where  $\hat{h}$  is the estimated value of the bandwidth for the hospital.
- Since the stay length must be a whole number we rounded up the  $x^*$  generated values.
- Once the simulated samples had been constructed, we applied the two-sample Kolmogorov-Smirnov test to determine whether the original data came from the same continuous distribution as the simulated data.

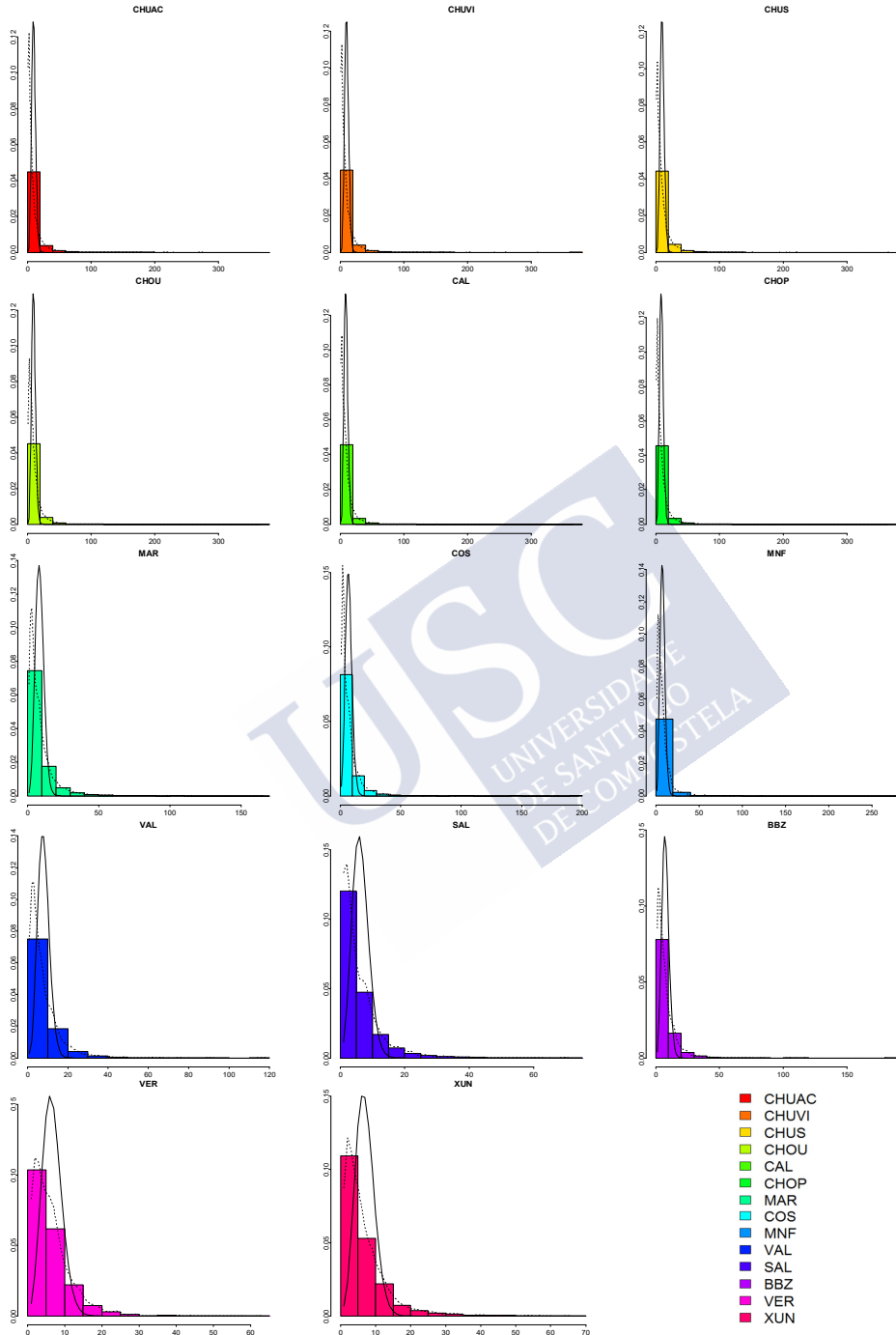


Figure 8. Histogram of stay lengths along with the estimated Poisson distribution (solid line) and the estimated non-parametric distribution (dashed line).

The  $p$ -values obtained seem to confirm that the non-parametric estimator reproduces the pattern of the original data better than the estimated Poisson distribution (), although only in some cases would the null hypothesis be accepted.

Table 10.  $P$ -values for the two-sample Kolmogorov-Smirnov test, one of them being the original data and the other either a sample of one or a sample generated from the non-parametric density estimator.

Hospital	CHUAC	CHUVI	CHUS	CHOU	CAL	CHOP	MAR	COS	MNF	VAL	SAL	BBZ	VER	XUN
$Pois(\hat{\lambda})$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\hat{f}_h$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0013	0.0000	0.1409	0.0376	0.0001	0.0012	0.3046	0.0369

### 3.4.4. Daily admissions

#### 3.4.4.1. Descriptive analysis

We analysed daily admissions in 2007 using a similar approach to that employed for stay length.

Table 11 shows the quartiles and the average for the number of daily admissions in the different hospitals. Obviously, there is a clear direct relationship between the daily admissions to a hospital and the number of beds it has in operation.

Table 11. Average and quartiles of daily admissions in the different hospitals.

Hospital	Mín.	1° Cuartil	Mediana	Media	3° Cuartil	Máx.
CHUAC	54	87	121	118.00	146	196
CHUVI	44	82	105	104.00	127	164
CHUS	35	78	96	95.58	119	155
CHOU	33	63	73	75.02	89	118
CAL	25	52	68	66.21	80	110
CHOP	19	45	59	58.47	72	101
MAR	11	32	39	38.93	46	68
COS	2	11	16	15.40	20	31
MNF	1	9	13	12.86	16	25
VAL	1	8	11	11.23	15	23
SAL	3	9	12	12.35	15	28
BBZ	1	7	10	10.13	13	29
VER	1	5	8	7.62	10	18
XUN	2	6	8	8.41	10	18

It seems reasonable to assume that there are two different patterns of admissions for working days and non-working days. We decided to treat all days in the months of July, August and December as non-working days, as well as Saturdays and Sundays. Figures 9 and 10 show that the number of admissions was clearly lower on non-working days. We used a series of Kolmogorov-Smirnov tests to confirm that this difference was significant in all the hospitals. In all cases the null hypothesis that the distribution of admissions was the same on non-working days and working days was rejected, hence in further analyses of admissions working days were analysed separately from non-working days.

#### 3.4.4.2. Clusters

We used the Kolmogorov-Smirnov test to determine whether the distribution of admissions was the same in pairs of hospitals (Tables 12 and 13). In the case of working days, the MNF-SAL and VER-XUN groupings appear. On non-working days the following associations were observed: CHUAC-CHUVI, MNF-VAL-SAL-BBZ and VER-XUN.

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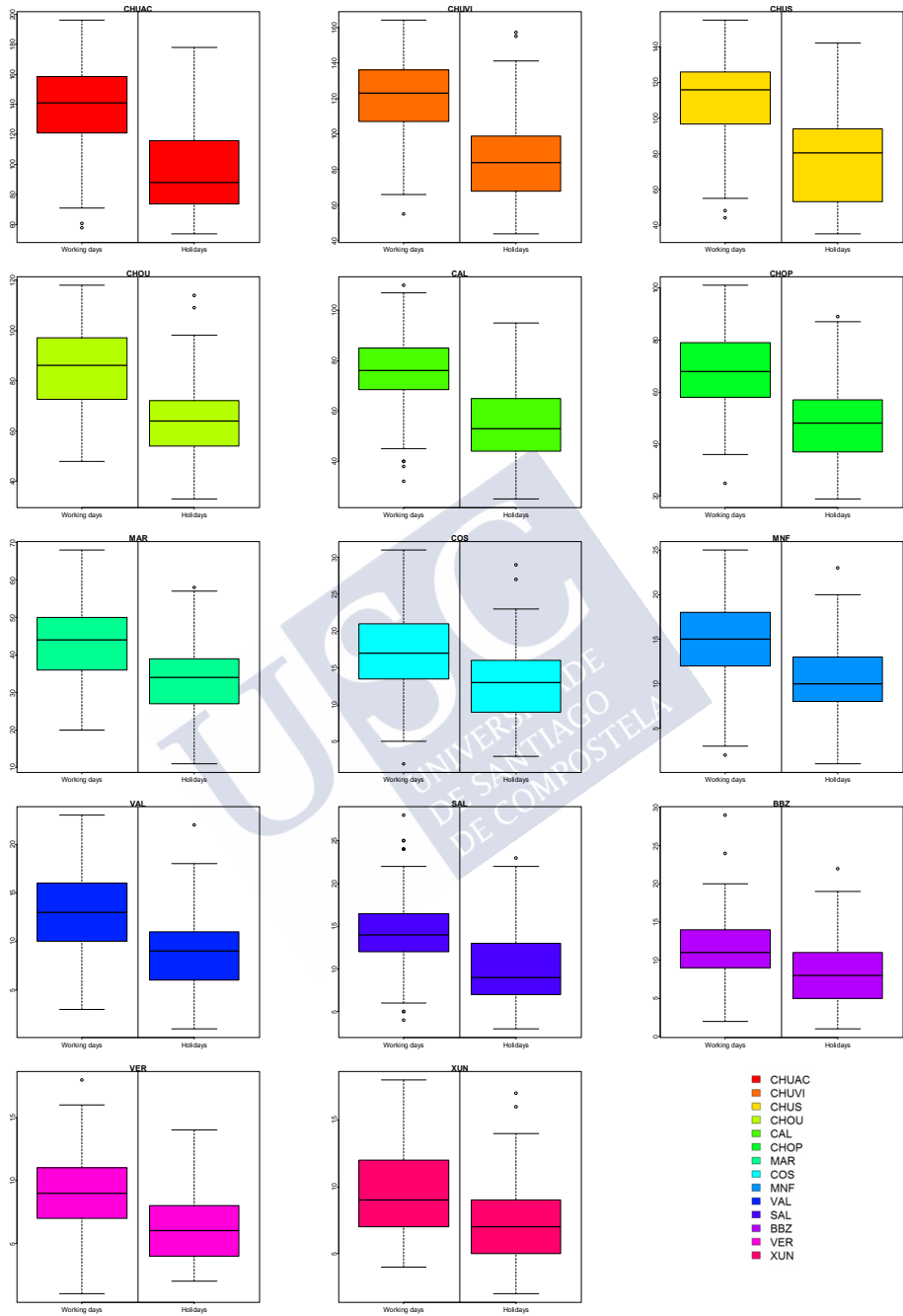


Figure 9. Box plots of daily admissions on working and non-working days for all hospitals.

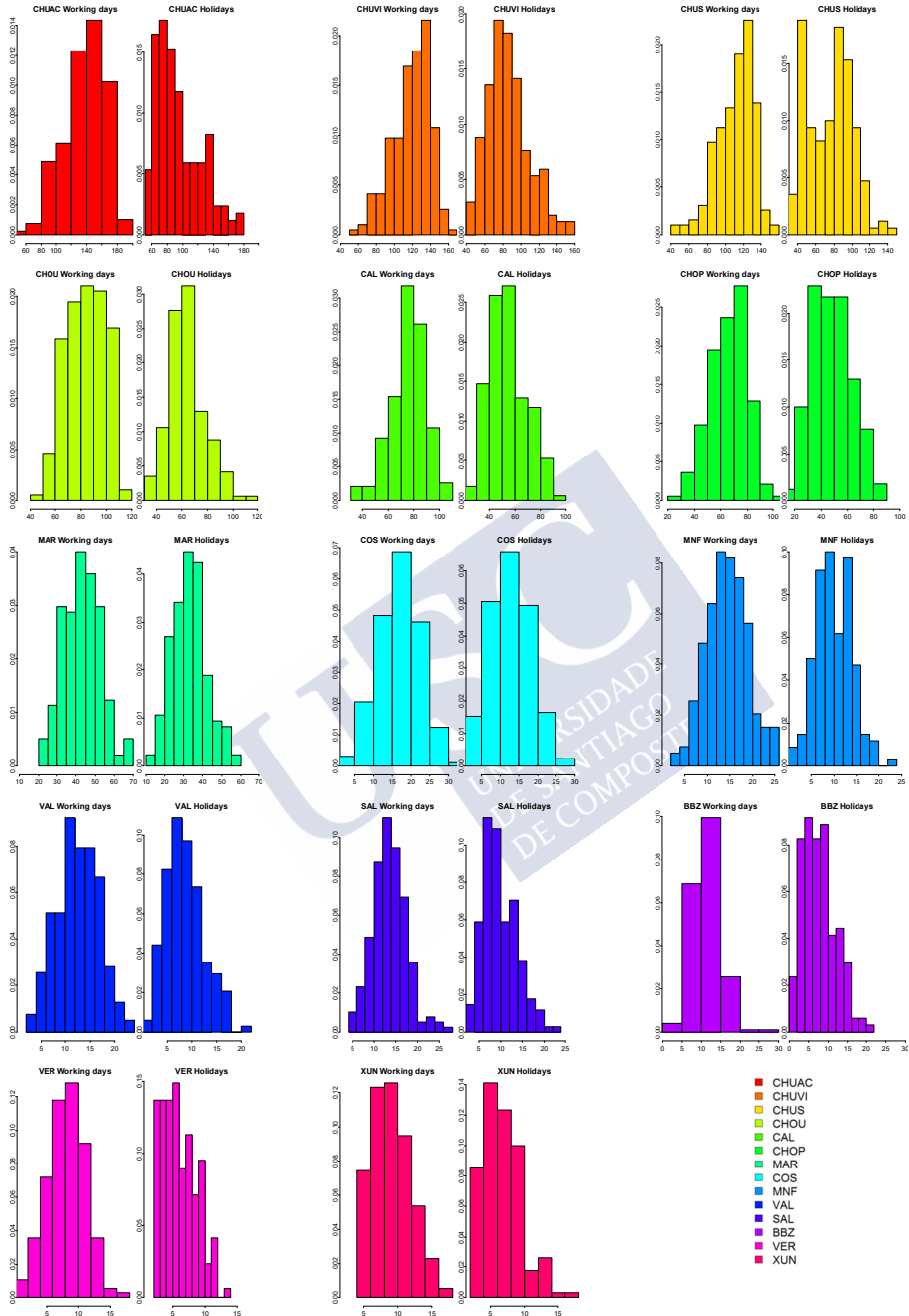


Figure 10. Histograms of daily admissions  $n$  working and non-working days for all hospitals.

### 3. INPATIENT WAITING LISTS: HOW MANY HOSPITAL BEDS ARE ENOUGH? A SIMULATION ANALYSIS

Table 12. *P*-values of the Kolmogorov-Smirnov test for admissions on working days.

Hospital	CHUAC	CHUVI	CHUS	CHOU	CAL	CHOP	MAR	COS	MNF	VAL	SAL	BBZ	VER	XUN
CHUAC	1	0.000	0.000	0	0	0	0	0	0.000	0.000	0.000	0.000	0.000	0.000
CHUVI	-	1.000	0.001	0	0	0	0	0	0.000	0.000	0.000	0.000	0.000	0.000
CHUS	-	-	1.000	0	0	0	0	0	0.000	0.000	0.000	0.000	0.000	0.000
CHOU	-	-	-	1	0	0	0	0	0.000	0.000	0.000	0.000	0.000	0.000
CAL	-	-	-	-	1	0	0	0	0.000	0.000	0.000	0.000	0.000	0.000
CHOP	-	-	-	-	-	1	0	0	0.000	0.000	0.000	0.000	0.000	0.000
MAR	-	-	-	-	-	-	1	0	0.000	0.000	0.000	0.000	0.000	0.000
COS	-	-	-	-	-	-	-	1	0.000	0.000	0.000	0.000	0.000	0.000
MNF	-	-	-	-	-	-	-	-	1.000	0.014	0.167	0.000	0.000	0.000
VAL	-	-	-	-	-	-	-	-	-	1.000	0.005	0.001	0.000	0.000
SAL	-	-	-	-	-	-	-	-	-	-	1.000	0.000	0.000	0.000
BBZ	-	-	-	-	-	-	-	-	-	-	-	1.000	0.000	0.000
VER	-	-	-	-	-	-	-	-	-	-	-	-	1.000	0.449
XUN	-	-	-	-	-	-	-	-	-	-	-	-	-	1.000

Table 13. *P*-values for the Kolmogorov-Smirnov test on non-working days.

Hospital	CHUAC	CHUVI	CHUS	CHOU	CAL	CHOP	MAR	COS	MNF	VAL	SAL	BBZ	VER	XUN
CHUAC	1.000	0.089	0	0	0.000	0.000	0	0	0.000	0.000	0.000	0.000	0.000	0.000
CHUVI	-	1.000	0	0	0.000	0.000	0	0	0.000	0.000	0.000	0.000	0.000	0.000
CHUS	-	-	1	0	0.000	0.000	0	0	0.000	0.000	0.000	0.000	0.000	0.000
CHOU	-	-	-	1	0.000	0.000	0	0	0.000	0.000	0.000	0.000	0.000	0.000
CAL	-	-	-	-	1.000	0.005	0	0	0.000	0.000	0.000	0.000	0.000	0.000
CHOP	-	-	-	-	-	1.000	0	0	0.000	0.000	0.000	0.000	0.000	0.000
MAR	-	-	-	-	-	-	1	0	0.000	0.000	0.000	0.000	0.000	0.000
COS	-	-	-	-	-	-	-	1	0.000	0.000	0.000	0.000	0.000	0.000
MNF	-	-	-	-	-	-	-	-	1.000	0.014	0.439	0.000	0.000	0.000
VAL	-	-	-	-	-	-	-	-	-	1.000	0.149	0.063	0.000	0.000
SAL	-	-	-	-	-	-	-	-	-	-	1.000	0.000	0.000	0.000
BBZ	-	-	-	-	-	-	-	-	-	-	-	1.000	0.002	0.013
VER	-	-	-	-	-	-	-	-	-	-	-	-	1.000	0.072
XUN	-	-	-	-	-	-	-	-	-	-	-	-	-	1.000

#### 3.4.5. Relationship between daily admissions and number of beds

We estimated the average number of daily admissions (and variability in daily admissions) for all hospitals using the mean / median data from 2007 (respectively, sd / mad). We wanted to determine whether there was a linear relationship between

admissions and the number of hospital beds and to do this we calculated two linear adjustments: one for working days and another for non-working days.

### 3.4.5.1. Average number of daily admissions and beds

Tables 14 and 15 contain the adjustments obtained for daily admissions on working days and non-working days respectively. In particular for the case of the average of the admissions the models that are obtained are the following ones

$$\text{Mean of inpatients in working days} \sim 4.5398 + 0.0943 * \text{number of beds}$$

$$\text{Mean of inpatients in non-working days} \sim 4.5398 + 0.0943 * \text{number of beds}$$

obtaining in both cases a good adjustment (99.45% and 98.84%, respectively). In addition, the intercept and the slope of the working days function were significantly different from zero ( $p < .05$ ). An  $F$ -test revealed that the slopes were not null ( $p < .05$ ).

Table14. Linear adjustment for the average / median (left / right) number of daily admissions on working days.

Coefficients:					Coefficients:						
	Estimate	Std. Error	t value	Pr(> t )		Estimate	Std. Error	t value	Pr(> t )		
(Intercept)	<b>4.539758</b>	1.408342	3.223	<b>0.0073 **</b>	(Intercept)	<b>3.961936</b>	1.435988	2.759	<b>0.0173 *</b>		
n_cam	<b>0.094335</b>	0.002033	46.411	<b>6.54e-15 ***</b>	n_cam	<b>0.097229</b>	0.002073	46.914	<b>5.75e-15 ***</b>		
---					---						
Signif. codes:	0	****	0.001	***	0.01	**	0.05	‘.’	0.1	‘ ’	1
Residual standard error:	3.613 on 12 degrees of freedom				Residual standard error:	3.684 on 12 degrees of freedom					
Multiple R-squared:	<b>0.9945</b> , Adjusted R-squared: 0.994				Multiple R-squared:	<b>0.9946</b> , Adjusted R-squared: 0.9941					
F-statistic:	2154 on 1 and 12 DF, p-value: <b>6.539e-15</b>				F-statistic:	2201 on 1 and 12 DF, p-value: <b>5.75e-15</b>					

Table15. Linear adjustment for the average / median (left / right) number of daily admissions on non-working days.

Coefficients:					Coefficients:						
	Estimate	Std. Error	t value	Pr(> t )		Estimate	Std. Error	t value	Pr(> t )		
(Intercept)	<b>4.069079</b>	1.438085	2.83	<b>0.0152 *</b>	(Intercept)	<b>4.23618</b>	2.00254	2.115	<b>0.056 .</b>		
n_cam	<b>0.066256</b>	0.002076	31.92	<b>5.64e-13 ***</b>	n_cam	<b>0.06432</b>	0.00289	22.256	<b>3.99e-11 ***</b>		
---					---						
Signif. codes:	0	****	0.001	***	0.01	**	0.05	‘.’	0.1	‘ ’	1
Residual standard error:	3.689 on 12 degrees of freedom				Residual standard error:	5.138 on 12 degrees of freedom					
Multiple R-squared:	<b>0.9884</b> , Adjusted R-squared: 0.9874				Multiple R-squared:	<b>0.9763</b> , Adjusted R-squared: 0.9744					
F-statistic:	1019 on 1 and 12 DF, p-value: <b>5.635e-13</b>				F-statistic:	495.3 on 1 and 12 DF, p-value: <b>3.991e-11</b>					

Test Average of admissions Average of admissions

Table 16. *P*-values for tests of the normality and independence of the residuals for working days.  
Test Average of admissions Average of admissions

Test	Media de los ingresos	Mediana de los ingresos
Shapiro-Wilk	0.8442	0.5102
Durbin-Watson	0.0032	0.0112

Table 17. *P*-values for tests of the normality and independence of the residuals for non-working days.

Test	Media de los ingresos	Mediana de los ingresos
Shapiro-Wilk	0.8424	0.2592
Durbin-Watson	0.0091	0.0027

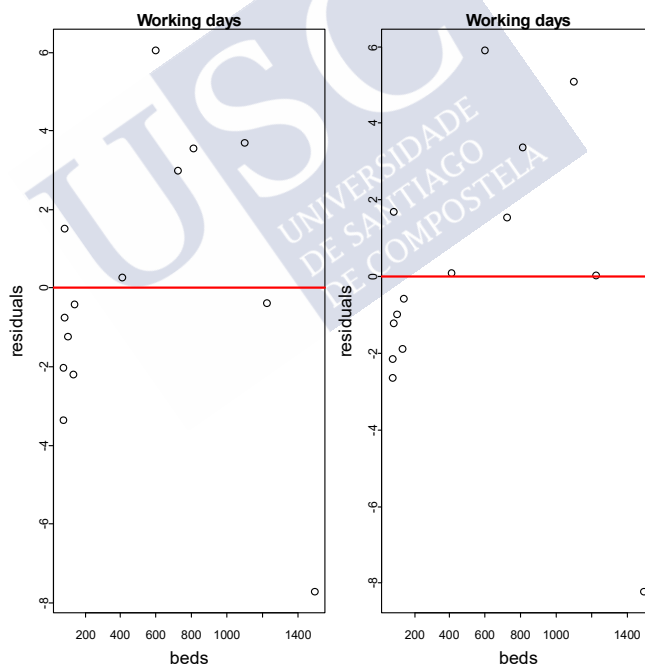


Figure 11. Graphs of the residuals for the average / median (left / right) number of admissions in front of the beds in the working days.

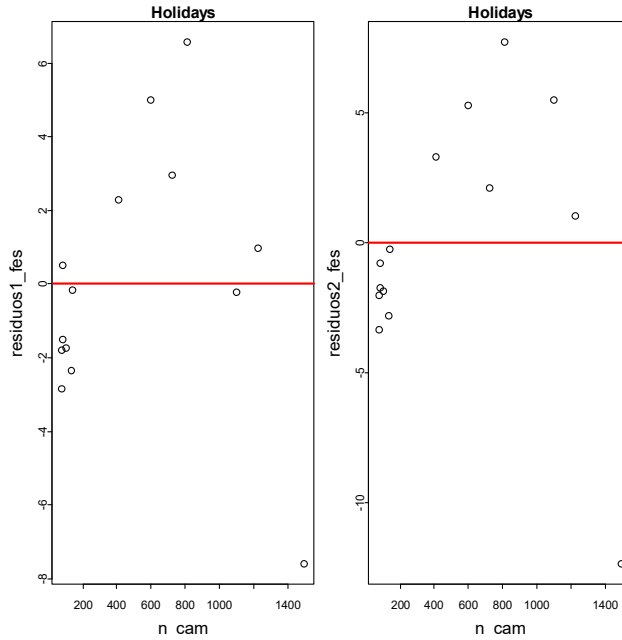


Figure 12. Graphs of the residuals for the average / median (left / right) number of admissions in front of the beds on non-working days.

Again, we needed to check that the residuals were normal, homoscedastic and independent. Tables 16 and 17 show the results of the normality and independence tests, which indicated that for both working and non-working days the residuals were normally distributed but not independent. It is not clear from Figures 11 and 12 whether the residuals were homoscedastic.

#### 3.4.5.2. Medium of daily admissions and beds

Tables 14 and 15 contain the linear regression adjustment for admissions on working and non-working days respectively. The models would be

$$\text{Median of inpatients in working days} \sim 3.9619 + 0.0972 * \text{number of beds}$$

$$\text{Median of inpatients in non-working days} \sim 4.2362 + 0.0643 * \text{number of beds}$$

obtaining in both cases a good adjustment (99.46% and 97.63%, respectively). In addition, the intercept for working days and both pending were significantly different

from zero ( $p < .05$ ).  $F$ -tests confirmed that the slopes were not null (both  $ps < .05$ ). We also checked the normality, homoscedasticity and independence of the residuals.

Tables 16 and 17 show that for both working and non-working days the residuals were normal, but not independent. Figures 11 and 12 show the residuals plotted against the number of beds and were used to analyse the homoscedasticity of the residuals. They suggest that the residuals were not homoscedastic in either case.

### 3.4.5.3. Standard deviation of daily admissions and beds

Tables 18 and 19 contain the adjustments obtained for daily admissions on working and non-working days respectively. In particular for the case of the average admissions and the models obtained are the following

*Standard Deviation of inpatients in working days*~ $2.5020 + 0.0161 * \text{number of beds}$

*Standard Deviation of inpatients in non-working days*~ $2.0305 + 0.0181 * \text{number of beds}$

obtaining in both cases a good adjustment (98.78% and 98.3%, respectively). In addition, the intercept and the slope for working days were significantly different from zero ( $p < .05$ ) and this was confirmed with an  $F$ -test ( $p < .05$ ). Again, we needed to check that the residuals were normal, homoscedastic and independent. Tables 20 and 21 show the results of the normality and independence tests, which were positive. On the other hand, Figures 13 and 14 show that it is not clear whether the residuals were homoscedastic.

Table 18. Linear adjustment for the sd / mad (left / right) of daily admissions on working days.

Coefficients:					Coefficients:				
	Estimate	Std. Error	t value	Pr(> t )		Estimate	Std. Error	t value	Pr(> t )
(Intercept)	2.5020049	0.3573112	7.002	1.43e-05 ***	(Intercept)	2.3747031	0.6028371	3.939	0.00197 **
n_cam	0.0160780	0.0005157	31.177	7.46e-13 ***	n_cam	0.0164986	0.0008701	18.963	2.59e-10 ***
---					---				
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1					Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1				
Residual standard error: 0.9167 on 12 degrees of freedom					Residual standard error: 1.547 on 12 degrees of freedom				
Multiple R-squared: 0.9878, Adjusted R-squared: 0.9868					Multiple R-squared: 0.9677, Adjusted R-squared: 0.965				
F-statistic: 972 on 1 and 12 DF, p-value: 7.458e-13					F-statistic: 359.6 on 1 and 12 DF, p-value: 2.595e-10				

Table19. Linear adjustment for the sd / mad (left / right) of daily admissions on non-working days. Sd test of admissions Mad of admissions

Coefficients:					Coefficients:				
	Estimate	Std. Error	t value	Pr(> t )		Estimate	Std. Error	t value	Pr(> t )
(Intercept)	2.0305408	0.4755016	4.27	0.00109 **	(Intercept)	2.298388	0.985318	2.333	0.0379 *
n_cam	0.0180558	0.0006863	26.31	5.56e-12 ***	n_cam	0.018120	0.001422	12.742	2.47e-08 ***
---					---				
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1					Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1				
Residual standard error: 1.22 on 12 degrees of freedom					Residual standard error: 2.528 on 12 degrees of freedom				
Multiple R-squared: 0.983, Adjusted R-squared: 0.9815					Multiple R-squared: 0.9312, Adjusted R-squared: 0.9254				
F-statistic: 692.2 on 1 and 12 DF, p-value: 5.564e-12					F-statistic: 162.4 on 1 and 12 DF, p-value: 2.472e-08				

Table20. P-values for tests of the normality and independence of the residuals for working days. Sd test of admissions Mad of admissions

Test	Sd de los ingresos	Mad de los ingresos
Shapiro-Wilk	0.9474	0.8027
Durbin-Watson	0.4415	0.7478

Table 21. P-values for tests of the normality and independence of the residuals for non-working days.

Test	Sd de los ingresos	Mad de los ingresos
Shapiro-Wilk	0.2721	0.0482
Durbin-Watson	0.9390	0.9520

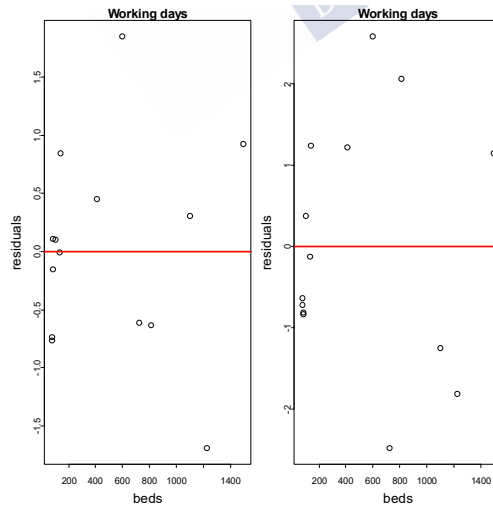


Figure 13. Graphs of the residuals for the sd / mad (left / right) of admissions in front of the beds in the working days.

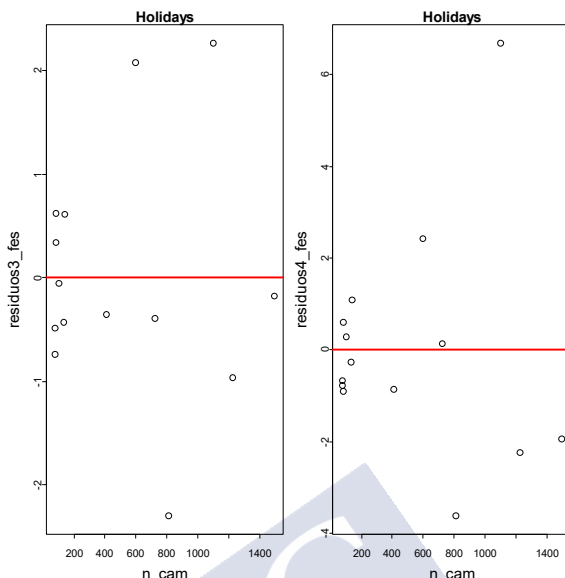


Figure 14. Graphs of the residuals for the sd / mad (left / right) of admissions in front of the beds in the non working days.

#### 3.4.5.4. Mad of daily admissions and beds

Tables 18 and 19 contain the adjustment of the linear regression for admissions on working and non-working days respectively.

The models would be

$$\text{Mad of inpatients in working days} \sim -2.3747 + 0.0165 * \text{number of beds}$$

$$\text{Mad of inpatients in non-working days} \sim -2.3747 + 0.0165 * \text{number of beds}$$

obtaining in both cases a good adjustment (96.77% and 93.12%, respectively). In addition, both slopes were significantly different from zero ( $p < .05$ ) and this was confirmed by the results of  $F$ -tests (both  $ps < .05$ ). We also checked the normality, homoscedasticity and independence of the residuals. Tables 20 and 21 show the residuals for non-working days were normally distributed and that in both cases they were independent. Figures 13 and 14 show the residuals plotted against the number of beds and were used to analyse the homoscedasticity of the residuals; they indicate that the residuals may not have been homoscedastic.

3.4.5.5. Graphical representation and conclusions

In Figure 15 and Figure 16 (Figure 17 and Figure 18, respectively), we represent the average / median (sd and mad, respectively) of daily admissions versus the number of beds, together with the previously fitted regression lines for working and non-working days.

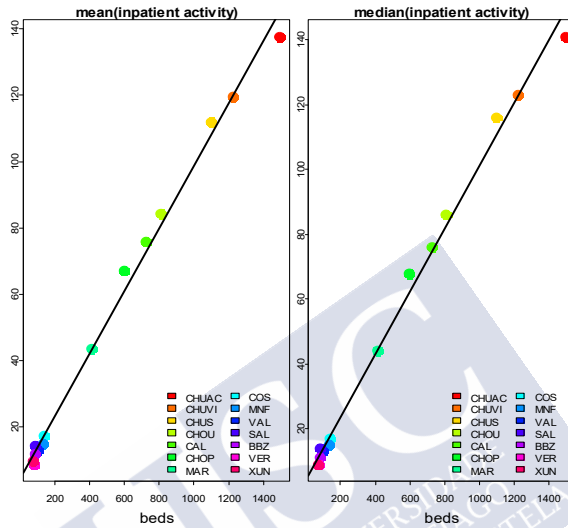


Figure 15. Average and median admissions on working days versus the number of beds and linear adjustments.

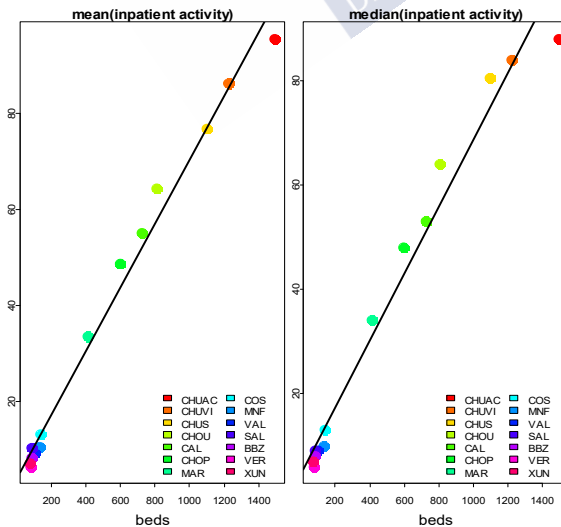


Figure 16. Average and median admissions on non-working days versus the number of beds and linear adjustments.

### 3. INPATIENT WAITING LISTS: HOW MANY HOSPITAL BEDS ARE ENOUGH? A SIMULATION ANALYSIS

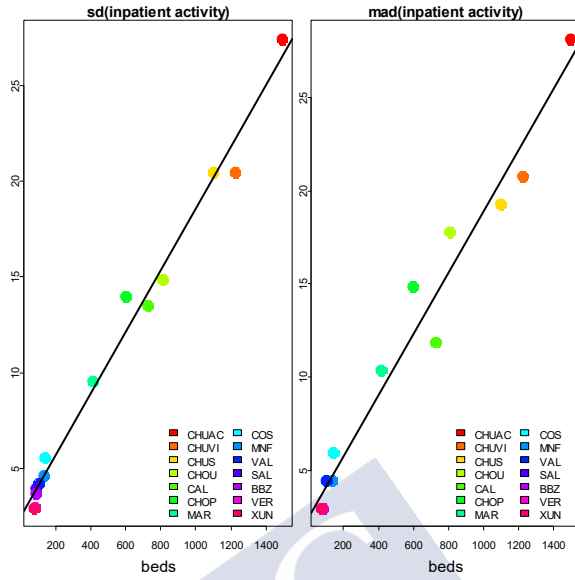


Figure 17. Sd and mad of working admissions versus the number of beds and linear adjustments.

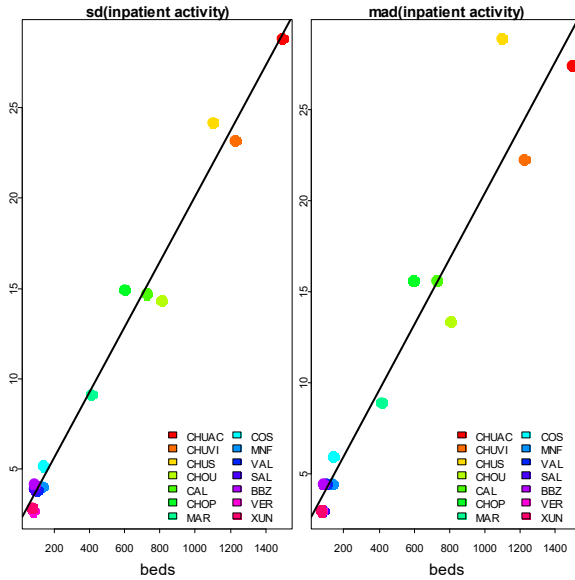


Figure 18. Sd and mad of admissions on non-working days compared with the number of beds and linear adjustments.

We can conclude that on both working and non-working days there is a clear linear relationship between the number of beds in the hospital and the pattern of daily admissions, as we had already assumed.

### 3.4.6. Simulation of new values

Figure 10 shows the daily admissions histograms for each hospital. We wanted to simulate admissions based on the daily admissions data for 2007. We did this by fitting parametric and non-parametric distributions to the original data.

#### 3.4.6.1. Parametric adjustment: normal distribution

We started by fitting a Poisson distribution to the data. As the estimates of the parameter  $\lambda$  were going to be greater than 10, we approximated the Poisson distribution with a normal distribution with adequate parameters and used the Kolmogorov-Smirnov test to determine whether the data were normally distributed.

Table22. Estimated values of the parameters of the normal distribution and  $p$ -value of the associated Kolmogorov-Smirnov test for working days.

Hospital	CHUAC	CHUVI	CHUS	CHOU	CAL	CHOP	MAR	COS	MNF	VAL	SAL	BBZ	VER	XUN
$\hat{\mu}$	137.749	119.610	112.015	84.415	75.928	67.000	43.574	17.338	14.897	13.026	14.169	11.810	8.626	9.497
$\hat{\sigma}$	27.376	20.440	20.445	14.839	13.512	13.930	9.536	5.585	4.621	4.248	3.984	3.707	3.001	2.950
<b>p-value</b>	0.108	0.179	0.050	0.300	0.156	0.319	0.760	0.251	0.591	0.250	0.190	0.028	0.188	0.040

Table23. Estimated values of the parameters of the normal distribution and  $p$ -value of the associated Kolmogorov-Smirnov test for non-working days.

Hospital	CHUAC	CHUVI	CHUS	CHOU	CAL	CHOP	MAR	COS	MNF	VAL	SAL	BBZ	VER	XUN
$\hat{\mu}$	95.453	86.141	76.724	64.235	55.053	48.682	33.594	13.171	10.518	9.165	10.265	8.189	6.446	7.165
$\hat{\sigma}$	28.747	23.093	24.084	14.295	14.679	14.864	9.070	5.159	3.987	3.821	3.907	4.177	2.705	2.873
<b>p-value</b>	0.002	0.281	0.048	0.212	0.171	0.412	0.686	0.509	0.132	0.058	0.000	0.055	0.010	0.032

Tables 22 and 23 show the estimated values for the mean and standard deviation in all the hospitals, and the contrast values. In most cases the results indicate the normality of the data.

### 3.4.6.2. Non-parametric adjustment: kernel estimator of density

Based on the assumption that the number of daily admissions is a continuous, random variable, we calculated a kernel estimator of its density function, taking as K function the density of a standard normal distribution.

The values of the bandwidth estimates for the different hospitals are shown in Tables 24 and 25.

Table 24. Estimated values of bandwidth for working days.

Hospital	CHUAC	CHUVI	CHUS	CHOU	CAL	CHOP	MAR	COS	MNF	VAL	SAL	BBZ	VER	XUN
$\hat{h}$	8.6045	6.4244	6.4260	4.6638	3.8602	4.3782	2.9973	1.7547	1.4037	1.3352	1.0528	1.1652	0.9358	0.9271

Table 25. Estimated values of bandwidth for non-working days.

Hospital	CHUAC	CHUVI	CHUS	CHOU	CAL	CHOP	MAR	COS	MNF	VAL	SAL	BBZ	VER	XUN
$\hat{h}$	9.2901	7.2740	7.7833	4.2081	4.7440	4.8037	2.8856	1.6671	1.2023	1.2023	1.2627	1.3514	0.8764	0.9285

### 3.4.6.3. Comparison: normal distribution vs non-parametric estimator

Figure 19 shows histograms of daily admissions to all hospitals on working and non-working days. The solid black lines are the density function of the corresponding estimated normals and the dashed black lines the non-parametric density estimates. In this case, the two estimates of the density function give similar results.

Finally, we simulated a data set with each of the density estimates considered and used the two-sample Kolmogorov-Smirnov test to determine . The  $p$ -values obtained indicate that in most cases the null hypothesis is accepted for both the normal and non-parametric approaches (Tables 26 and 27 respectively).

Table 26.  $P$ -values for a two-sample Kolmogorov-Smirnov test comparing the original data for working day admissions with a sample or a sample generated from the non-parametric density estimator.

Hospital	CHUAC	CHUVI	CHUS	CHOU	CAL	CHOP	MAR	COS	MNF	VAL	SAL	BBZ	VER	XUN
$N(\hat{\mu}, \hat{\sigma})$	0.5277	0.4490	0.3771	0.4490	0.2566	0.2566	0.9597	0.8540	0.9855	0.9597	0.5277	0.9855	0.5277	0.9966
$\hat{f}_h$	0.7791	0.7791	0.6963	0.5277	0.3129	0.8540	0.9996	1.0000	0.8540	1.0000	0.1670	0.7791	0.9597	0.4490

Table 27.  $P$ -values for a two-sample Kolmogorov-Smirnov test comparing the original data for non-working day admissions with a sample or a sample generated from the non-parametric density estimator.

Hospital	CHUAC	CHUVI	CHUS	CHOU	CAL	CHOP	MAR	COS	MNF	VAL	SAL	BBZ	VER	XUN
$N(\hat{\mu}, \hat{\sigma})$	0.0375	0.3631	0.1494	0.5224	0.7908	0.6116	0.9303	0.7029	0.3631	0.6116	0.0506	0.6994	0.9272	0.9303
$\hat{f}_h$	0.3631	0.5224	0.5224	0.0506	0.7908	0.8688	0.8688	0.9918	0.9987	0.5224	0.9711	1.0000	0.9912	0.8688

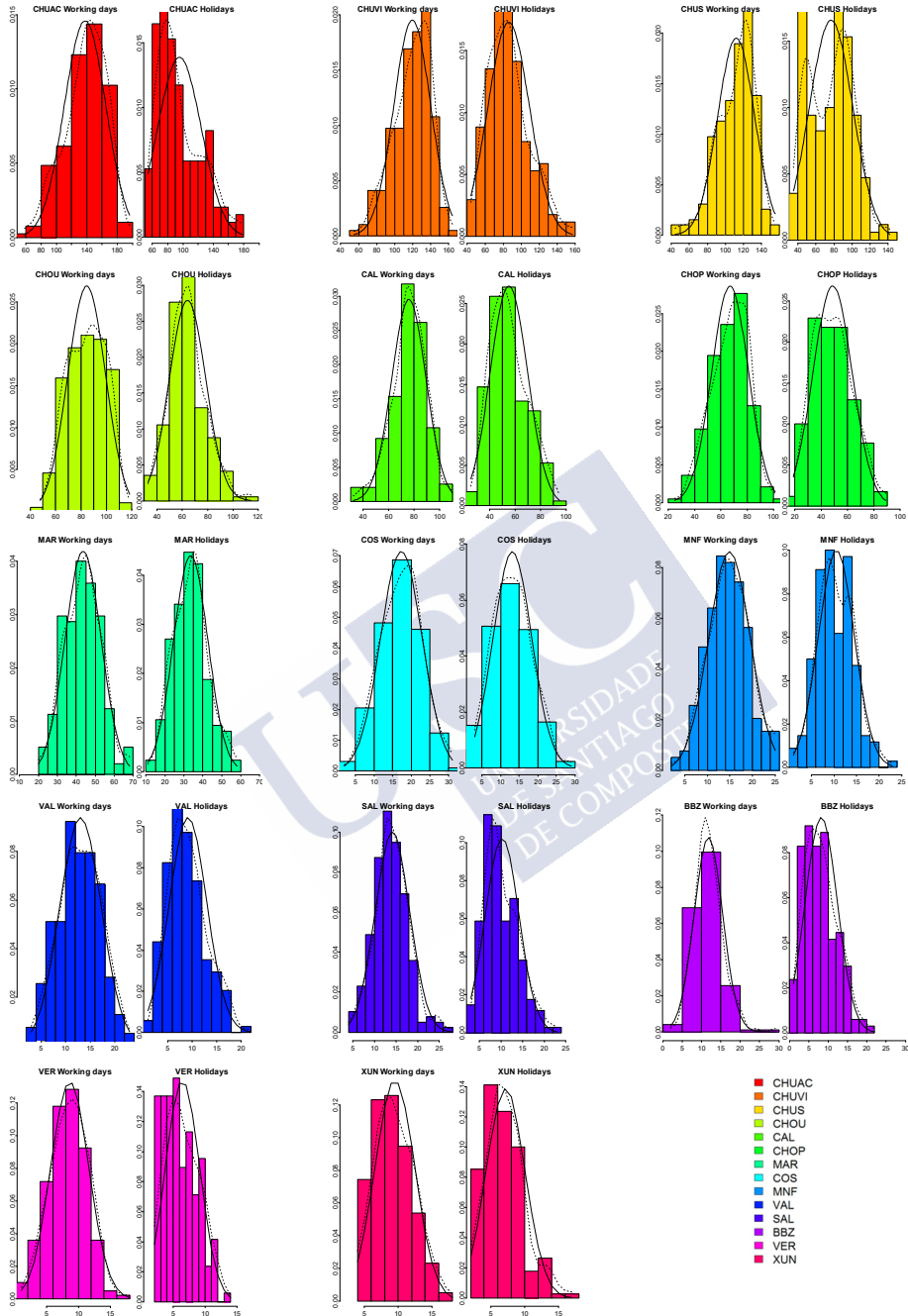


Figure 19. Histograms of daily admissions for all hospitals on working and non-working days, together with the estimated normal density function (solid line) and the estimated non-parametric density function (dashed line).

#### 3.4.6.4. Simulation study

We wanted to simulate admissions to Galician hospitals and the duration of patients stays in order to analyse waiting lists and bed occupancy. We considered different numbers of beds in order to evaluate how this parameter affects occupancy and waiting lists. In particular, if  $n_i$  is the number of hospital beds  $i$ , the values for the number of beds were taken as:  $\{n_i, n_i + 5\%n_i, n_i + 10\%n_i, \dots, n_i + 50\%n_i\}$

We simulated data for the years 2007-2010, performing 500 iterations, in each of which the daily number of admissions and length of stay for each admission were simulated. During this process we constructed indices of the waiting list and daily occupancy. To avoid starting the simulation with empty hospitals (an unrealistic situation), the process began to simulate August 1, 2006.

Patient stays were simulated using a non-parametric kernel-type estimator of the density (with the previously calculated bandwidth). Admissions were simulated by extracting normal values with different parameters, second the day is working or festive.

As we saw earlier, the number of beds in a hospital seems to be related to its daily admissions, which is why we carried out two different studies. As a first approximation we simulated admissions without taking into account the number of beds (without elasticity), using the normal distributions we had already fitted to the data to generate the numbers of admissions.

However, as previously mentioned, the number of beds in a hospital seems to affect its daily admissions, so to take this into account we adapted the admissions simulation procedure. Using a linear adjustment we obtained a normal distribution that we subsequently used to generate averages

$c_1 + c_2 * n^q$  de camas and standard deviations  $c_3 + c_4 * n^q$  de camas, using coefficients  $c_j$  obtained from the earlier linear adjustments (with elasticity).

#### 3.4.6.5. Comparison of simulated and actual admissions

Figure 20 shows the 500 simulated daily admissions for the actual number of beds at each centre in 2007 (grey lines) and the actual admissions during 2007 (black line). The revenues were generated without taking into account the elasticity.

Figure 21 presents the results obtained when elasticity was taken into account. In general, simulations of admissions using both techniques follow the pattern of actual admissions in 2007. However, we must take into account that when the adjustment of the regression line considered in the case of elasticity. It is not very good for a specific hospital, the simulated admissions may seem "displaced".

For example, in the cases of SEE and XUN, average daily admissions were lower than the value assigned by the adjusted regression line; in other words simulated admissions tend to be higher than actual (observed) admissions, shown in Figure 21.

#### 3.4.6.6. Waiting lists

The averages of the 500 simulated waiting lists for each number of beds are shown in Figure 22 (without elasticity) and in Figure 23 (with elasticity).

It can be seen that, in general, when simulating the data without elasticity, waiting lists are reduced by increasing bed numbers, sometimes to the point where they disappear, whereas when we simulate the data with elasticity increasing bed numbers appears to have less effect on waiting lists.

There were also hospitals for which the waiting list soared as much in the simulation without elasticity as in the simulation with elasticity. In all cases in which the waiting list increased continuously this was mitigated by increasing the number of beds in the hospital.

Some of the differences between the simulations with and without elasticity may be due to the fact that in the former case the average from which the number of admissions is generated is that obtained from the real data, whereas in the latter the value assigned by the number of beds of interest to the adjusted straight line is used.

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This adjusted value may be higher than the observed average (e.g. the waiting lists in SEE and XUN did not increase exponentially in the simulation without elasticity, but did in the simulation with elasticity) or lower (e.g. the SAL waiting list increases dramatically in simulation without elasticity but only moderately in the simulation with elasticity).

#### CLUSTER 1: CHUAC, CHUVI and CHUS

##### Without elasticity

- CHUAC. With the number of beds there was in 2007 there should not be a waiting list.
- CHUVI and CHUS. Assuming the number of beds there was in 2007 the waiting lists peak at 150 and 400 patients, respectively. Increasing the number of beds by just 5% is sufficient to reduce the waiting lists drastically and with larger increases they are almost eliminated.

##### With elasticity

- CHUAC. Presents waiting list peaks at 60 assuming the 2007 number of beds. Increasing bed numbers slightly reduces waiting lists.
- CHUVI and CHUS. As in the without-elasticity simulation the waiting lists peak at much high numbers than at CHUAC, but increasing bed numbers has little impact.

#### CLUSTER 2: CHOU, CAL, CHOP and MAR

##### Without elasticity

- CHOU and CHOP. The waiting lists have high peaks (400 and 250 respectively), which are considerably reduced by increasing bed numbers.
- CAL and MAR. Waiting lists are minimal with the current number of beds (peaks of 25 and 12, respectively). The lists disappear with a slight increase in the number of beds.

##### With elasticity

- CHOU. The peaks are reduced to 80 patients. The increase in the number of beds helps to reduce waiting lists.
- CAL, CHOP and MAR. Waiting lists peak at less than 10. It also begins to appreciate the effect of the number of beds.

### CLUSTER 3: COS, MNF, VAL, SAL, BBZ, SEE and XUN

#### Without elasticity

- COS, MNF, SEE and XUN. Waiting lists practically non-existent with the number of beds registered in 2007. Peaks are 10, 4, 1 and 10 respectively.
- VAL, SAL and BBZ. With the current number of beds the waiting lists never disappear. The increase is particularly spectacular at SAL. The increases in waiting lists are eliminated by increasing bed numbers.

#### With elasticity

- COS and SAL. Waiting lists have moderate peaks (10 and 35 respectively) and increasing bed numbers reduces waiting lists appreciably.
- COS, MNF, VAL, SAL, BBZ, SEE and XUN. There is a clear trend for waiting lists to increase, except when bed numbers are increased dramatically. The increase in waiting lists is moderate in the cases of MNF and SEE (peaks of 200 and 400 respectively) and dramatic in the cases of VAL, BBZ and XUN (to over 1500 patients).

### IOD

The changes in waiting lists are reflected in the daily occupation index (% of beds occupied on a given day).

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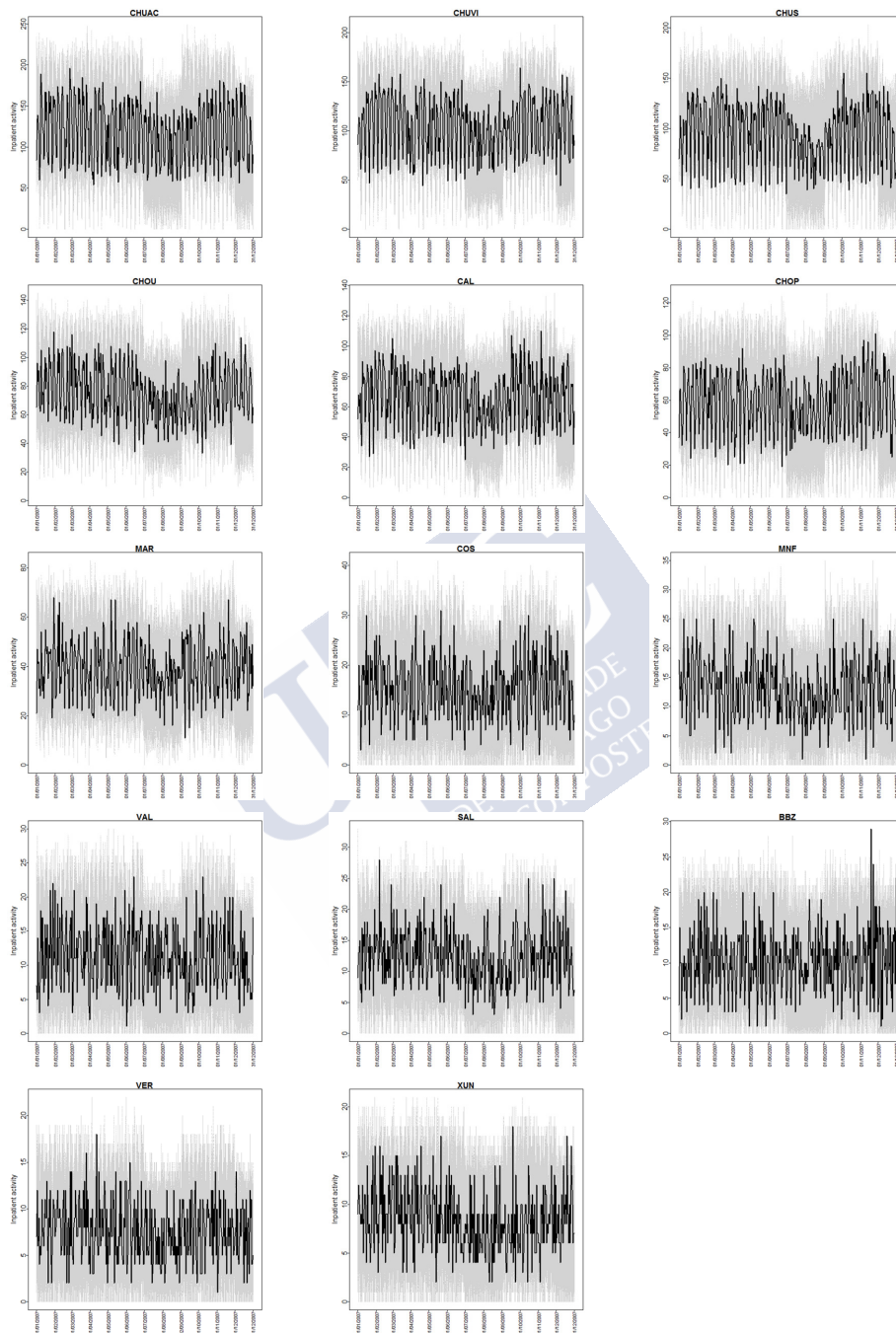


Figure 20. Simulated (grey lines) and actual admissions (black line) for 2007 in the various hospitals.

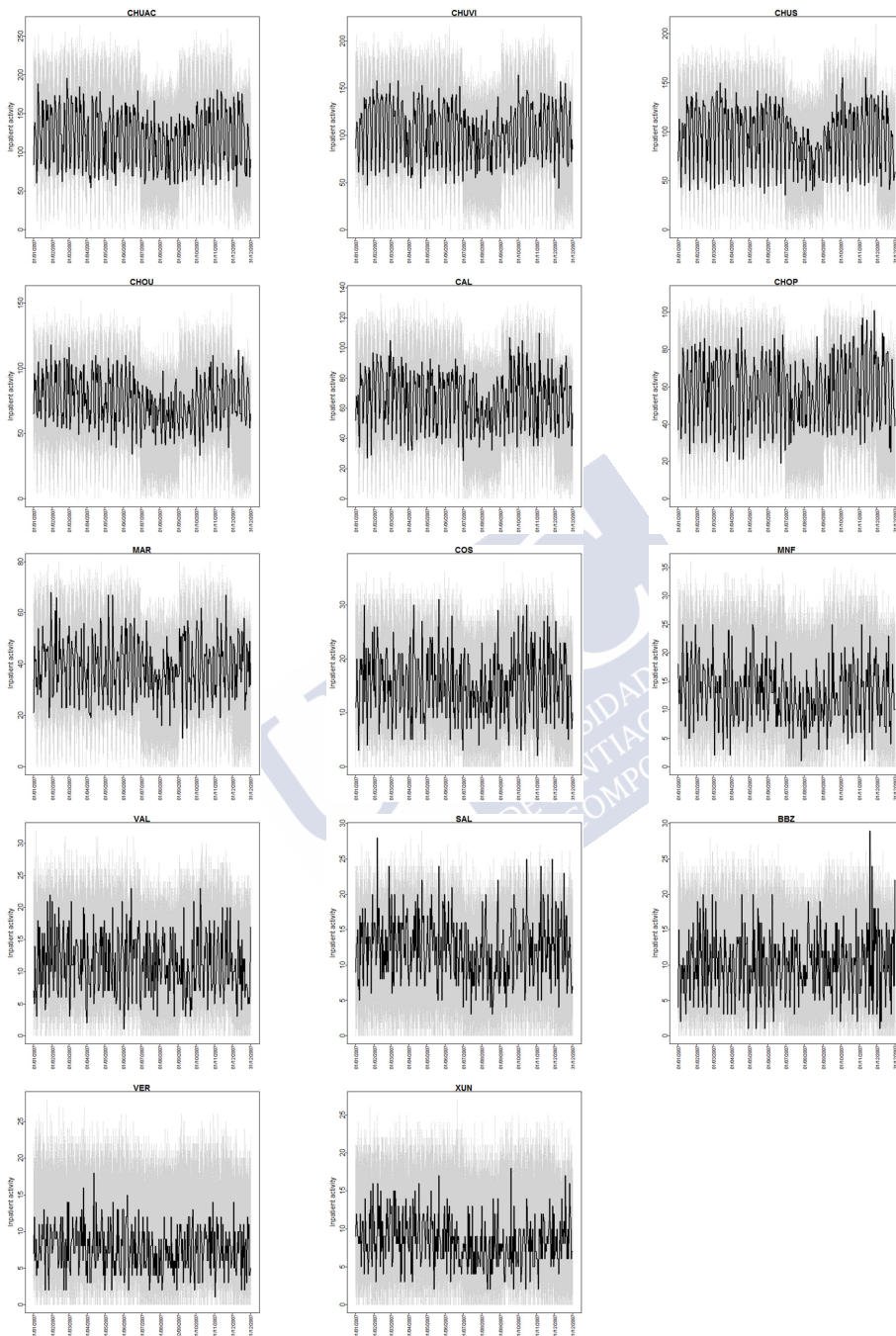


Figure 21. Simulated admissions (grey lines) and real admissions (black line) for the year 2007 in the different hospitals.

### 3. INPATIENT WAITING LISTS: HOW MANY HOSPITAL BEDS ARE ENOUGH? A SIMULATION ANALYSIS

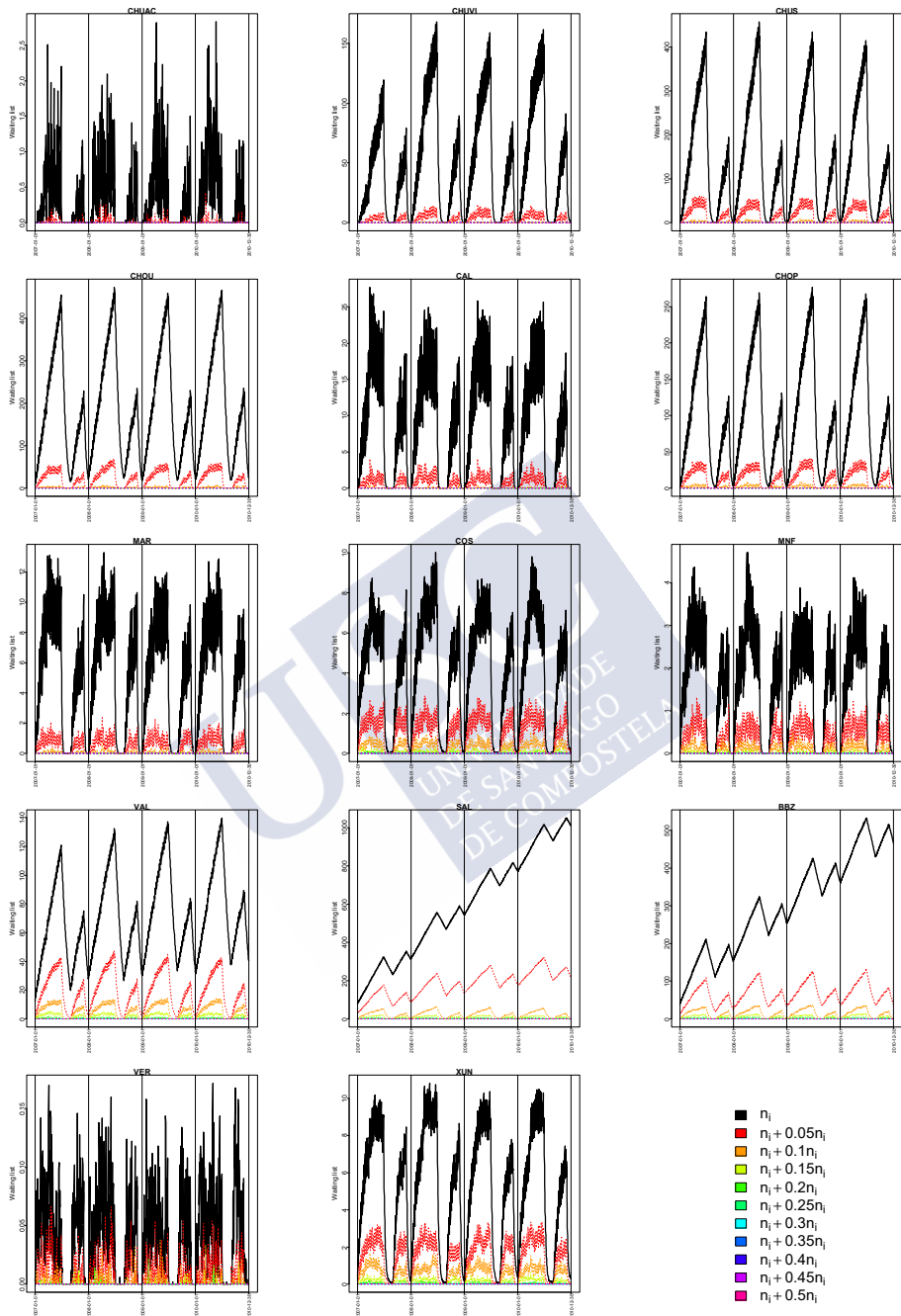


Figure 22. Effect of number of beds on simulated waiting lists for the period 2007-2010.

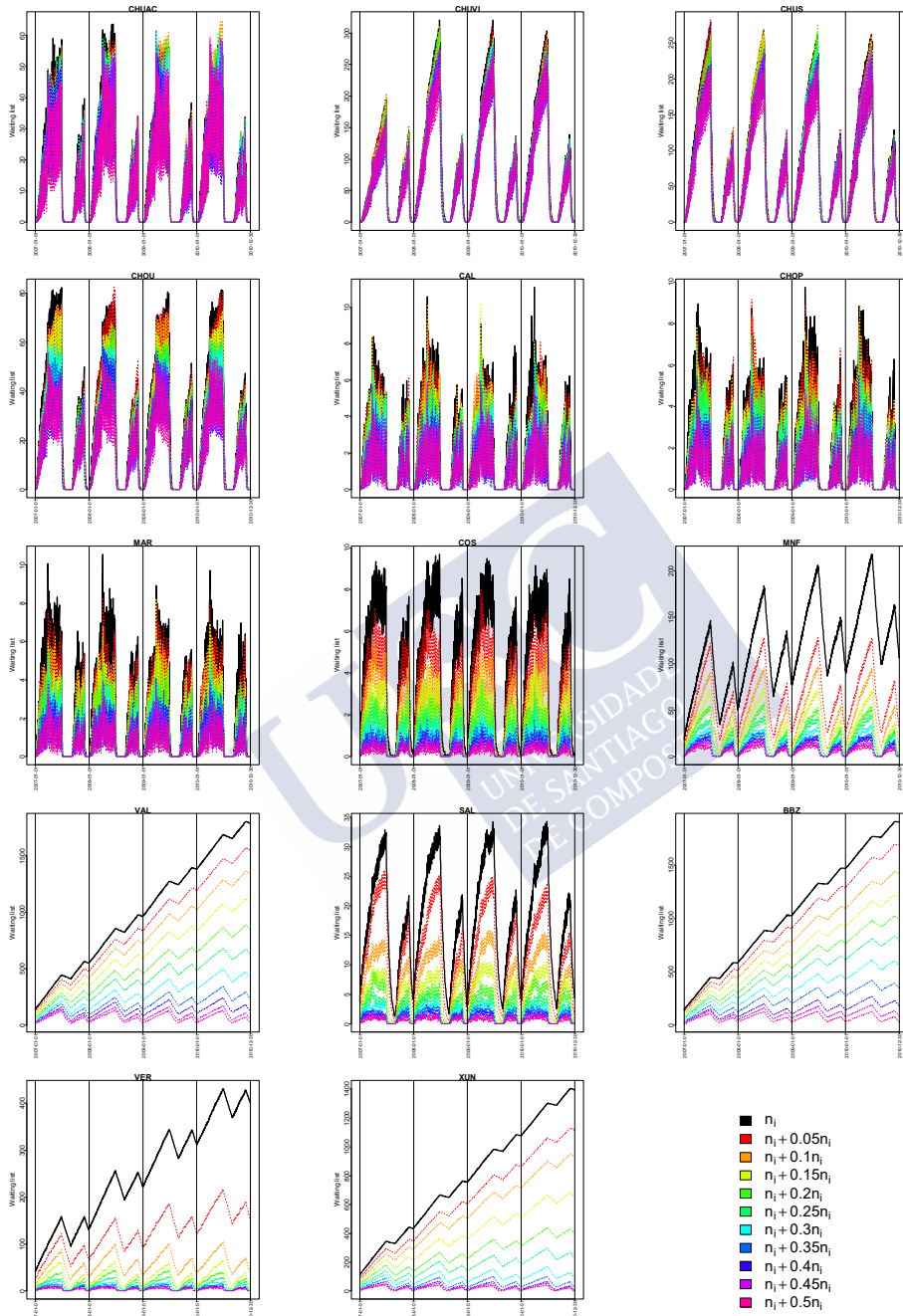


Figure 23. Effect of number of beds on simulated waiting lists for the period 2007-2010.

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Figure 24. Effect of number of beds on simulated IOD for the period 2007-2010.

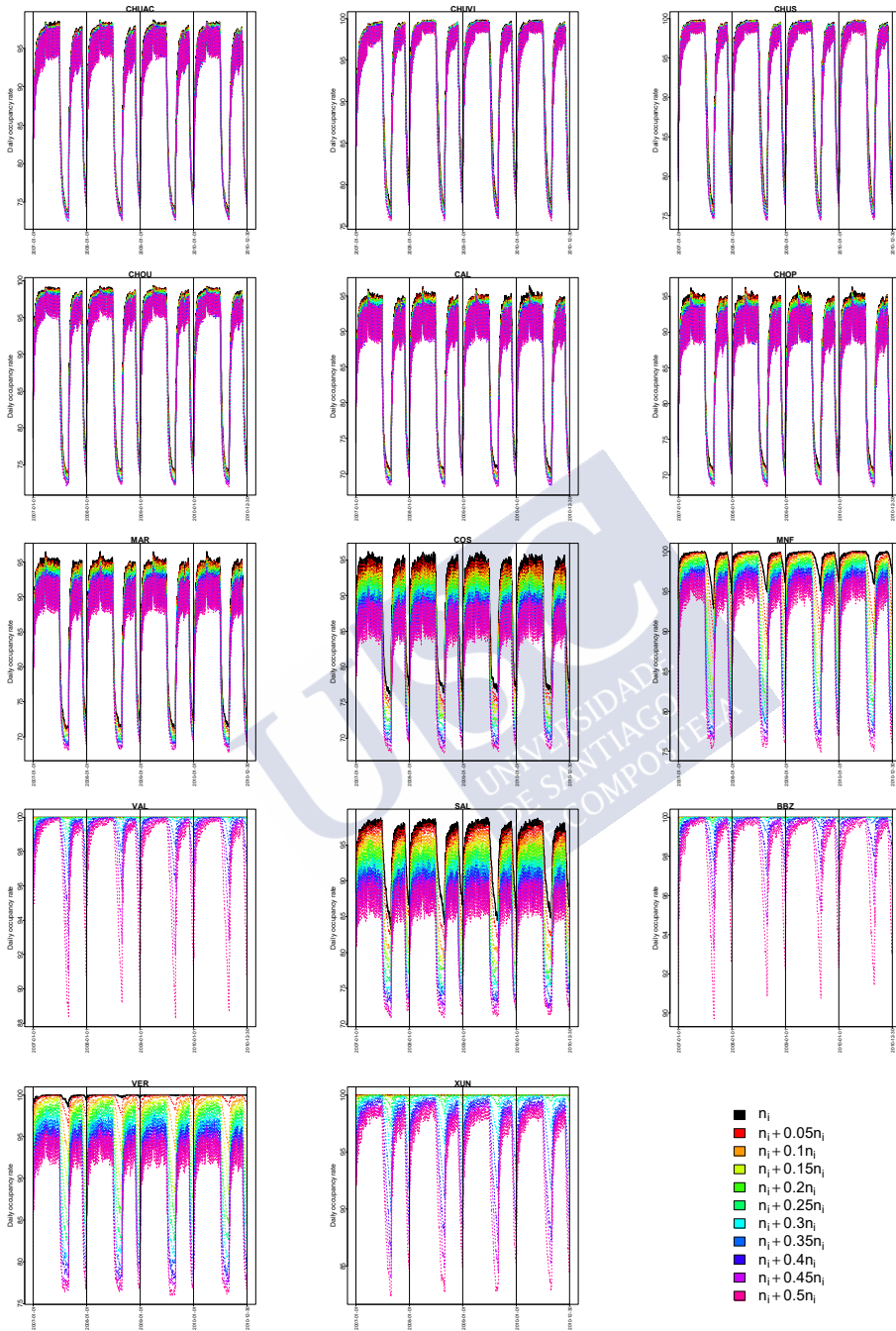


Figure 25. Effect of number of beds on simulated IOD for the period 2007-2010.

## 4. DISCUSSION AND CONCLUSIONS

Following McKee et al (1999)<sup>105</sup> “understanding the components of the health care production function, a coordinated cost control strategy can be developed. However, without the added power of simulation modeling, these strategies suffer a serious flaw. Using the delivery of obstetrical services as an example, assume a detailed analysis of the four cost components has been completed. An accurate birth volume forecast has been developed based on female population demographics and fertility rates. The important obstetrical practice patterns have been analyzed and clinical targets have been established by the medical staff. The inpatient unit has been sized using probabilistic models (e.g., Poisson Process). Using optimization techniques, nurse staff schedules have been prepared based on patient acuity needs and staffing policies. While the resulting nurse staff schedule is a significant achievement over more traditional approaches because it links nurse staff requirements and physical capacity with clinical practice patterns and patient demand, an extremely important dimension is missing from this solution. The problem is the data on which these analyses are based are at least a year old. The level of capacity resources (facility size, equipment, and staff) derived from these analyses are based on what occurred a year or more ago. Things may have changed. For example, patient volume may have increased or decreased due to enrollment or population shifts. Clinical practice patterns may have changed due to shift in the provider mix or new evidence concerning the quality of outcomes. A union may have been successful in organizing the nursing staff. Once a simulation model has been constructed and validated using historical data, the model is a powerful tool to explore the consequences of changes in parameters. The health care production function is a dynamic rather than a static process. The key to managing total costs is the simultaneous management of these four cost components. Simulation models have been designed to mimic the actual flow of patients through an inpatient

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<sup>105</sup> McKee, T.C.; Ward, T.J.; Isken, M.W. (1999) From Theory to Practice – Developing Decision Support Tools to Manage Cost and Quality of Obstetrical Care, Paper Presented at 11TH Annual Quest for Quality & Productivity in Health Services Conference sponsored by the Society for Health Systems of the Institute of Industrial Engineers, Washington D.C.

and an outpatient unit as closely as possible. By modeling the patient care paths, the relationship among patient volume, clinic practice, facility size, and staffing can be fully explored”.

Moreover, for Di Giorgio et al (2016)<sup>106</sup> “low-resource countries can greatly benefit from even small increases in efficiency of health service provision, supporting a strong case to measure and pursue efficiency improvement in low- and middle-income countries (LMICs). However, the knowledge base concerning efficiency measurement remains scarce for these contexts. The study shows that current estimation approaches may not be well suited to measure technical efficiency in LMICs and offers an alternative approach for efficiency measurement in these settings. Authors developed a simulation environment which reproduces the characteristics of health service production in LMICs, and evaluated the performance of Data Envelopment Analysis (DEA) and Stochastic Distance Function (SDF) for assessing efficiency. Authors found that an ensemble approach (ENS) combining efficiency estimates from a restricted version of DEA (rDEA) and restricted SDF (rSDF) is the preferable method across a range of scenarios. This is the first study to analyze efficiency measurement in a simulation setting for LMICs. Their findings aim to heighten the validity and reliability of efficiency analyses in LMICs, and thus inform policy dialogues about improving the efficiency of health service production in these settings”.

This paper studies the potential of additive flexible models as an instrument for calculating the production functions of hospitals and of medical and surgical specialties. The results obtained with the flexible model AM have been compared with those of the two functions most used in the field of health care; the Cobb–Douglas function and the TransLog function.

In this work, the variable Year, as a representation of the changes in production technology, does not show any statistical significance for the global model, which would indicate that technological changes are neutral in relation to output. The use of

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<sup>106</sup> Di Giorgio, L.; Flaxman, A.D.; Moses, M.W.; Fullman, N.; Hanlon, M.; Conner, R.O.; Wollum, A.; Murray, C.J.L. (2016) Efficiency of Health Care Production in Low-Resource Settings: A Monte-Carlo Simulation to Compare the Performance of Data Envelopment Analysis, Stochastic Distance Functions, and an Ensemble Model, PLOS ONE | DOI:10.1371/journal.pone.0147261 January 26, 2016

the variable time as an approach to technological change has been used in multiple studies, but following Blank (2009)<sup>107</sup>, innovations are disseminated slowly to reach all hospitals and, therefore, we can find different hospitals working with different technologies in the same period of time.

An example of the above is the work of Meyer (2007)<sup>108</sup> on the application of an economic function of production in hospitals with different levels of integration in their information systems. The study included 17 public hospitals of the Public Assistance of the Paris Region that were followed up in the period 1998–2005. Using an extended Cobb–Douglas production function, the annual output was correlated with three inputs: the capital factor, the labour factor, and information technology.

The calculations done for two subgroups of hospitals, divided according to the level of integration of information technologies, indicate that the higher the level of integration of the information system, the greater is its positive influence on the level of hospital production.

The results related to the work factor present different tendencies according to the hospital clusters, and the services should be analyzed in relation to other studies that address this issue. In this respect, the increase in income, surgical interventions and consultations in the period 1995–1999 in a regional hospital was compared with the increase in the number of hospital professionals during the same period of time. The data show that, for each category of health professionals, there are global decreases in productivity, defined as per capita consultations. However, several services show a different behaviour, with increases in productivity (Bratlid, 2000<sup>109</sup>).

Another study analyzes the role of medical staff characteristics in determining the different dimensions of hospital production. Employing a set of production functions

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<sup>107</sup> Blank Jos L.T., Van Hulst Bart L. 2009. Productive innovations in hospitals: an empirical research on the relation between technology and productivity in the Dutch hospital industry. *Health Economics*, 18: 665-679

<sup>108</sup> Meyer R, Degoulet P, Omnes L. 2007. Impact of health care information technology on hospital productivity growth: a survey in 17 acute university hospitals. [Stud Health Technol Inform](#); 129(Pt 1):203-7

<sup>109</sup> Bratlid D. 2000. *Manpower resources and patient treatment at a regional hospital*. [Tidsskr Nor Laegeforen](#). Oct 20;120(25):3021-6

with a flexible functional form, and adjusting for the complexity of the hospital, the authors examine the contribution to the hospital's production of the physicians and other factors, as well as the influence that the physicians of the different clinical services have on the productivity of the remaining physicians, and other factors of work and capital. The study also examines the possibilities of substitution between the factors of hospital production. The authors have identified that physicians are an important production factor that should be taken into account in studies of the cost and production functions of hospitals.

In the results of our study model for specialties, we have observed the possible existence of economies of scale in certain services, but not in all. Thirteen studies suggest that the hospital's production can be subdivided into several independent processes, each related to a specific process or service. A specific production function can be used to study the existence and, where appropriate, the magnitude of the scale effects for each service or process. If the economies of scale and the optimal configuration of the production of the hospital clinical services exist, this may be due to the existence of learning curves. Several studies on the production of hospital services, in an individualized way, identify a common limitation to all of them: they assume that the production of a hospital is divisible and, therefore, that hospitals do not organize their productive factors to obtain a global product.

A multi-centric study conducted by Tey (2006)<sup>110</sup> to resize the space in a hospital with the aim of incorporating a cataract unit resulted in a modest capital investment in the reconditioning of the physical space and in the number of workers in the ophthalmology service. This dedication to the pathology of cataracts can increase the quality and quantity indexes in the number of surgeries performed and a decrease in surgery waiting time of between one year and three months.

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<sup>110</sup> [Tey A](#), [Grant B](#), [Harbison D](#), [Sutherland S](#), [Kearns P](#), [Sanders R](#). (2007) Redesign and modernisation of an NHS cataract service (Fife 1997-2004): multifaceted approach. [BMJ](#). Jan 20;334(7585):148-52

Following Rezapoor A. et al (2014)<sup>111</sup>, some results of their study were that “among the affecting production variables in the studied industry, the inputs of physicians, active beds, and other personal had the positive effect and nurses as an input had the negative impact. Among the mentioned inputs, the most positive effects belonged to the capital factor or active beds and the minimum impact was related to physicians. According to the findings of the present study, the elasticity of physicians' production was estimated 0.017 that indicated 1 percent increase of physicians' number in the surveyed industry can lead to 0.017 percent increase in the number of inpatient admissions. According to recent research findings, the production elasticity of active beds was estimated 1.02 which indicated that 1 percent increase in the number of the active beds in the studied industry can lead to more than one percent increase in the number of the inpatient admissions. The results also showed that the coefficient of the Cobb- Douglas production function for the studied industry was estimated to be 1.143, which indicates that the industry total has the increasing returns to scale”.

Attending the results of the study by Mohammadi and Meskarpour-Amiri (2016)<sup>112</sup>, it is revealed that “in Iranian public hospitals, the elasticity of inpatient service level in terms of specialized human resources (0.88) is higher than beds (0.18). Based on this, the production of selected public hospitals in the country is most affected by the specialized manpower and reacts with more sensitivity to decrease and increase in specialized human resources rather than beds. The 10% increase in net working hours of specialized human resources (doctors and nurses), will increase the service production in country's public hospitals by 9%”.

Related to the study by Romley and Sood (2013)<sup>113</sup>, in terms of public policy, their finding that “the returns to hospital care are positive suggest that broad-based

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<sup>111</sup> Rezapoor, A., Shokuh, M.H., Saeed Bagheri Faradonbeh, S.B, Yousefzadeh, N. (2014) Studying Effects of Production Factors in Hospitals Affiliated with Tehran University of Medical Sciences (2008 -2011), Journal of Health Policy and Sustainable Health Vol. 1, No. 2, Spring

<sup>112</sup> Hamid Mohammadi, H., Meskarpour-Amiri, M. (2016) Estimation Production Function of Inpatient Services and Input Productivity: A Cross-Sectional Study of Iran Selected Public Hospitals, Hospital Practices and Research, Aug:1(3):91-93

<sup>113</sup> Romley, J.A., Sood, N. (2013) Identifying the Health Production Function: The Case of Hospitals, NBER Working Paper No. 19490, October, JEL No. D24,I1,I12

reductions in hospital spending could have adverse effects on patient outcomes. However, it is important to note some caveats for interpreting our findings. First, their findings do not indicate that there are no avenues for curtailing spending without hurting patient outcomes. In other words, it is possible that some spending is “wasteful,” and reducing such spending will not hurt patient outcomes. The impacts of reductions in medical spending are likely to be context-specific, and to be influenced by where and how spending is reduced. Second, our study is motivated by productivity differences; however, it does not shed light on the root causes of such differences. Understanding whether such differences arise from failures of management or governance, from differences in specialization or skills, or from inadequate public policies can shed light on the extent to which productivity differences can be eliminated. How difficult or easy is it to improve the productivity of low-performing hospitals or regions, and how this can be done, are largely unknown. Finally, their study followed the literature in focusing on variation in intensity across areas, and it is certainly possible that there are large inefficiencies within regions or even within hospitals. Identifying such inefficiencies and effective policy solutions are important directions for future research”.

Finally, and in relation to the model analyzed in this study, we must consider that an additive structure allows the factors to be added in indexes. Thus, the elasticities or substitution ratios can be derived directly and, in addition, the GAMs allow the calculation of the second derivative. In summary, GAMs are easily interpretable in the field of health economics. In addition, the additive models allow us to evaluate the behaviour of the linear models and therefore to evaluate their application and behaviour.

The results of this study suggest that the Flexible Additive Model is a promising methodology for the study applied in the field of health economics, due to its better fit for the prediction of the behaviour of the variables of hospital production functions. In addition, this methodology can be extended to studies on cost, demand and utility functions in the field of health economics.

On the other hand, supposing that an increase in the number of beds does not modify the process of generation of new values for inpatient activity or the length of stays, the waiting list disappears and the daily occupancy rate is drastically reduced when the number of beds rises (even for the 5% increase). This result would be extended to every cluster and every hospital.

Nevertheless, if we consider the modified simulation process which takes into account the beds effect in the behavior of the inpatient activity, we must remark that there are no significant differences in terms of waiting lists and occupancy rates when we increase the number of beds of the hospitals in Cluster 1 (highest case-mix). For hospitals in Cluster 2 (middle case-mix) the waiting list decreases when the number of beds rises. For hospitals in Cluster 3 (lowest case-mix), the waiting list increases as the number of beds rises.

Kroneman and Siegers<sup>114</sup> in their study of the effect of hospital bed reduction in bed use in ten European countries found that admission rates appear to be sensitive to bed supply, with a positive elasticity of 1.44; countries with higher bed supply show higher admission rates; the same has been found in the present study where admission rates appear to be sensitive to bed supply.

It seems that there are no significant differences in terms of waiting lists and occupancy rates when the number of beds in teaching hospitals increased. In that sense, Zeraati et al. (2005)<sup>115</sup> suggested that an increase in the supply of hospital beds tends to generate additional demand, either in the form of more patients admitted or patients treated for longer periods of time or some combination of the two. This is known as Roemer's Law, following the study by Roemer (1961)<sup>116</sup> who reported on a natural experiment where a sudden increase in hospital beds in one country, with no changes in other factors, led to a sharp increase in usage rates.

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<sup>114</sup> Kroneman, M.; Siegers, J.J. (2004) The effect of hospital bed reduction on the use of beds: a comparative study of 10 European countries, *Social Science & Medicine* 59, 1731–1740

<sup>115</sup> Zeraati, H.; Zayeri, F.; Babae, G.; Khanafshar, N.; Ramezanzadeh, F.; (2005) Required hospital beds estimation: a simulation study, *Journal of Applied Sciences* 5, 1189–1191

<sup>116</sup> Roemer, M.I. (1961) Bed supply and hospital utilization: a national experiment, *Hospitals: JAHA* 35, 988–993

Supply-side policies may also be disappointing in their effects on waiting times for small rural hospitals. The same results have been found by Siciliani and Hurst (2003)<sup>117</sup> who show that a common experience is to take measures aimed at reducing waiting times by increasing activity, only to find that after a brief period demand has increased and waiting times have reverted to levels similar to those before the introduction of the measures, since demand responds positively to reductions in waiting times. The same conclusions<sup>118</sup> have been shown by Donald et al. (2003)<sup>118</sup> who found that, although staff and other capacity constraints can make it more difficult to reduce waiting times, increases in capacity will not necessarily achieve sustained reductions in waiting times because changes in working practices and accurate measurement of how capacity is used are also required in order to ensure that changes elsewhere in the health system do not reduce the effect of the additional capacity.

On the other side, our study shows that higher capacity in terms of more beds is associated with lower waiting times for medium size hospitals. Equally, Siciliani and Hurst (2003)<sup>119</sup> found that on the supply side, the pronounced and prolonged reductions in long waiting times for coronary revascularization surgery in Denmark have been brought about by significant increases in activity, backed up by increases in capacity. The same results were found by Martin and Smith (1999)<sup>120</sup> and Lindsay and Feigenbaum (1984)<sup>121</sup>, showing that econometric evidence suggests that higher capacity, in terms of increased numbers of beds and physicians, is associated with lower waiting times.

Although simulation offers an opportunity to research the effect of many different alternatives in situations where actual experiments are impossible, or too costly, time-

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<sup>117</sup> Siciliani, L.; Hurst, J. (2003) Explaining waiting times variations for elective surgery across OECD countries, OECD Health Working Papers 7

<sup>118</sup> Donald, J.; Huby, C.; Maxwell, D. (2003). The outpatient waiting times problem (and the solutions), waiting for elective admission, Review of national findings, Audit Commission, London

<sup>119</sup> Siciliani, L.; Hurst, J. (2003) Explaining waiting times variations for elective surgery across OECD countries, OECD Health Working Papers 7

<sup>120</sup> Martin, S.; Smith, P.C. (1999). Rationing by waiting lists: an empirical investigation, *Journal of Public Economics* 71 141–164

<sup>121</sup> Lindsay, C.M.; Feigenbaum, B. (1984). Rationing by waiting lists, *American Economic Review* 74, 404–417

consuming or risky, according to Álvarez and Centeno (1999)<sup>122</sup> the primary reason for the reluctance of the health care industry to accept simulation was the management's reluctance to reduce a complex process in the health care field to a model representation.

Even more, following the systematic review by Naiker et al, (2018)<sup>123</sup> “research in this area continues to be challenging because there are numerous factors that affect waiting times. Previous studies have focused on resource, operational and process areas. Implementing the changes recommended by computer simulation studies and the challenges associated with ensuring the sustainability of these changes continue to evolve”.

Facing the study by Lade et al (2015)<sup>124</sup>, “the only way that the service demand can be met with ease is to increase the service capacity (and raising the efficiency of the existing capacity if possible) to the exiting level. The capacity might be built to such high level as can always meet the peak demand with no queues. But adding to capacity may be a costly affair and uneconomic after a stage because then it shall remain idle to varying degrees when there are no or few customers. A manger, therefore, has to decide on an appropriate level of service which is neither too low nor too high. Providing too low service would cause excessive waiting which has a cost in terms of customer frustration, loss of goodwill in the long run, direct cost of idle employees (where, for example, the employees have to wait near the store to obtain the supplies of materials, parts or tools needed for their work), or loss associated with poor employee morale resulting from being idle. On the other hand, too high a service level would result in very high set up cost and idle time for the service stations, thus, the goal of queuing modeling is the achievement of an economic balance between the cost of providing service and the cost associated with the wait required for that service”.

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<sup>122</sup> Álvarez, A.; Centeno, M. (1999). Enhancing simulation models for emergency rooms using VBA, Winter Simulation Conference, 1685–1693

<sup>123</sup> Naiker, U., FitzGerald, G., Dulhunty, J.M., Rosemann, M., (2018) Time to wait: a systematic review of strategies that affect out-patient waiting times, Australian Health Review, 42, 286–293

<sup>124</sup> Lade, I.P., Sakhare, V.P., Shelke, M.S., Sawaitul, P.B. (2015) Reduction of Waiting Time by Using Simulation & Queuing Analysis, International Journal on Recent and Innovation Trends in Computing and Communication, Volume: 3 Issue: 2

The study by Simwita and Helgheim (2016)<sup>125</sup> focused on exploring the entire orthopedic care process. The authors explore “the efficiency of the process, focusing on the inadequate utilization of surgeons and high patient wait times. The focus is identifying process inhibitors that lead to poor utilization of surgeons and showing how surgeon utilization can be improved and also investigating the effect of improved utilization on future increasing demand. Discrete event simulation was used to explore the base scenario that represents the observed orthopedic care process and to develop a proposal scenario that can be used to improve surgeons utilization as well as reducing patient waiting time. The simulation results from the base scenario reveal long patient wait times and poor surgeon utilization. Poor surgeon utilization has several negative effects, such as long patient wait times, as well as morbidity and mortality”. The authors suggested a proposal scenario that demonstrates a change that may lead to the improvement of the orthopedic care process, without any increase in resources.

Wijeyesundera et al (2014)<sup>126</sup> applied discrete event modelling, using data from the Placement of Aortic Transcatheter Valves (PARTNER) trials. They compared “transcatheter aortic valve replacement (TAVR) with medical therapy in the inoperable cohort, and compared TAVR to conventional aortic valve surgery in the high-risk cohort. One-year mortality and wait-time deaths were calculated in different scenarios by varying TAVR wait times from 10 days to 180 days, while maintaining a constant wait time for surgery at a mean of 15.6 days. Authors found that increasing wait times for TAVR might have important clinical implications of the effectiveness of this procedure in inoperable and high-risk severe AS populations. Wait-time strategies to minimize delays in access to TAVR will be associated with a reduction of complications during waiting, and improvement of clinical outcomes”.

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<sup>125</sup> Simwita, Y.W., Helgheim, B. I., (2016) Improving surgeon utilization in an orthopedic department using simulation modeling, *Journal of Healthcare Leadership*, 8 41–50

<sup>126</sup> Wijeyesundera, H.C., Wong, W.W.L., Bennell, M.C., Fremes, S.E., Radhakrishnan, S., Peterson, M., Ko, D.T. (2014) Impact of Wait Times on the Effectiveness of Transcatheter Aortic Valve Replacement in Severe Aortic Valve Disease: A Discrete Event Simulation Model, *Canadian Journal of Cardiology*: 30, 1162-1169

Attending the study by O'Brien et al (2015)<sup>127</sup>, authors performed a sensitivity analysis and found “how wait times change as the result of changing the number of ORs, the service time and surgical volume. The parameters of the program were adjusted according to the characteristics of individual hospitals and Queuing theory was also used as part of a broad approach to smooth patient flow and concluded that Monte Carlo simulation can guide decisions on how to balance resources for elective and non-elective surgical procedures”.

Facing the study by Greenroyd et al (2017)<sup>128</sup>, the tool presented in their paper provides a “discrete-event simulation tool for analysing a range of patient schedules across nine metrics, including: patient waiting, clinic room utilisation, waiting room utilisation, staff hub utilisation, clinician utilisation, patient facing time, clinic over-run, post-clinic waiting, and post-clinic patients still being examined. This allows clinic managers to analyse a number of scheduling solutions to find the optimum schedule for their department by comparing the metrics and selecting their preferred schedule. Also provided is an analysis of the impact of variations in appointment durations and their impact on how a simulation tool provides results. This analysis highlights the need for multiple simulation runs to reduce the impact of non-representative results from the final schedule analysis”.

## CONCLUSIONS

1. The study demonstrated the usefulness of simulation techniques for examining a hospital system and, in particular, the daily inpatient activity and the length of the stay. Furthermore, it has been very useful to analyse the drop in waiting list numbers due to clustering under the new-bed allocation; as well as the development of bed configurations more directly suited to clustering.

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<sup>127</sup> O'Brien, Antognini, J.M., Joseph F. Antognini, J.F., Khatri, V. (2015) How many operating rooms are needed to manage non-elective surgical cases? A Monte Carlo simulation study, *BMC Health Services Research*, 15:487

<sup>128</sup> Greenroyd, F.L., Hayward, R., Price, A., Demian, P., Sharma, S. (2017) Maximising patient through put using discrete event simulation. IN: *Proceedings of the 7th International Conference on Simulation and Modeling Methodologies, Technologies and Applications - Volume 1: SIMULTECH*, pp. 204-214, Madrid, Spain

2. The results in this study suggest that AM is a promising technique for the areas of research and application in health economics. In addition, the results allow us to characterize the domains in which our approach can be effective, such as those related to demand, costs and useful functions in health care.

3. The flexibility of additive models offers interesting advantages for research in other areas of health economics. Even when the data handled are susceptible to more traditional techniques, AMs can, as in the present study, provide useful verification of the validity of less sophisticated methods, such as Cobb-Douglas and the Translogarithmic models.

4. On the other hand, if hospital admission patterns are affected by the number of beds, each increase in the number of beds, including notable increases, lead to no significant differences in hospital occupation rates or hospitalization rates and waiting lists.



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## SUPPLEMENTARY MATERIALS





**PUBLICATIONS****Publications derived from the present doctoral thesis:**

Reyes-Santías, F., Cadarso-Suárez, C.; Rodríguez-Alvarez, MX. (2011): Estimating hospital production functions through flexible regression models. *Mathematical and Computer Modelling*, Vol 54, Issues 7-8, pp 1760-1764.

Reyes-Santías, F., Cadarso-Suárez, C.; Martínez-Calvo, A. (2013): Applying a simulation model in order to manage waiting lists for hospital inpatient activity in an EU region. . *Mathematical and Computer Modelling*, Vol 57, Issues 7-8, pp 1840-1846.

Antelo, M.; Reyes Santías, F.; Martínez Calvo, A. (2015): Bed capacity and surgical waiting lists: a simulation analysis. *European Journal of Government and Economics*, vol.4 nº 2.

Antelo, M.; Reyes Santías, F.; Cadarso-Suárez, C.; Rodríguez-Álvarez, M.X. (2017): Comparing Some Production Functions for Inpatient Health Services in Selected Public Hospitals in Spain. *Hospital Topics*, 95:3, pp. 63-71.



## **Annexe 1**





Links publicaciones:

Reyes-Santías, F., Cadarso-Suárez, C.; Rodríguez-Alvarez, MX. (2011): Estimating hospital production functions through flexible regression models. *Mathematical and Computer Modelling*, Vol 54, Issues 7-8, pp 1760-1764.

<https://www.sciencedirect.com/science/article/pii/S089571771100762X>







## **Annexe 2**



Reyes-Santías, F., Cadarso-Suárez, C.; Martínez-Calvo, A. (2013): Applying a simulation model in order to manage waiting lists for hospital inpatient activity in an EU region. . Mathematical and Computer Modelling, Vol 57, Issues 7-8, pp 1840-1846.

<https://www.sciencedirect.com/science/article/pii/S089571771100762X>





## Annexe 3





Antelo, M.; Reyes Santías, F.; Martínez Calvo, A. (2015): Bed capacity and surgical waiting lists: a simulation analysis. European Journal of Government and Economics, vol.4 n° 2.

<https://core.ac.uk/download/pdf/44597775.pdf>





**Annexe 4**





Antelo, M.; Reyes Santiás, F.; Cadarso-Suárez, C.; Rodríguez-Álvarez, M.X. (2017): Comparing Some Production Functions for Inpatient Health Services in Selected Public Hospitals in Spain. *Hospital Topics*, 95:3, pp. 63-71.

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