Structural Change and Economic Dynamics 67 (2023) 293–302

Contents lists available at ScienceDirect

Structural Change and Economic Dynamics

journal homepage: www.elsevier.com/locate/strueco

An alternative for tracing the path between supply and use tables in current and constant prices

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ARTICLE INFO

JEL codes:
C89
D57
E31

Keywords:
Price deflation
Limited information
Input-output

ABSTRACT

Supply and use tables (SUT) in constant prices ensure coherent volume and price information across economic accounts. They are needed to appropriately measure technological change and connect physical and monetary models. To estimate SUTs in constant prices, researchers normally apply commodity-specific deflators to SUTs. From an economics perspective, deflators are undoubtedly cell-specific since exchanges of a commodity occur in different markets and institutional contexts. RAS can be used to calculate such cell-specific deflators. But deflating SUTs via RAS can become impossible due to excessive information requirements. This article revisits Path-RAS and applies it to build SUTs in constant prices. Our methodology lowers information requirements, enables cell-specific deflators, and avoids *ad hoc* adjustments. Additional information about specific industries or products can be included if existing and non-conflicting. We provide an empirical application based on the current 27 European Union countries to explore the accuracy of our estimations considering different information scenarios.

1. Why bothering? The relevance of SUTs and IO data in constant prices

Supply and use tables (SUT) play a central role in the United Nations' (2009) system of national accounts. They also provide the basis for the construction of input-output (IO) tables. As a framework, they ensure (i) systematic bookkeeping of aggregated and disaggregated macroeconomic data, (ii) consistency between the production of goods and services, as well as income accounts, and (iii) coherent gross domestic product (GDP) figures, not only within a nation from both production and expenditure perspectives but also across nations (Mahajan et al., 2018). Nevertheless, the relevance of SUTs and IO data in constant prices goes beyond descriptive statistical criteria towards modelling. Since they report linkages from commodities to industries and vice versa, SUTs enable links between other economic datasets that report information either by commodity or industry. In this vein, SUTs can be used to relate national accounts to jobs and occupations, land use, energy consumption, pollution, waste generation, water usage, amongst a wider range of possibilities.

According to de Boer and Rodrigues (2020), interest in price deflation can be traced back as far as the 18th century. Apparently Dutot (1738) was a pioneer in calculating indexes, when he did so for several commodities by accounting for price variations as far back as 1515. Since then, a vast literature on indexes has emerged that suggests economist consider both price and quantity changes (Balk, 2008). IO models seem to be no exception (Oosterhaven, 2023). According to Leontief (1951), they were initially conceived from both a physical and a monetary perspective. In fact, the first precedent of an IO price model (Leontief, 1937) appeared soon after the first IO table was published (Leontief, 1936). amongst the many applications of SUTs and IO data in constant prices, we can highlight two: measuring technological change and linking physical and monetary models. Both applications appear as highly interrelated in the literature (Hoekstra and van den Bergh, 2003; Su and Ang, 2012).

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https://doi.org/10.1016/j.strueco.2023.08.008
Received 19 December 2022; Received in revised form 4 August 2023; Accepted 18 August 2023
Available online 20 August 2023
0954-349X/

On the one hand, measuring technological change is key to economic growth decompositions with an interindustry perspective. Economic growth decompositions include, at least, growth accounting (supply-side), structural decomposition analysis (demand-side) and shift-share analysis to include an interregional perspective (Oosterhaven, 2021).

- If used for growth accounting, IO data needs to be measured in constant prices to ensure that the total factor productivity measures technological change and not increases in the value of intermediate inputs, labour and capital (Miller and Blair, 2022, pp. 724–726; Peterson, 1979).
- Most empirical structural decomposition analyses use deflated IO data (Savona and Ciarli, 2019). The rationale behind this choice follows. If we consider current prices only, relative price changes could relocate value while the distribution of the volume of output follows a different path. For instance, Henriques and Kander (2010) find that, when measured in constant prices, tertiarization phenomena are less intensive than suggested by previous literature. Sánchez Chóliz and Duarte (2006) also argue for the use of IO data in constant prices, particularly when a time period with high inflation rates is to be analysed.
- Finally, shift-share analysis can yield misleading results if divergent price dynamics appear across regions. Note, structural decomposition analysis can be combined with shift-share analysis (Lahr and Dietzenbacher, 2017; Lahr and Ferreira, 2021) as well. In these cases, price deflation is required across time and regions.

On the other hand, price deflation plays a relevant role to link the physical and the monetary perspective in environmentally extended IO analysis. Hubacek and Giljum (2003) and Giljum et al. (2004) argue that physical tables yield different and more accurate results since they better capture the physical reality of economic exchanges. In a reply, Suh (2004) points that physical IO models might only tell us that some products are less expensive and others more costly per unit of physical measurement. The latter author also pointed that physical IO tables suffer from operational issues including statistical bias that attaches to sectoral aggregation as well as problems that arise from sectoral inconsistencies across tables over time. Hoen (2002) make a similar point and defends monetary models in constant prices as a viable alternative to physical models.

For all industries producing physical commodities, a bridge between physical and monetary models should exist and should not be hard to calculate since value equals mass multiplied by price per unit mass (Hoekstra and van den Bergh, 2006). Establishing the equivalence between physical and monetary flows requires prices, either actual or estimated (Többen, 2017). Despite this straightforward relationship, Weisz and Duchin (2006) suggest that physical and monetary tables cannot be translated from one to another using a single price for all deliveries of an industry or commodity. Cell-specific deflators are needed. Dietzenbacher (2005), however, notes that this is not the only issue one faces when linking physical and monetary models. Appropriate waste treatment has also found to be fundamental when one links the two model types.

2. Alternatives for SUT deflation

2.1. Brief description of the accounting framework

For the purpose of this paper,[1] the SUT framework only covers the supply and use tables in basic prices. For k products and l industries a SUT can be summarised as in table 1.[2] Matrices $\mathbf{U}^d$ and $\mathbf{U}^m_{\mathbf{ij}}$ represent intermediate product consumption by industries domestically produced (d) and imported (m) respectively. Both have dimension ($k \times l$). Matrices $\mathbf{F}^d$ and $\mathbf{F}^m$ represent domestically produced and imported commodity shipments to final demand. They have dimensions $(k \times \varphi)$ were φ represents the number of final demand components. Matrix $\mathbf{W}$ stands for gross value added (GVA) and has dimensions $(p \times l)$ where p stands for the number of value-added components. Matrix $\mathbf{V}$ represents the supply that each industry provides for each commodity and has dimensions $(l \times k)$. Matrix $\mathbf{M}$ stands for the commodity flows imported from different origins and has dimensions $(o \times k)$ where o denotes the number of import origins.

Table 1
Supply and use table with disaggregated domestic and imported flows.

	Products	Industries	Final demand	Sum
Products		$\mathbf{U}^d$	$\mathbf{F}^d$	$\mathbf{q}$
Products		$\mathbf{U}^m$	$\mathbf{F}^m$	$\mathbf{m}$
Value added		$\mathbf{W}$		$\mathbf{w}$
Industries	$\mathbf{V}$			$\mathbf{g}$
Imports	$\mathbf{M}$			$\mathbf{o}$
Sum	$\mathbf{q}' + \mathbf{m}'$	$\mathbf{g}'$	$\mathbf{f}'$	

In addition, vectors $\mathbf{q}$ and $\mathbf{m}$ denote total supply and total use (domestic and imported) by commodity. Vector $\mathbf{g}$ represents gross output by industry. Vectors $\mathbf{f}$ and $\mathbf{w}$ stand for the sum of each component of final demand and value-added matrices. Finally, $\mathbf{o}$ contains total imports by origin.

2.2. Double deflation

Initially conceived to estimate real GDP (United Nations, 1973), the double-deflation method (DD) is widely used for obtaining IO data in constant prices (Li and Kuroko, 2016). In fact, some global IO models such as the World Input-Output Dataset (WIOD) use double deflation to obtain matrices measured in previous year prices (Dietzenbacher et al., 2013).

Double deflation is based on the idea that it is difficult, if not impossible, to obtain price indices for the different GVA components (Ahmad, 1999). In contrast, some economic measures like gross output, imports or final demand are often published officially in both volume and monetary terms. Given the SUT framework in Table 1, let $\boldsymbol{\pi}^\mathbf{q}$, $\boldsymbol{\pi}^\mathbf{m}$, $\boldsymbol{\pi}^\mathbf{g}$ and $\boldsymbol{\pi}^\mathbf{f}$ be the price deflators associated with vectors $\mathbf{q}$, $\mathbf{m}$, $\mathbf{g}$ and $\mathbf{f}$, respectively. Furthermore, let $\overline{\mathbf{u}}^d_{\bullet\mathbf{j}}$ and $\overline{\mathbf{u}}^m_{\bullet\mathbf{j}}$ be the column sums of domestic and imported use matrices $\overline{\mathbf{U}}^d$ and $\overline{\mathbf{U}}^m$ in constant prices. Table 2 illustrates how deflators can be applied to deflate a SUT by means of double deflation.

To obtain a full GVA matrix $\overline{\mathbf{W}}$, a vector of implicit GVA deflators $\boldsymbol{\pi}^\mathbf{w}$ can be calculated. Each element of $\boldsymbol{\pi}^\mathbf{w}$ is a ratio. The difference between an industry's deflated gross output and its deflated intermediated consumption is divided by GVA in current prices:

$$\boldsymbol{\pi}^\mathbf{w} = \left(\overline{\mathbf{g}} - \overline{\mathbf{u}}^d_{\bullet\mathbf{j}} - \overline{\mathbf{u}}^m_{\bullet\mathbf{j}}\right)\mathrm{diag}(\mathbf{W}'\mathbf{i})^{-1} \tag{1}$$

Double deflation is theoretically sound and preferred to single-

[1] In most countries, a full SUT framework consists of a supply table in basic prices combined with a transformation matrix to purchaser's prices. A SUT also includes a use table measured in both basic and purchaser's prices. Gross value added is measured in basic prices to complete the accounting framework.

[2] Matrices are denoted in upper-case bold font; vectors in lower-case bold font; and scalars are denoted in italic font. Vectors are columns by definition. Superscript $'$ indicates transposition. A bar above the variable, $\overline{\mathbf{x}}$, denotes constant prices. A circumflex, $\widehat{\mathbf{x}}$, indicates that the vector has been transformed into a square diagonal matrix, i.e., one with elements on the main diagonal and zeros elsewhere. For composite vectors we use "diag" as the circumflex equivalent. A summation vector of ones is denoted by $\mathbf{i}$. Note, summation vectors vary their dimensions across equations.

Table 2
Supply and use table in constant prices using double deflation.

	Products	Industries	Final demand	Sum
Products		$\overline{\mathbf{U}}^{d} = \widehat{\pi}^{\mathbf{q}}\mathbf{U}^{d}$	$\overline{\mathbf{F}}^{d} = \widehat{\pi}^{\mathbf{q}}\mathbf{F}^{d}\widehat{\pi}^{\mathbf{f}}$	$\overline{\mathbf{q}}$
Products		$\overline{\mathbf{U}}^{m} = \widehat{\pi}^{\mathbf{m}}\mathbf{U}^{m}$	$\overline{\mathbf{F}}^{\mathbf{m}} = \widehat{\pi}^{\mathbf{m}}\mathbf{F}^{m}\widehat{\pi}^{\mathbf{f}}$	$\overline{\mathbf{m}}$
Value added		$\overline{\mathbf{W}} = \mathbf{W}\widehat{\pi}^{\mathbf{w}}$		$\mathbf{w}$
Industries	$\overline{\mathbf{V}} = \widehat{\pi}^{\mathbf{g}}\mathbf{V}\widehat{\pi}^{\mathbf{q}}$			$\overline{\mathbf{g}} = \widehat{\pi}^{\mathbf{g}}\mathbf{g}$
Imports	$\overline{\mathbf{M}} = \mathbf{M}\widehat{\pi}^{\mathbf{m}}$			$\overline{\mathbf{o}}$
Sum	$\overline{\mathbf{q}}' + \overline{\mathbf{m}}'$	$\overline{\mathbf{g}}' = \mathbf{g}\widehat{\pi}^{\mathbf{g}}$	$\overline{\mathbf{f}}'$	

deflation methods since it yields a balanced SUT and thus balanced real GDP figures from both income and spending perspectives (Oulton et al., 2018). But it does have some pitfalls. These are particularly related with intermediate transactions, which are the core of IO models.

First, double deflation implicitly assumes that an industry or commodity category is linked to a single commodity that corresponds to the price index applied. The reality, however, is that any given element of a SUT presents a composite commodity that embodies a mix unique to that specific industry and/or commodity transaction. Thus, double deflation necessarily induces some degree of aggregation bias (Dietzenbacher and Hoen, 1999). Double deflation also assumes that all exchanges of an industry or commodity have the same price dynamics; this neglects the fact that different market and institutional contexts undoubtedly affect price changes of the composite commodity differently (Folloni and Miglierina, 1994). GVA estimates are particularly sensitive to the manner in which they are deflated, due to measurement errors inherent in $\overline{\mathbf{q}}$, $\overline{\mathbf{g}}$, $\overline{\mathbf{m}}$ and $\overline{\mathbf{f}}$ (Wolff, 1994).

Moreover, from Eq. (1) one can probe that double deflation can induce a sign flip in an industry's value-added figure. Such an issue arises when the deflated intermediate consumption (domestic and imported) of an industry exceeds its deflated gross output. For a given industry j, we get:

$$\begin{aligned} &\text{if} \quad \overline{u}^{d}_{\bullet j} + \overline{u}^{m}_{\bullet j} > \overline{g}_j \\ &\text{then} \quad \pi^{\mathrm{w}}_j = \frac{\overline{g}_j - \overline{u}^{d}_{\bullet j} - \overline{u}^{m}_{\bullet j}}{w_j} < 0 \quad \forall w_j > 0 \end{aligned} \tag{2}$$

Therefore, the value-added deflator would induce a sign flip. In such cases *ad hoc* adjustments are required.

2.3. Biproportional techniques

Dietzenbacher and Hoen (1998) suggest that deflating via RAS ultimately applies cell-specific deflators. They focus on the intermediate transaction matrix given a set of industry output, final demand and import vectors in constant prices. Within IO analysis, basic RAS is a popular technique, if not the most popular, for matrix updating and balancing (Lahr and de Mesnard, 2004). Note that several other approaches exist that can deal with problems to which RAS is typically applied (Jackson and Murray, 2004). Some have been used to balance SUTs (Nicolardi, 2013; Rampa, 2008). Temursho et al. (2011), however, suggest that GRAS (Günlük-Şenesen and Bates, 1988; Junius and Oosterhaven, 2003) presents the best balance between accuracy and speed/simplicity. Thus, in what remains of this paper, we consider GRAS to be the *status quo* of the state of the art in matrix balancing.[3] While originally developed for balancing and updating symmetric IO tables, GRAS and other biproportional techniques can also be applied to SUTs (Serpell, 2018).

To implement GRAS,[4] we split a benchmark matrix $\mathbf{Z}_{(0)}$ into matrices $\mathbf{Z}^{+}_{(0)}$ and $\mathbf{Z}^{-}_{(0)}$. On the one hand, matrix $\mathbf{Z}^{+}_{(0)}$ contains only the positive elements of $\mathbf{Z}_{(0)}$. On the other, matrix $\mathbf{Z}^{-}_{(0)}$ contains the absolute values of all negative elements. Therefore, we have $\mathbf{Z}_{(0)} = \mathbf{Z}^{+}_{(0)} - \mathbf{Z}^{-}_{(0)}$. Vectors $\boldsymbol{\mu}$ and $\boldsymbol{\upsilon}$ are the row and column sum targets for the balanced matrix $\mathbf{Z}^{*}$. GRAS derivation arrives at a second-order equation. For rows and columns with positive and negative elements, the definition of coefficients r_i and s_j considers the positive root of the second-order equations. For the cases where no positive or negative elements are in a row or column, scalars are defined as in standard RAS (Temursho et al., 2013). The algorithm runs iteratively until the conditions:

$$\begin{aligned} &\left|\left[\widehat{\mathbf{r}}\mathbf{Z}^{+}\widehat{\mathbf{s}} - (\widehat{\mathbf{r}})^{-1}\mathbf{Z}^{-}(\widehat{\mathbf{s}})^{-1}\right]\mathbf{i} - \boldsymbol{\mu}\right| < \varepsilon \\ &\left|\left[\widehat{\mathbf{r}}\mathbf{Z}^{+}\widehat{\mathbf{s}} - (\widehat{\mathbf{r}})^{-1}\mathbf{Z}^{-}(\widehat{\mathbf{s}})^{-1}\right]'\mathbf{i} - \boldsymbol{\upsilon}\right| < \varepsilon \end{aligned} \tag{3}$$

are fulfilled for a sufficiently small value of ε—a pre-determined level of tolerated error.

The biggest obstacle for GRAS implementation is that data on commodity vectors, $\mathbf{q}$ and $\mathbf{m}$, are rarely available. The same problem appears when it comes to price deflation: we might lack a complete set of deflators for each row and column in the SUT. To overcome this issue, a first balancing alternative appeared when Beutel's (2002, pp. 114–118) algorithm was adapted for a SUT framework (SUT-Euro). SUT-RAS proposed by Temursho and Timmer (2011) solves some SUT-Euro limitations. It uses SUT balancing equations (1) to endogenously derive targets for $\mathbf{q}$ and $\mathbf{m}$ given an industry output target data ($\mathbf{g}$). Further, it can be applied to rectangular matrices and manages negative values just like GRAS. SUT-RAS also can adopt additional information as long as constraints do not conflict (Valderas Jaramillo et al., 2019). In fact, SUT-RAS and GRAS are apparently equivalent as long as $\mathbf{q}$ and $\mathbf{m}$ are exogenously set (Temursho, 2021).

Despite the solid theoretical foundation and promising empirical results, the scarcity of price indices has tended to prevent the use of RAS-based deflation approaches from the user's point of view. SUT-RAS data requirements —gross output by industry— are reasonable when updating via current prices. Admittedly, many developed countries can estimate gross output by industry in constant prices. They can even give deflated estimates for a number of commodities, final demand components and trade. But data on industry output (either in current or constant prices) are often not available for many developing countries. At the regional level, data scarcity can become an even more acute problem. Furthermore, SUT-RAS cannot assign different reliability measures to supply and use tables separately.

2.4. The aim of this paper: Relaxing information requirements for SUT deflation

Data on prices and volumes are increasingly published at higher levels of disaggregation in developed countries. But data availability for developing nations and regions remains a prime constraint for the compilation of official IO statistics in constant prices (Tandon and Ahmed, 2016). The aim of this paper is to present a methodological alternative for SUT deflation. We intend to address some improvement opportunities spotted in double-deflation and biproportional techniques.

Particularly we focus on:

a) Relaxing information requirements to enable SUT deflation where data are scarce and yet permit the application of additional information if available and non-conflicting.
b) Obtaining cell-specific deflators, as opposed to the one-price-fits-all approach implicit in double deflation.
c) Transparently managing possible incoherencies that can arise during the deflation process to avoid *ad hoc* solutions.

To such ends, we revisit Path-RAS approach introducing major

[3] For the sake of simplicity, we do not consider subsequent GRAS extensions (Lenzen et al., 2009, 2014; among others).

[4] We standardize notation across all sections of this paper. As a result, we do not follow notation in preceding GRAS literature.

modifications in the original algorithm (Pereira López et al., 2013; Pereira López and Rueda Cantuche, 2013). This is the prime contribution of this paper. As such, we detail it in Section 2. Still, it essentially remains a modification of Path-RAS. In Section 3 we present an empirical application based on the EU-27 countries. Section 4 concludes, states our limitations and points to possible future research avenues.

3. Our methodological proposal: the modified Path-RAS

3.1. Basic accounting equalities

The following basic accounting equalities must hold for a SUT to be balanced:

$$\begin{aligned} \mathbf{q} &= \mathbf{U}^d\mathbf{i} + \mathbf{F}^d\mathbf{i} = \mathbf{V}'\mathbf{i} = \mathbf{q} \\ \mathbf{m} &= \mathbf{U}^m\mathbf{i} + \mathbf{F}^m\mathbf{i} = \mathbf{M}'\mathbf{i} = \mathbf{m} \\ \mathbf{g} &= \mathbf{U}^{d'}\mathbf{i} + \mathbf{U}^{m'}\mathbf{i} + \mathbf{W}'\mathbf{i} = \mathbf{V}\mathbf{i} = \mathbf{g} \end{aligned} \tag{4}$$

Total commodity supply, both domestically produced and imported, must equal total use of each commodity. Total inputs by industry must equal total outputs by industry.

To facilitate comprehension of this section, we specify the industry and commodity structures for both purchases and sales in a SUT and relate them to the basic accounting equalities (4). On the one hand, using industry structures, we consider matrices $\mathbf{A}^d = \mathbf{U}^d(\widehat{\mathbf{g}})^{-1}$, $\mathbf{A}^m = \mathbf{U}^m(\widehat{\mathbf{g}})^{-1}$ and $\mathbf{C} = \mathbf{V}'(\widehat{\mathbf{g}})^{-1}$. On the other hand, regarding commodity structures, we can define matrices $\mathbf{B}^d = (\widehat{\mathbf{q}})^{-1}\mathbf{U}^d$, $\mathbf{B}^m = (\widehat{\mathbf{m}})^{-1}\mathbf{U}^m$ and $\mathbf{D} = \mathbf{V}(\widehat{\mathbf{q}})^{-1}$.

Substituting $\mathbf{A}^d$, $\mathbf{A}^m$, $\mathbf{C}$, $\mathbf{B}^d$, $\mathbf{B}^m$ and $\mathbf{D}$ in (4) we get:

$$\begin{aligned} \mathbf{q} &= \mathbf{A}^d\mathbf{g} + \mathbf{F}^d\mathbf{i} = \mathbf{C}'\mathbf{g} = \mathbf{q} \\ \mathbf{m} &= \mathbf{A}^m\mathbf{g} + \mathbf{F}^m\mathbf{i} = \mathbf{M}'\mathbf{i} = \mathbf{m} \\ \mathbf{g} &= \mathbf{B}^{d'}\mathbf{q} + \mathbf{B}^{m'}\mathbf{m} + \mathbf{W}'\mathbf{i} = \mathbf{D}\mathbf{q} = \mathbf{g} \end{aligned} \tag{5}$$

As in (4), total commodity supply, both domestically produced and imported, must equal total use by commodity. Total inputs by industry must equal total industry outputs. In this way, SUTs remain balanced.

3.2. Minimum information requirements

Our method operates equivalently to GRAS. A benchmark matrix $\mathbf{Z}_{(0)}$ is modified to obtain a new matrix $\mathbf{Z}^*$ using targets up-to-date margins $\boldsymbol{\mu}$ and $\boldsymbol{\upsilon}$. Different from GRAS, row and column targets are endogenously calculated during each iteration using the basic accounting equalities. There is, however, a minimum set of data needed to start the balancing process:

- $\mathbf{q}^*_{\bullet\bullet}$ = overall supply and use by products.
- $\mathbf{g}^*_{\bullet\bullet}$ = overall input and output by industries.
- $\mathbf{f}^*_{\bullet\bullet}$ = overall final demand.
- $\mathbf{m}^*_{\bullet\bullet}$ = overall imports.
- $\mathbf{w}^*_{\bullet\bullet} = \mathbf{f}^*_{\bullet\bullet} - \mathbf{m}^*_{\bullet\bullet}$ = overall gross value added.

Note, requirements can be further simplified using national accounts' identities. For instance, $\mathbf{q}^*_{\bullet\bullet} = \mathbf{g}^*_{\bullet\bullet}$ holds by definition.

3.3. Our workflow

3.3.1. Step 0: Defining a starting point

Let iterations $n = 0, 1, \cdots, N$ be indicated by subscript (n) associated with matrices and vectors. Superscripts *A, C, B, D* in vectors refer to the industry/commodity structures in (5) to estimate new target vectors after every iteration. Superscript 0 indicates the starting point.

The process initiates with the obtention of output by industry $\mathbf{g}^{(0)}_{(1)}$, final demand components $\mathbf{f}^{(0)}_{(1)}$ and total imports by origin $\mathbf{m}^{(0)}_{(1)}$ estimates that will be considered as the starting point. To do so, we define a diagonal matrix $\widehat{\pi}^{(0)}_{(1)}$ with dimensions $(l + \varphi + o) \times (l + \varphi + o)$. This matrix contains deflators defined as ratios between the given pieces of information in constant prices and their counterparts in matrix $\mathbf{Z}_{(0)}$.

As an example, for the minimum information scenario:

$$\begin{bmatrix} \overline{\mathbf{g}}^{(0)}_{(1)} \\ \overline{\mathbf{f}}^{(0)}_{(1)} \\ \overline{\mathbf{o}}^{(0)}_{(1)} \end{bmatrix} = \widehat{\pi}^{(0)}_{(1)} \begin{bmatrix} \mathbf{g}_{(0)} \\ \mathbf{f}_{(0)} \\ \mathbf{o}_{(0)} \end{bmatrix}$$
$$\boldsymbol{\pi}^{(0)}_{(1)} = \left[\operatorname{diag} \begin{bmatrix} \sum \mathbf{g}_{(0)} \\ \sum \mathbf{f}_{(0)} \\ \sum \mathbf{o}_{(0)} \end{bmatrix} \right]^{-1} \begin{bmatrix} \mathbf{g}^*_{\bullet\bullet} \\ \mathbf{f}^*_{\bullet\bullet} \\ \mathbf{m}^*_{\bullet\bullet} \end{bmatrix} \tag{6}$$

Deflators can be specific for some industries, products, final demand components, import origins or calculated according to aggregated information. Once calculated deflators are expanded (if needed) to achieve vectors with the appropriate dimensions.

3.3.2. Step 1. Path AC

3.3.2.1. Industry balancing. The first step balances industries using the starting-point targets in Eq. (6). For the supply matrix, we calculate row targets $\boldsymbol{\mu}^{(0)}_{(1)}$; for the use matrix, column targets $\boldsymbol{\upsilon}^{(0)}_{(1)}$:

$$\boldsymbol{\mu}^{(0)}_{(1)} = \begin{bmatrix} \overline{\mathbf{g}}^{(0)}_{(1)} \\ \overline{\mathbf{o}}^{(0)}_{(1)} \end{bmatrix} \quad \boldsymbol{\upsilon}^{(0)}_{(1)} = \begin{bmatrix} \overline{\mathbf{g}}^{(0)}_{(1)} \\ \overline{\mathbf{f}}^{(0)}_{(1)} \end{bmatrix} \tag{7}$$

By industry balancing, we mean that matrices $\mathbf{V}$ and $\mathbf{M}$ are row-scaled. Conversely, matrices $\mathbf{U}^d$, $\mathbf{U}^m$ and $\mathbf{W}$ are column-scaled.

Formally:

$$\begin{bmatrix} \mathbf{V}_{(1)} \\ \mathbf{M}_{(1)} \end{bmatrix} = \widehat{\mathbf{r}} \begin{bmatrix} \mathbf{V}^+_{(0)} \\ \mathbf{M}^+_{(0)} \end{bmatrix} + (\widehat{\mathbf{r}})^{-1} \begin{bmatrix} \mathbf{V}^-_{(0)} \\ \mathbf{M}^-_{(0)} \end{bmatrix}$$
$$\begin{bmatrix} \mathbf{U}^d_{(1)} \\ \mathbf{U}^m_{(1)} \\ \mathbf{W}_{(1)} \end{bmatrix} = \begin{bmatrix} \left[\mathbf{U}^d_{(0)}\right]^+ \\ \left[\mathbf{U}^m_{(0)}\right]^+ \\ \mathbf{W}^+_{(0)} \end{bmatrix} \widehat{\mathbf{s}} + \begin{bmatrix} \left[\mathbf{U}^d_{(0)}\right]^- \\ \left[\mathbf{U}^m_{(0)}\right]^- \\ \mathbf{W}^-_{(0)} \end{bmatrix} (\widehat{\mathbf{s}})^{-1} \tag{8}$$

Coefficients vectors $\mathbf{r}$ and $\mathbf{s}$ are calculated using the GRAS algorithm. Note, the GRAS algorithm is sing preserving Lenzen et al., 2014). Therefore, when balancing we do not induce undesired sign flips in value-added by industry. This is one of the main differences between our method and double deflation (see Eqs. (1) and (2).

3.3.2.2. Targets for commodities and value-added component. To conclude step 1, our algorithm endogenously calculates targets for $\mathbf{q}$, $\mathbf{m}$ and $\mathbf{w}$. Substituting $\overline{\mathbf{g}}^{(0)}_{(1)}$ in (5) we get:

$$\begin{aligned} \mathbf{q}^{(A)}_{(2)} &= \mathbf{A}^d_{(1)}\overline{\mathbf{g}}^{(0)}_{(1)} + \mathbf{F}^d_{(1)}\mathbf{i} = \mathbf{U}^d_{(1)}\mathbf{i} + \mathbf{F}^d_{(1)}\mathbf{i} \\ \mathbf{m}^{(A)}_{(2)} &= \mathbf{A}^m_{(1)}\overline{\mathbf{g}}^{(0)}_{(1)} + \mathbf{F}^m_{(1)}\mathbf{i} = \mathbf{U}^m_{(1)}\mathbf{i} + \mathbf{F}^m_{(1)}\mathbf{i} \\ \mathbf{q}^{(C)}_{(2)} &= \mathbf{C}'_{(1)}\overline{\mathbf{g}}^{(0)}_{(1)} = \mathbf{V}'_{(1)}\mathbf{i} \\ \mathbf{m}^{(C)}_{(2)} &= \mathbf{M}'_{(1)}\mathbf{i} \\ \mathbf{w}^{(A)}_{(2)} &= \mathbf{W}_{(1)}\mathbf{i} \end{aligned} \tag{9}$$

Since $\overline{\mathbf{g}}^{(0)}_{(1)}$ has been substituted in different structural equations, it is most likely that $\mathbf{q}^{(A)}_{(2)} \neq \mathbf{q}^{(C)}_{(2)}$ and $\mathbf{m}^{(A)}_{(2)} \neq \mathbf{m}^{(C)}_{(2)}$. Target vectors for $\mathbf{q}$ and $\mathbf{m}$ are derived as convex combination between vectors obtained through paths *A* and *C*. Formally:

$$\begin{aligned}\mathbf{q}_{(2)}^{(AC)} &= \alpha \mathbf{q}_{(2)}^{(A)} + (1-\alpha)\mathbf{q}_{(2)}^{(C)} \\ \mathbf{m}_{(2)}^{(AC)} &= \alpha \mathbf{m}_{(2)}^{(A)} + (1-\alpha)\mathbf{m}_{(2)}^{(C)} \\ &\text{with} \quad 0 \le \alpha \le 1\end{aligned} \tag{10}$$

Values for α can be interpretated as the degree of reliability assigned to the information contained in the $\mathbf{A}^d$, $\mathbf{A}^m$ and $\mathbf{C}$ matrices.

To conclude this step, vectors $\mathbf{q}_{(2)}^{(AC)}$, $\mathbf{m}_{(2)}^{(AC)}$ and $\mathbf{w}_{(2)}^{(A)}$ might need to be corrected to ensure balances defined in Eq. (4). If additional information is available, this correction can require the introduction of subset constraints to be applied to the target vectors during the next iteration. Following the example given in step 0, new deflators can be calculated:

$$\begin{aligned}\boldsymbol{\pi}_{(2\mathbf{\upsilon})}^{(AC)} &= \frac{\mathbf{q}_{\bullet\bullet}^{*} + \mathbf{m}_{\bullet\bullet}^{*}}{\sum \mathbf{q}_{(2)}^{(AC)} + \sum \mathbf{m}_{(2)}^{(AC)}} \\ \boldsymbol{\pi}_{(2\boldsymbol{\mu})}^{(AC)} &= \left[\operatorname{diag}\begin{bmatrix}\sum \mathbf{q}_{(2)}^{(AC)} \\ \sum \mathbf{m}_{(2)}^{(AC)} \\ \sum \mathbf{w}_{(2)}^{(A)}\end{bmatrix}\right]^{-1}\begin{bmatrix}\mathbf{q}_{\bullet\bullet}^{*} \\ \mathbf{m}_{\bullet\bullet}^{*} \\ \mathbf{w}_{\bullet\bullet}^{*}\end{bmatrix}\end{aligned} \tag{11}$$

Finally, targets for iteration $n = 2$ are derived. For the supply table, column targets $\boldsymbol{\upsilon}_{(2)}^{(AC)}$; for the use table, row targets $\boldsymbol{\mu}_{(2)}^{(AC)}$:

$$\begin{aligned}\boldsymbol{\upsilon}_{(2)}^{(AC)} &= \left[\overline{\mathbf{q}}_{(2)}^{(AC)} + \overline{\mathbf{m}}_{(2)}^{(AC)}\right] = \widehat{\pi}_{(2\upsilon)}^{(AC)}\left[\mathbf{q}_{(2)}^{(AC)} + \mathbf{m}_{(2)}^{(AC)}\right] \\ \boldsymbol{\mu}_{(2)}^{(AC)} &= \begin{bmatrix}\overline{\mathbf{q}}_{(2)}^{(AC)} \\ \overline{\mathbf{m}}_{(2)}^{(AC)} \\ \overline{\mathbf{w}}_{(2)}^{(A)}\end{bmatrix} = \widehat{\pi}_{(2\mu)}^{(AC)}\begin{bmatrix}\mathbf{q}_{(2)}^{(AC)} \\ \mathbf{m}_{(2)}^{(AC)} \\ \mathbf{w}_{(2)}^{(A)}\end{bmatrix}\end{aligned} \tag{12}$$

These vectors combine information contained in the $\mathbf{A}^d$, $\mathbf{A}^m$ and $\mathbf{C}$ matrices. They also ensure balance in the SUT framework. In addition, subset constraints can be included.

3.3.3. Step 2. Path BD

3.3.3.1. Commodity balancing. By commodity balancing we mean that matrices $\mathbf{V}$ and $\mathbf{M}$ are column-scaled. Conversely, matrices $\mathbf{U}^d$, $\mathbf{U}^m$ and $\mathbf{W}$ are row-scaled.

Formally:

$$\begin{aligned}\begin{bmatrix}\mathbf{V}_{(2)} \\ \mathbf{M}_{(2)}\end{bmatrix} &= \begin{bmatrix}\mathbf{V}_{(1)}^{+} \\ \mathbf{M}_{(1)}^{+}\end{bmatrix}\widehat{\mathbf{s}} + \begin{bmatrix}\mathbf{V}_{(1)}^{-} \\ \mathbf{M}_{(1)}^{-}\end{bmatrix}(\widehat{\mathbf{s}})^{-1} \\ \begin{bmatrix}\mathbf{U}_{(2)}^{d} \\ \mathbf{U}_{(2)}^{m} \\ \mathbf{W}_{(2)}\end{bmatrix} &= \widehat{\mathbf{r}}\begin{bmatrix}\left[\mathbf{U}_{(1)}^{d}\right]^{+} \\ \left[\mathbf{U}_{(1)}^{m}\right]^{+} \\ \mathbf{W}_{(1)}^{+}\end{bmatrix} + (\widehat{\mathbf{r}})^{-1}\begin{bmatrix}\left[\mathbf{U}_{(1)}^{d}\right]^{-} \\ \left[\mathbf{U}_{(1)}^{m}\right]^{-} \\ \mathbf{W}_{(1)}^{-}\end{bmatrix}\end{aligned} \tag{13}$$

Coefficients vectors $\mathbf{r}$ and $\mathbf{s}$ are calculated using the GRAS algorithm. Again, since GRAS preserves the signs in the matrix, we avoid any undesired sign flips in value-added.

3.3.3.2. Targets for industry output, final demand components, and imports. To conclude step 2, targets for $\mathbf{g}$, $\mathbf{f}$ and $\mathbf{o}$ are calculated endogenously. We also obtain estimates for total final demand components and total imports by origin. Substituting $\overline{\mathbf{q}}_{(2)}^{(AC)}$ and $\overline{\mathbf{m}}_{(2)}^{(AC)}$ in (5) we get:

$$\begin{aligned}\mathbf{g}_{(3)}^{(B)} &= \mathbf{B}_{(2)}^{d'}\overline{\mathbf{q}}_{(2)}^{(AC)} + \mathbf{B}_{(2)}^{m'}\overline{\mathbf{m}}_{(2)}^{(AC)} + \mathbf{W}_{(2)}^{'}\mathbf{i} \\ \mathbf{g}_{(3)}^{(D)} &= \mathbf{D}_{(2)}\overline{\mathbf{q}}_{(2)}^{(AC)} \\ \mathbf{f}_{(3)}^{(B)} &= \mathbf{F}^{d'}_{(2)}\mathbf{i} + \mathbf{F}^{m'}_{(2)}\mathbf{i} \\ \mathbf{o}_{(3)}^{(D)} &= \mathbf{M}_{(2)}\mathbf{i}\end{aligned} \tag{14}$$

Since $\overline{\mathbf{q}}_{(2)}^{(AC)}$ and $\overline{\mathbf{m}}_{(2)}^{(AC)}$ have been substituted in different structural equations, it is most likely that $\mathbf{g}_{(3)}^{(B)} \neq \mathbf{g}_{(3)}^{(D)}$. Target vector for $\mathbf{g}$ is derived as convex combination between vectors obtained through paths B and D. Formally:

$$\begin{aligned}\mathbf{g}_{(3)}^{(BD)} &= \beta \mathbf{g}_{(3)}^{(B)} + (1-\beta)\mathbf{g}_{(3)}^{(D)} \\ &\text{with} \quad 0 \le \beta \le 1\end{aligned} \tag{15}$$

where β can be interpretated as the degree of reliability assigned to the information contained to the information in the $\mathbf{B}^d$, $\mathbf{B}^m$ and $\mathbf{D}$ matrices.

Vectors $\mathbf{g}_{(3)}^{(BD)}$, $\mathbf{f}_{(3)}^{(B)}$ and $\mathbf{o}_{(3)}^{(D)}$ might need to be rectified to ensure balances defined in Eq. (4). If additional information is available, this rectification can be used to introduce subset constraints for next iteration. Following the example given in step 0, new deflators can be calculated:

$$\begin{aligned}\boldsymbol{\pi}_{(3\boldsymbol{\mu})}^{(BD)} &= \left[\operatorname{diag}\begin{bmatrix}\sum \mathbf{g}_{(3)}^{(BD)} \\ \sum \mathbf{o}_{(3)}^{(D)}\end{bmatrix}\right]^{-1}\begin{bmatrix}\mathbf{g}_{\bullet\bullet}^{*} \\ \mathbf{m}_{\bullet\bullet}^{*}\end{bmatrix} \\ \boldsymbol{\pi}_{(3\boldsymbol{\upsilon})}^{(BD)} &= \left[\operatorname{diag}\begin{bmatrix}\sum \mathbf{g}_{(3)}^{(BD)} \\ \sum \mathbf{f}_{(3)}^{(B)}\end{bmatrix}\right]^{-1}\begin{bmatrix}\mathbf{g}_{\bullet\bullet}^{*} \\ \mathbf{f}_{\bullet\bullet}^{*}\end{bmatrix}\end{aligned} \tag{16}$$

Subsequently, targets for iteration $n = 3$ are derived. For the supply table, we define row targets $\boldsymbol{\mu}_{(3)}^{(BD)}$; for the use table, column targets $\mathbf{v}_{(3)}^{(BD)}$:

$$\begin{aligned}\boldsymbol{\mu}_{(3)}^{(BD)} &= \begin{bmatrix}\overline{\mathbf{g}}_{(3)}^{(BD)} \\ \overline{\mathbf{o}}_{(3)}^{(D)}\end{bmatrix} = \widehat{\pi}_{(3\mu)}^{(BD)}\begin{bmatrix}\mathbf{g}_{(3)}^{(BD)} \\ \mathbf{o}_{(3)}^{(D)}\end{bmatrix} \\ \boldsymbol{\upsilon}_{(3)}^{(BD)} &= \begin{bmatrix}\overline{\mathbf{g}}_{(3)}^{(BD)} \\ \overline{\mathbf{f}}_{(3)}^{(B)}\end{bmatrix} = \widehat{\pi}_{(3\upsilon)}^{(BD)}\begin{bmatrix}\mathbf{g}_{(3)}^{(BD)} \\ \mathbf{f}_{(3)}^{(B)}\end{bmatrix}\end{aligned} \tag{17}$$

These vectors combine information contained in the $\mathbf{B}^d$, $\mathbf{B}^m$ and $\mathbf{D}$ matrices. They also ensure balance in the SUT framework. In addition, subset constraints can be included.

3.3.4. The iterative process

In each iteration, let $\mathbf{z}_{i\bullet}$ and $\mathbf{z}_{\bullet j}$ stand for the SUT row and column sum vectors. In addition, let vectors $\boldsymbol{\mu}_{(n)}$ and $\boldsymbol{\upsilon}_{(n)}$ be defined as:

$$\boldsymbol{\mu}_{(n)} = \begin{bmatrix}\boldsymbol{\mu}_{(n)}^{(AC)} \\ \boldsymbol{\mu}_{(n)}^{(BD)}\end{bmatrix} \boldsymbol{\upsilon}_{(n)} = \begin{bmatrix}\boldsymbol{\upsilon}_{(n)}^{(AC)} \\ \boldsymbol{\upsilon}_{(n)}^{(BD)}\end{bmatrix} \tag{18}$$

To achieve a unique solution, steps 1 and 2 are repeated iteratively until $n = N$ when:

$$\begin{aligned}\max\left|\boldsymbol{\mu}_{(N)} - \mathbf{z}_{i\bullet}\right| &< \varepsilon \\ \max\left|\boldsymbol{\upsilon}_{(N)} - \mathbf{z}_{\bullet j}\right| &< \varepsilon\end{aligned} \tag{19}$$

is fulfilled for a sufficiently small ε.

4. Empirical application

4.1. Data and error measurement

The main challenge faced when empirically testing deflation alternatives is the absence of "true" IO data measured in constant prices. In this paper, we circumvent this difficulty using survey-based data in current prices. Our rationale for this choice follows. If a balancing method can accurately update a matrix $\mathbf{Z}_{(0)}$ to obtain matrix $\mathbf{Z}^*$ given a set of marginal totals, we can expect the result to be as accurate as if this same information is given in constant prices. In other words, we assume if a method is good for updating, it will be good for deflating. We understand this is a strong assumption since no full set of IO data in current prices are ever available on the basis of a complete census of establishments. Purebred survey-based IO data is never cost-effective (Lahr, 1993), not even in the case of national statistical agencies.[5] We understand that our findings are imperfect from this perspective, but it is the best that one can do, given the resources at hand. Thus, our findings in this section should be absorbed with appropriate caution.

Our dataset includes 2010 and 2015 SUTs for current 27 European Union (EU) countries retrieved from the FIGARO model.[6] See Remond-Tiedrez & Rueda Cantuche (2019) for a complete report on the model. Supply $\mathbf{V}$, and use $\mathbf{U}^d$, $\mathbf{U}^m$ matrices account for $k = l = 64$ products and industries. Import matrix $\mathbf{M}$ has $o = 1$ import origins. Therefore $\mathbf{M} = \mathbf{m}$ according to our notation. Final demand matrices $\mathbf{F}^d$, $\mathbf{F}^m$ have $\varphi = 6$ components: (i) household consumption, (ii) collective consumption, (iii) government spending, (iv) gross fixed capital formation and (v) inventory variations, (vi) exports. Matrix $\mathbf{W}$ has $\rho = 4$ different rows: (a) taxes less subsidies on products, (b) gross operating surplus (c) compensation of employees and (d) other net taxes on production. To simplify how results are reported, let:

$$\mathbf{U} = \begin{bmatrix} \mathbf{U}^d \\ \mathbf{U}^m \end{bmatrix} \quad \mathbf{F} = \begin{bmatrix} \mathbf{F}^d \\ \mathbf{F}^m \end{bmatrix} \tag{20}$$

SUTs in basic prices are organised following the scheme depicted in table 1. We use years 2010 and 2015 since those are years in which EU member countries must report symmetric IO tables alongside SUTs (Eurostat, 2014).

To measure the accuracy of our estimates, we use the weighted average percentage error (WAPE) by Mínguez et al. (2009) which was further refined by Temursho et al. (2011). This measure is defined as follows. The advantage of using weighted measures is that the errors on large cells are emphasized. Conversely, differences generated in small entries are understated. Let $\mathbf{X}^* = \{x^*_{ij}\}$ be a subset of target matrix $\mathbf{Z}^*$ (e.g., $\mathbf{X}^* = \mathbf{V}^*$). WAPE is formally defined as:

$$\text{WAPE} = \sum_{i=1}^{k}\sum_{j=1}^{l}\left(\frac{\left|x^*_{ij}\right|}{\sum_i\sum_j x^*_{ij}}\right)\frac{\left|x^{(t)}_{ij} - x^*_{ij}\right|}{\left|x^*_{ij}\right|} \times 100 \tag{21}$$

To facilitate comparisons between methodologies across countries we use an accuracy gain measure. We consider the initial distance $\text{WAPE}^{(0)}$ defined as the WAPE between $\mathbf{Z}_{(0)}$ and $\mathbf{Z}^*$ matrices. We then see how each of the methods we test reduces that initial distance. Let t stand for a specific deflation alternative (e.g., double deflation). The accuracy gains (Δ) for each deflation alternative t with respect to the initial distance is defined as:

$$\Delta^{(t)} = \frac{\text{WAPE}^{(0)} - \text{WAPE}^{(t)}}{\text{WAPE}^{(0)}} \times 100 \tag{22}$$

Hence, the closer we get to 100%, the better the result will be.

4.2. Deflation alternatives: Two Path-RAS settings, double deflation, and GRAS

We evaluate our methodological proposal for two different information settings. The first (Path-RAS-1) makes use of the minimum information requirements as stated in Section 2.1. For the second setting (Path-RAS-2), we assume output by industry to be fully known. In both cases we arbitrarily set $\alpha = \beta = 0.5$.

To contrast Path-RAS performance, we applied double deflation to our dataset following the developments of section 1.3.1. In addition, we approximately reproduce the "column-row-column" deflation practice reported by Eurostat (2008, pp. 247–250). To do so, we use GRAS. This situation (deflation using standard GRAS) is hardly reproducible from the user's point of view, especially for countries/regions with less available data. Nevertheless, by using GRAS we simulate a reference point and observe how close we can get when making use of less information. Table 3 summarises the information sets used by each deflation alternative. All target information is from 2015 SUTs. On the one hand, label "Sum" is used when only the overall sum of a vector is known. On the other hand, label "Vector" means that all elements of that vector are considered.

4.3. Empirical outcomes

In this section we elaborate on Jensen's (1980) two different concepts of accuracy for IO models. On the one hand, partitive accuracy refers to the degree in which each part of the estimated model resembles its true counterpart. In this regard, we measure the cell-by-cell accuracy of our estimated SUTs. We measure each block within the SUT separately. On the other hand, the concept of holistic accuracy focuses on how the estimated models reproduce the main features of the true model (Fournier Gabela, 2020). To assess holistic accuracy, we derive IO symmetric tables from our estimated SUTs. We use these IO tables to calculate the corresponding coefficients and the Leontief inverse matrices and measure how close they are to the true models. In addition, we use the Leontief inverse matrices to simulate hypothetical demand models.

4.3.1. Partitive accuracy

Figs. 1 and 2 illustrate the results regarding partitive accuracy. Two clarifying comments must be made before we start discussing our findings. First, double deflation yields, by definition, the same results as GRAS for the $\mathbf{M}$ and $\mathbf{V}$ matrices. This is because a set of consistent targets is imposed for all products, industries and for total imports by origin (see Section 2.2). Second. in the case of Romania (RO), the supply matrix $\mathbf{V}$ is a diagonal matrix with no secondary production. Therefore, all alternatives that consider exogenously given $\mathbf{g}^*$ targets (Path-RAS-2, double deflation, and GRAS) precisely generate the 2015 matrix.

As for the results, on the one hand, Path-RAS-1 yields the poorest performance for matrices $\mathbf{V}$, $\mathbf{M}$, $\mathbf{U}$ and $\mathbf{F}$ as expected. Using only the minimum information requirements, this alternative ensures that estimated tables respect the limited set of targets. Accuracy gains are generally modest compared to those obtained with the remaining alternatives. In some cases, the overall error is even slightly greater after balancing than before.

On the other hand, Path-RAS-2 generally performs as well as double deflation and GRAS. Thus, results suggest, despite fewer information

Table 3
Information used for each deflation alternative.

	$\mathbf{g}^*$	$\mathbf{o}^*$	$\mathbf{q}^*$	$\mathbf{m}^*$	$\mathbf{f}^*$	$\mathbf{w}^*$
Path-RAS-1	Sum	Sum	Sum	Sum	Sum	Sum
Path-RAS-2	Vector	Vector	Sum	Sum	Sum	Sum
DD	Vector	Vector	Vector	Vector	Vector	Sum
GRAS	Vector	Vector	Vector	Vector	Vector	Vector

[5] Governments interpolate information for firms that do not reply and answer to some questions that establishments fail to supply. Published government data are far from perfect, despite our hopes and beliefs.

[6] All data is available at: https://ec.europa.eu/eurostat/web/esa-supply-use-input-tables/data/database

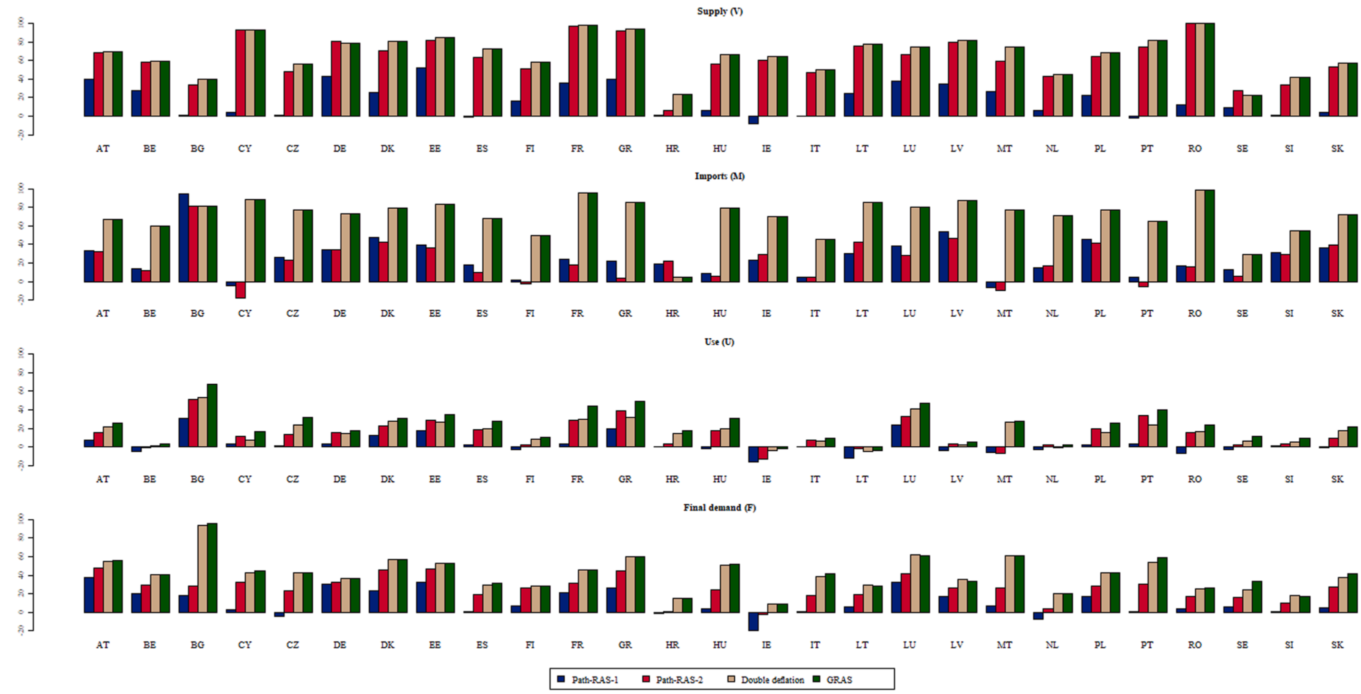

Fig. 1. Accuracy gains for each deflation alternative. Matrices **V, M, U** and **F**.
Source: own elaboration.

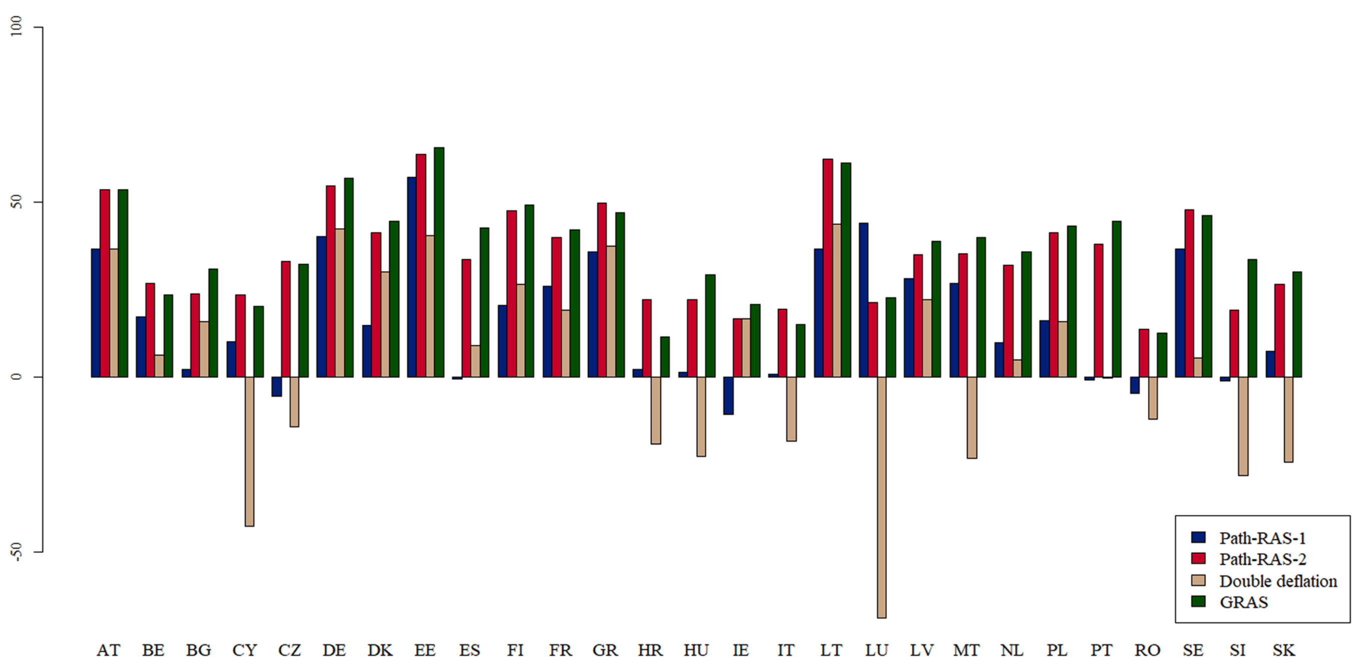

Fig. 2. Accuracy gains for each deflation alternative. Value-added matrix (**W**).
Source: own elaboration.

requirements, that Path-RAS is as accurate as alternatives that are more data-demanding. Moreover, Path-RAS-2 seems to perform better for **V** and **U** matrices. This is in line with Dietzenbacher and Hoen's (1998) findings that intermediate transactions with cell-specific deflation would be substantially better. Overall results are likely to improve as additional information constrains matrices **M** and **F**. Note that volume data on final demand and imports tend to be more widely available than are data for intermediate demand or intermediate inputs.

GVA estimates deserve a separate comment. For eighteen out of twenty-seven countries that we analysed, Path-RAS-1 outperformed double deflation in estimating GVA. Double deflation, making use of more information, yields relatively smaller accuracy gains. In ten cases, accuracy gains for GVA using double deflation were negative. Indeed, Path-RAS-2 yielded better results in all cases; accuracy gains are in line with those via GRAS. This result is highlighted since GRAS requires total GVA components to be established exogenously. While the exogenous specification of GVA is reasonable when updating matrices in current prices, it is nigh unto impossible for matrix price deflation. Thus, the

Path-RAS modification appears to be a truly viable innovation in this regard.

4.3.2. Holistic accuracy

Admittedly, the concept of holistic accuracy can be addressed in various ways. For the purpose of this paper, we partially reproduce the experiment by Dietzenbacher and Temursho (2012). These authors combine the final demand vectors of various IO tables to generate a hypothetical final demand vector $\tilde{\mathbf{y}}$. Using this vector, they estimate the gross output using the Leontief inverse matrices derived from their deflated and non-deflated models. It is to be noted that when we get to know gross output in constant prices, calculations regarding real income, energy use, etc. become possible too. In their experiment deflated and non-deflated IO tables yield quite similar results. They do point, however, that (i) sector-level differences are notable and (ii) their test used potentially peculiar data and, thus, should be taken with some caution. Moreover, other work on other countries (Tandon and Ahmed, 2016) shows substantial differences in impacts over time by industry.

We follow Eurostat (2008, pp. 293–371) to derive symmetric IO models from the true SUTs and those estimated using GRAS, double deflation and the two Path-RAS scenarios. We produce product-by-product input-output tables based on the industry technology assumption: each industry has its own specific way of production, irrespective of its product mix. See Rueda Cantuche (2011) for further discussion on the transformation of SUTs into symmetric IO tables. We derive the technical coefficient ($\mathbf{A}$) and the Leontief inverse ($\mathbf{L}$) matrices using the fundamental relationships of the IO demand models (see Miller and Blair, 2022, pp. 10–35). For our hypothetical final demand vector, we use the average row sum across countries of the true final demand matrices ($\mathbf{F}^*\mathbf{i}$). We calculate hypothetical gross output vectors using the basic equation of the IO demand models:

$$\tilde{\mathbf{q}} = \mathbf{L}\tilde{\mathbf{y}} \tag{23}$$

For the purpose of error measurement, the true output array is the result of multiplying the true Leontief inverse ($\mathbf{L}^*$) by the hypothetical final demand vector ($\tilde{\mathbf{y}}$). We use WAPE as the error measure as in Section 4.3.1. The lower the error, the better the estimate is.

Table 4 summarizes our results for the $\tilde{\mathbf{q}}$ vectors in each country as well as for the $\mathbf{A}$ and $\mathbf{L}$ matrices. We report the GRAS results separately to observe how close to reality would our estimates be in case of having a much more complete set of price information (see table 3). The errors observed when estimating our hypothetical output vectors ($\tilde{\mathbf{q}}$) appear to be quite close between the different tested alternatives, with the exception of Cyprus (CY). This means that, within our dataset, we can approximately replicate the output of an IO demand model using less information. Double deflation yields better results in a majority of cases. Nevertheless, Path-RAS-2 surpass its double deflation competitor on average across all countries. As for the $\mathbf{A}$ and $\mathbf{L}$ matrices, Path-RAS seems to perform better than double deflation despite utilising less information. Even in the minimum information scenario (Path-RAS-1), the error level is not so far away from more data demanding alternatives, GRAS estimates included. As expected, introducing additional information generally leads to better results. For the $\mathbf{A}$ matrices, Path-RAS-2 outperforms double deflation and Path-RAS-1 in 20 countries out of 27. For the $\mathbf{L}$ matrices, it presents lower errors in 18 countries.

5. Conclusions

In this paper, we develop an alternative way to estimate supply and use tables (SUT) in constant prices. Our proposal modifies the Path-RAS approach for SUT updating. It requires less information than do known predecessor approaches. Most importantly, we reduce needs for gross value added (GVA) price indexes. We believe these features can facilitate SUT deflation in regions and nations where data is relatively scarce. The approach yields cell-specific deflators, capturing IO price dynamics more realistically. Therefore, it could be used to circumvent some issues related with physical and monetary IO models. Moreover, our approach does not introduce *ad hoc* adjustments, providing constraints are non-conflicting. Hence, we provide a transparent and reproducible method both from the users and the producers point of view. Despite been

Table 4
WAPE between estimated and observed values. **A** and **L** matrices, and hypothetical output.[71] **Source:** own elaboration.

	A				L				$\tilde{\mathbf{q}}$			
	GRAS	DD	Path-RAS-1	Path-RAS-2	GRAS	DD	Path-RAS-1	Path-RAS-2	GRAS	DD	Path-RAS-1	Path-RAS-2
AT	17.85	19.39	18.08	**17.88**	6.07	6.66	**6.02**	6.06	2.61	**2.59**	3.14	3.02
BE	40.78	43.23	41.22	**41.01**	12.71	13.59	**12.67**	12.91	5.48	5.40	**5.22**	5.94
BG	14.92	**16.54**	23.17	18.91	5.19	**5.79**	8.60	6.79	1.22	**0.93**	6.11	4.28
CY	133.03	329.82	**58.25**	151.29	38.91	97.11	**16.01**	45.63	12.47	42.18	**7.97**	15.61
CZ	25.35	27.50	28.05	**27.35**	8.56	9.77	9.78	**9.35**	3.71	**4.53**	5.84	5.03
DE	23.45	24.69	25.87	**23.96**	8.94	9.79	10.23	**9.32**	3.93	4.13	5.68	**4.09**
DK	22.94	25.83	26.79	**23.64**	6.98	8.19	8.42	**7.11**	2.49	**2.75**	4.21	3.09
EE	28.11	32.41	28.40	**27.89**	7.92	9.26	8.10	**7.86**	3.38	3.60	3.54	**3.29**
ES	27.29	30.11	30.24	**27.99**	9.18	10.55	10.84	**9.62**	4.63	**5.21**	7.14	5.31
FI	36.83	**37.69**	38.08	37.94	11.61	**11.94**	12.45	12.16	5.43	**5.47**	7.01	6.18
FR	10.94	14.76	12.85	**10.78**	4.12	5.97	5.02	**4.11**	2.47	2.57	3.88	**2.51**
GR	20.66	38.51	27.31	**22.49**	6.36	13.82	8.67	**7.34**	3.12	5.16	5.45	**4.52**
HR	71.30	82.04	74.17	**72.81**	23.87	27.30	24.66	**24.08**	16.46	**16.96**	19.44	18.93
HU	30.98	32.95	33.33	**31.44**	7.47	8.34	8.30	**7.58**	3.69	4.48	5.38	**3.85**
IE	67.66	76.49	71.46	**67.14**	20.47	24.44	23.57	**22.19**	12.98	**14.76**	16.30	15.93
IT	43.31	43.80	44.15	**43.42**	14.23	14.62	15.01	**14.54**	3.23	**3.57**	5.77	4.10
LT	84.01	**89.77**	95.25	93.48	22.34	**24.31**	26.29	24.89	9.69	10.91	13.73	**10.84**
LU	51.53	64.67	50.88	**48.83**	14.02	18.97	14.37	**13.08**	6.42	7.22	8.38	**6.10**
LV	69.43	73.63	69.71	**68.25**	21.16	23.11	21.05	**20.61**	8.56	**7.92**	9.53	7.97
MT	65.54	70.95	**55.74**	58.51	20.73	20.72	**16.01**	17.85	11.40	11.38	**8.18**	11.11
NL	34.34	34.63	33.57	**33.09**	10.57	10.62	10.23	**10.07**	5.15	**4.98**	5.59	5.08
PL	19.99	22.08	21.93	**19.71**	6.93	7.98	7.87	**6.78**	3.31	**3.18**	5.07	3.39
PT	18.64	21.64	19.87	**18.74**	6.42	7.81	7.25	**6.41**	2.82	3.49	4.57	**3.40**
RO	40.98	45.59	45.97	**40.66**	15.52	17.57	18.47	**15.82**	8.49	**8.68**	14.06	8.72
SE	45.89	**54.77**	60.28	59.74	13.65	**17.12**	18.87	18.69	4.23	11.38	10.21	**9.37**
SI	45.21	**45.95**	46.72	47.22	12.97	**13.54**	13.54	13.86	4.17	**3.93**	5.47	6.03
SK	52.17	53.08	53.49	**51.84**	17.22	17.47	17.46	**17.07**	7.35	**8.10**	9.19	8.77
Mean	42.34	53.80	42.03	43.93	13.12	16.90	13.32	13.77	5.88	7.61	7.63	6.91
Best	—	5	2	20	—	5	4	18	—	15	3	9

conceived for the purpose of deflation, our algorithm might be used as a RAS variant in different contexts. For instance, we have shown that it is possible to use our method to update supply and use tables when information is scarce. In addition, our Path-RAS might be used to obtain regional supply and use tables from a national one in a similar vein as in Malizia and Bond (1974).

Even though Path-RAS demands comparatively modest amounts of data, it yields promising results. Admittedly, when available information is minimal, Path-RAS performs no better than its "competition." But it appears to outshine double deflation when it comes to estimating GVA. Moreover, when industry output is constrained, Path-RAS performs in line with GRAS, which requires GVA margins to be exogenously defined. According to our results this feature also holds when we use the deflated SUTs to build IO demand models. This suggests our modification to Path-RAS yields results that are sufficiently accurate despite using less information. Estimates for **V** and **U** appear to be relatively more accurate. This result seems coherent with the lower errors observed for the **A** and **L** matrices. Hence, we could be confirming the appropriateness of cell-specific deflators to measure intermediate transactions in constant prices. In addition, the balancing process is multiplicative, so it does not induce undesired sign flips.

The prime limitation of our work is the empirical test. Thus, our approach needs further empirical assessments. A broader coverage of countries could reduce biases associated with the use of peculiar data. Alternatively, accuracy could be evaluated using random SUTs as in Bonfiglio & Chelli (2008). We also hope to extend our analysis by deriving IO more tables from the deflated SUTs. We, thus, should be able to analyse how different deflation methods relate to such standard work as, for example, impact analysis, multiplier analysis or structural decomposition analysis. A possible extension of the research presented here could be the inclusion of techniques that systematically manage conflicting information. Finally, we hope future research will identify optimal reliability values (α, β) associated with the industry and commodity technology matrices.

Funding sources

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Author statement

We now provide a brief description of each author's contribution to the research article:

CRediT authorship contribution statement

Xesús Pereira López: Conceptualization, Writing – review & editing. **Fernando de la Torre Cuevas:** Conceptualization, Formal analysis, Methodology, Investigation, Software, Writing – original draft.

Declaration of Competing Interest

None.

Data availability

Data will be made available on request.

[7] In bold, the most accurate alternative between double deflation, Path-RAS-1 and Path-RAS-2.

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