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Jet quenching for measuring  
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TESE DE DOUTORAMENTO

**JET QUENCHING FOR  
MEASURING THE QCD  
COLLECTIVITY TEMPORAL  
STRUCTURE**

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# Abstract

In this thesis we use the medium-induced gluon radiation spectrum to study the quark gluon plasma generated at the LHC. In particular, we employ a new computational algorithm that exactly resums the QCD potential without truncations — either in the opacity expansion or the perturbative tails. First, we compare the fully resummed spectrum with different analytical approaches, improving our understanding of the physics behind the emission process. Then, we find an efficient way of computing the radiation spectrum in a realistically evolving media described by a hydrodynamical simulation, a crucial step to implement the framework in phenomenology. Finally, we analyze the impact that the propagation during the initial stages of the collision, which is usually neglected, has on jet quenching observables.

*Keywords:* Heavy ion collisions (HICs), quark gluon plasma (QGP), jet quenching, initial stages.



## Resumo

Nesta tese usamos o espectro de radiación de gluóns inducido polo medio para estudar o plasma de quarks e gluóns xerado no LHC. En particular, empregamos un novo algoritmo computacional que resume o potencial de QCD exactamente, sen truncamentos — nin na expansión de opacidade nin nas colas perturbativas. Primeiro, comparamos o espectro completamente resumado con diferentes aproximacións analíticas, mellorando a nosa comprensión da física que goberna o proceso de emisión. Despois, atopamos un método eficiente de calcular o espectro de radiación nun medio que evoluciona de xeito realista seguindo unha simulación hidrodinámica, un paso crucial para implementar o formalismo na fenomenoloxía. Finalmente, analizamos o impacto que ten a propagación durante os estádios iniciais da colisión, que é normalmente desprezada, en observables de jet quenching.

*Palabras chave:* Colisións de ións pesados (HICs), plasma de quarks e gluóns (QGP), supresión de jets, estádios iniciais.



## Resumen

En esta tesis usamos el espectro de radiación de gluones inducido por el medio para estudiar el plasma de quarks y gluones generado en el LHC. En particular, empleamos un nuevo algoritmo computacional que resume el potencial de QCD exactamente, sin truncamientos — ni en la expansión de opacidad ni en las colas perturbativas. Primero, comparamos el espectro completamente resumado con diferentes aproximaciones analíticas, mejorando nuestra comprensión de la física que gobierna el proceso de emisión. Después, encontramos un método eficiente de calcular el espectro de radiación en un medio que evoluciona de forma realista siguiendo una simulación hidrodinámica, un paso crucial para implementar el formalismo en la fenomenología. Finalmente, analizamos el impacto que tiene la propagación durante los estadios iniciales de la colisión, que es normalmente despreciada, en observables de jet quenching.

*Palabras clave:* Colisiones de iones pesados (HICs), plasma de quarks y gluones (QGP), supresión de jets, estadios iniciales.



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## Extended Abstract

QCD has been proved in all kind of kinematic regimes since it was formulated in the last century. Over the last decades one of the most active research areas is the study of QCD under extreme density and temperature conditions, where new states of the phase diagram can be found. In particular, at high energies the quarks and gluons inside the nuclear matter deconfine and create the QGP, a hot and dense medium that filled the early universe a few microseconds after the Big Bang. Nowadays we study the QGP and its properties in the laboratory by employing heavy ion collisions in colliders all around the world, like LHC and RHIC.

Highly energetic partons travelling through the QGP will experience modifications w.r.t. the vacuum propagation. These modifications are usually encompassed under the term jet quenching. The main objective of this thesis is to study the QGP employing jet quenching techniques. To be more precise, the jet quenching tool we will employ throughout the whole manuscript is the medium-induced gluon radiation spectrum. In this thesis we analyze the following specific topics.

First, we discuss the improvement of the computation of in-medium gluon emission. This radiation spectrum is the building block of a great number of phenomenological studies. However, the difficulties that arise when we try to compute it (having to solve a path integral with imaginary potential) force us to usually employ approximations, such as the Harmonic Oscillator (HO) or the opacity expansion. In recent years, a lot of theoretical effort has been put into improving this computation. In this thesis we make use of a previously developed formalism that evaluates the radiation spectrum with full resummation over multiple scatterings, which is then employed in several original studies. The evaluation of the spectrum with this method requires solving numerically the Schwinger-Dyson recursive equations satisfied by the emission kernel and momentum broadening, instead of computing them directly. This approach has many advantages compared to others, among which we highlight the flexibility to use any elastic collision rate model and/or medium profile.

After introducing the method for evaluating the spectrum, we use it to improve the current understanding of the physics behind the emission process. As stated in the previous paragraph, it is common to employ approximations when computing the in-medium radiation spectrum. Different approaches rely on distinct approximations, encoding different physical interpretations of the emission process. By using our full spectrum as a reference scenario w.r.t. which compare several analytical formalisms, we can validate the range of applicability of their assumptions across all energy regimes. We will pay special attention to the low energy region (also known as Bethe-Heitler regime), where our understanding of the physics governing the emission process is more limited. As there is no analytical framework that successfully describes the spectrum in this region, here, for the first time, we derive the low energy limit of the all-order spectrum which approximates the full result with high accuracy in the Bethe-Heitler regime. With these results we are able to achieve a proper physical interpretation of the emission process for all gluon energies.

Following the spirit of overcoming the commonly used approximations, we devote a chapter to compute the radiation spectrum in a realistically expanding QGP described by a hydrodynamical simulation, surpassing in that way the common assumption employed in many analytical approaches of considering the background medium as a collection of static scattering centres, usually called the brick approximation. This is a fundamental step to be able to implement our framework in phenomenological studies that describe experimental data with high precision. Computing the full solution of the spectrum using an expanding medium instead of a brick is much more computationally demanding. In order to overcome this situation, we investigate the use of scaling laws, simple relations between parameters that allows us to compare the spectrum computed in different scenarios. The classical formulation of scaling laws for the HO approach employs as reference scenario the static case, and it allowed the multiple soft scattering approximation to be used in many phenomenological works with high success. However, the performance of these scaling laws in the full case is not good enough. Here we will explore the use of scaling laws using as reference another scenario proposed by us, a power-law expanding medium, and we compare the performance of both approaches to find which one provides the best approximation of the fully resummed spectrum.

Finally, we use the medium-induced radiation spectrum evaluated in evolving media to study the role played by the initial stages of the heavy ion collision on jet quenching. Jet quenching studies have typically neglected the effects of the early stages of the collision, arguing that the length of the medium ( $\sim 5 - 10$  fm) is much larger than that of the initial stages ( $\sim 0.2 - 1$  fm), so their effects should be minimal. However, recently it was shown that initial stages may play a crucial role when we want to describe many jet quenching observables simultaneously. By comparing several scenarios with different treatment of the propagation during the initial stages we are able to isolate their effect in medium-induced radiation. We will then use these in-medium emission spectra to compute proxies of the inclusive suppression of charged particles and the high momentum azimuthal asymmetry for every scenario, achieving different results in every case. This simultaneous description of two jet quenching observables for different treatments of the early stages of the propagation will manifest the importance of properly accounting for initial stages on jet quenching studies.

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# 1

## Introduction

In this introductory chapter we start by briefly addressing some general points concerning Quantum Chromodynamics (QCD) under extreme temperature and density conditions. After that we present in detail the main topic we will discuss in this thesis, jet quenching, which is an umbrella term that encompasses any modification experienced by jets propagating through strongly interacting media. As exhaustively studying every jet quenching aspect is a task impossible to achieve in a thesis of this extension, we state here which jet quenching topics will be treated in detail in subsequent chapters of the thesis.

### 1.1 QCD under extreme conditions

#### QCD at high energies and densities

One of the main goals of the particle physics community is the study of strong interactions described by Quantum Chromodynamics (QCD) under extreme conditions. In particular, in the 1970's the existence of a new state of nuclear matter at large energies and densities was predicted for the first time [5–9]. This new universal state of matter at high temperature and density is called quark gluon plasma (QGP) and it is formed by free quarks and gluons that are not confined inside mesons and hadrons. The QGP filled the early universe a few microseconds after the Big Bang and understanding its properties has been one of the most active areas over the last decades. The QGP is also going to be the main object of interest of this thesis.

As the QGP is essentially a hot and dense medium formed by deconfined matter, if we create QGP in an experiment we can investigate the confined/deconfined phase transition

and get information of the QGP equation of state. The best experimental tool we have nowadays for creating the QGP are high energy heavy ion collisions, specially Au-Au at the Relativistic Heavy Ion Collider (RHIC) [10] and Pb-Pb at the Large Hadron Collider (LHC) [11]. In this kind of events the energy density achieved at the collision point is so extreme that protons and neutrons break down, freeing quarks and gluons inside them and generating an extremely short lived medium. Due to this short lifetime we can not measure the QGP directly and we have to employ different probes generated in the same event and that are modified by the underlying medium to study its properties.

Essentially we can classify these probes in two groups: soft probes and hard probes. Soft probes refer to those processes with small energy scale  $Q^2 \simeq \Lambda_{\text{QCD}}^2$ , being  $Q^2$  the relevant energy scale of the problem and  $\Lambda_{\text{QCD}} \approx 200$  MeV the QCD scale that separates weak and strongly coupled regimes. Thus, these kind of processes explore the non-perturbative region of QCD, which is especially relevant for understanding hadrons, their creation and structure. Also, low energy processes see the medium as a whole, so we can access information of bulk medium properties [12–14]. However, even though they are very interesting phenomena, we can not apply perturbative techniques in order to study them theoretically and will not be discussed in this thesis.

On the other hand we have hard probes, the ones that possess a large relevant energy scale  $Q^2 \gg \Lambda_{\text{QCD}}^2$ , which can be transverse momentum or mass. Hard probes are suitable to be employed to analyze the perturbative region of QCD, providing information about the effects of the medium on perturbative processes through the theoretical description of experimental data. These kind of studies allow us to learn about the medium properties, characterizing in that way the microscopic structure of the QGP (see [15] for a recent review on hard probes).

One of the most interesting features about hard probes is the fact that they are produced during the initial stages of the collision ( $\tau \lesssim 0.1$  fm/c), thus they can propagate through all of the stages of the medium from the very beginning and give us information about them [16]. Understanding the formation of the QGP during the initial stages of the collision is crucial to improve the current knowledge of the thermalization process. This is particularly relevant nowadays, as one of the main open questions in the field is the (possible) creation of QGP in small systems [17–19]. A proper understanding of thermalization during the early stages is essential to understand these shocking results.

There are many different kinds of hard probes, but the one we will focus on throughout the development of this thesis are jets and the modifications they undergo when propagating through strongly interacting matter, commonly known as jet quenching. Analyzing the effects that initial stages may have on jet quenching is the most ambitious goal of the current thesis, which is discussed in chapter 5.

## Lagrangian formulation of QCD

The study of strong interactions requires the employment of QCD, the non-Abelian Quantum Field Theory (QFT) which leaves unchanged its Lagrangian under  $SU(3)$  group

transformations, being color the associated conserved quantity. QCD describes color interactions between quarks and gluons, the building blocks of all hadronic matter. The QCD Lagrangian which determines their dynamics is given by [20–24]

$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{q}_i^f(x) [i\gamma^\mu D_\mu(x) - m_f]_{ij} q_j^f(x) - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}, \quad (1.1)$$

where  $q_i^f(x)$  ( $\bar{q}_i^f(x)$ ) is the quark (antiquark) fermionic field, with color  $i \in \{1, 2, 3\}$  in the fundamental representation of  $SU(3)$  and flavor  $f \in \{u, d, s, c, b, t\}$ ,  $m_f$  is the corresponding quark mass and  $\gamma^\mu$  are the Dirac gamma matrices.  $D_\mu$  stands for the covariant derivative, which ensures the Lagrangian invariance under local gauge transformations

$$D_\mu(x) = \partial_\mu - igA_\mu^a(x)t^a, \quad (1.2)$$

with  $g$  the strong coupling constant,  $A_\mu^a$  the gluon gauge field in the adjoint color representation  $a \in \{1, 2, \dots, 8\}$  and  $t^a$  the  $SU(3)$  color matrices in the fundamental representation. Finally,  $F_{\mu\nu}^a$  is the gauge field tensor given by the following expression in terms of the gauge field

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c, \quad (1.3)$$

where we are using the color group structure constants  $f^{abc}t^c = -i[t^a, t^b]$ . The quantization of this Lagrangian (1.1) allow us to build a set of Feynman rules which can be used to study any QCD perturbative process we like in terms of Feynman diagrams [20, 21].

In this Lagrangian eq. (1.1) it is specially relevant the appearance of nonlinear terms, which are the ones proportional to the color matrices  $t^a$  in the covariant derivative (1.2), and the ones proportional to the structure constant  $f^{abc}$  in the gauge field tensor (1.3). These kind of contributions only appear in non-Abelian theories to assure that gauge invariance is preserved, and they are directly responsible of the most fascinating properties of QCD. For instance, asymptotic freedom is explained due to the presence of these terms, as we discuss in the following section.

## Running coupling constant and pQCD

The non-Abelian nature of QCD leads to some of the most interesting properties of the theory. The first important characteristic refers to nature being color neutral at low energies, with quarks and gluons confined inside hadrons and only being able to escape at high energies. This is called confinement and it implies that we can not measure quarks and gluons directly. The second remarkable feature allows us to perform perturbative calculations employing a finite number of Feynman diagrams, and it is called asymptotic freedom. Asymptotic freedom is the fact that the QCD running coupling constant,  $\alpha_s = g^2/(4\pi)$ , becomes smaller the higher the energy scale is, thus at large energies (small distances) quarks and gluons are almost free particles and standard perturbation techniques are possible in this region. In the opposite limit, as energy decreases (distance

increases) the coupling constant becomes larger (non-perturbative regime) and at some point interactions are so strong that partons can not separate from each other [24, 25].

The behaviour of the running coupling constant is explained due to the non-Abelian nature of QCD. As gluons own the color charge they mediate, they can interact with themselves and we have three and four gluon interaction vertices in the theory. This fact has some important consequences and, in particular, if we compute the coupling constant at one loop order we find that it can be expressed as [24, 26]

$$\alpha_s(Q^2) \equiv \frac{g^2(Q^2)}{4\pi} = \frac{1}{\beta \log \frac{Q^2}{\Lambda_{\text{QCD}}^2}}, \quad (1.4)$$

with  $Q^2$  the relevant energy scale of the problem,  $\Lambda_{\text{QCD}} \approx 200 \text{ MeV}$  the QCD scale and  $\beta = (11N_c - 2n_f)/(12\pi)$ , being  $N_c = 3$  the number of colors and  $n_f$  the number of active flavors (6 if all of them are active). We see in eq. (1.4) that for large scales ( $Q^2 \gg \Lambda_{\text{QCD}}^2$ )  $\alpha_s$  is negligible, thus asymptotically the theory is free QCD, being the physical degrees of freedom quarks and gluons. On the other hand, if we have small scales ( $Q^2 \simeq \Lambda_{\text{QCD}}^2$ ) the denominator of eq. (1.4) is close to 0, hence  $\alpha_s$  diverges and the theory is so strongly coupled that we can not apply perturbative techniques. In this regime quarks and gluons are confined inside hadrons. The running of the QCD coupling constant has been measured in many independent experiments with high precision, being in complete agreement experimental results and theoretical predictions [27].

As stated in the previous paragraph, for high energies we can simplify QCD employing perturbative expansions and describe processes using only a few Feynman diagrams. This simplified version of QCD employed in the high energy regime is known as perturbative QCD (pQCD) and it is the basic tool that we use to describe the propagation of high energy particles in all kind of scenarios. pQCD has been tested in many different conditions for decades in accelerators all around the world with high success [28–31]. Some of the most important examples of colliders that highly contributed to improve our knowledge about QCD are: the Alternating Gradient Synchrotron (AGS) at the Brookhaven National Laboratory (BNL), the Super Proton Synchrotron (SPS) at Conseil Européen pour la Recherche Nucléaire (CERN), the Hadron–Electron Ring Accelerator (HERA) at Deutsches Elektronen-Synchrotron (DESY), RHIC at BNL and LHC at CERN.

## Hard probes in heavy ion collisions: jet quenching

Asymptotic freedom is employed at high energies to justify the use of pQCD to study strong processes analytically. As it is required to have a large energy scale to utilize pQCD, hard probes in heavy ion collisions are the kind of signals we are interested in employing to characterize the QGP. One of the most important characteristics about these kind of probes is that they are usually analyzed by studying their modifications with respect to vacuum. In practice this means that we use p-p collisions as a reference. Comparisons between results in nucleus-nucleus and proton-proton collisions allow us to

isolate the effects induced by the medium, which can be used to extract information about its properties. Among all of the available hard probes in this thesis we will focus on jets, as previously stated.

Roughly speaking, a jet is a parton which propagates (either in medium or vacuum) at high energy radiating many particles in a cone-shape. These particles later hadronize and are experimentally measured by detectors. Jets are also one kind of hard probe which can resolve the time evolution of the medium [32, 33]. Moreover, we have high precision data of vacuum jets measured in p-p collisions which can be used to compare with, so we can accurately isolate medium contributions to extract QGP properties [34, 35]. Due to these reasons, we choose jets as the tool that we employ to investigate the QGP.

These jets when traveling through a dense medium like the QGP experience modifications with respect to vacuum propagation. We use jet quenching as an umbrella term that includes all of the possible effects that jets undergo due to interactions with the medium. Among these modifications here we focus on the ones that make jets lose energy compared to the case where these interactions do not exist.

There are two kinds of energy loss processes: radiative energy loss and collisional energy loss. Radiative energy loss refers to the emission of quarks and gluons by a high energy parton which, due to interactions with the medium, go outside the jet cone-shape. This leads to these particles not being measured by the detector, thus resulting in a smaller energy assigned to the jet when comparing w.r.t. the case where we have no medium. On the other hand, collisional energy loss denotes parton-medium interactions which result in the medium directly absorbing some of the parton energy. Radiative energy loss is dominant for light quarks and gluons, whereas that collisional energy loss is only relevant for heavy quarks, thus it will not be discussed in this thesis.

Jet quenching was first conjectured in 1982 by Bjorken [36] and measured experimentally at RHIC 20 years later [37]. Currently, jet quenching is a well-established hard probe measured in several kinds of conditions. For instance, we have many experimental data about the single-inclusive particle spectra [38–43] and two-particle correlations [44–47]. Moreover, other reflections of jet quenching have been seen in several jet analyses [48–51].

Jet quenching is the main tool that we are going to employ in this thesis to investigate the QGP and its properties. We use the following section 1.3 to introduce some of the jet quenching techniques that we will employ in subsequent chapter of this manuscript.

## 1.2 Objectives and methodology

After this more general part of the introduction we include here, to comply with the regulations for doctoral studies of the University of Santiago de Compostela, the main goals of the thesis and a summary of the methodology that we will employ as two separated sections.

### 1.2.1 Objectives and hypothesis

The main assumption in which the current thesis relies on is the fact that we consider that the propagation of high energy partons through the QGP is well-described employing jet quenching techniques. This thesis has three main objectives:

- Upgrade our current knowledge about the gluon emission process inside the medium. We do so by comparing a recently developed approach which computes the medium-induced gluon radiation with full resummation over multiple scatterings to several analytical formalisms that rely on different assumptions regarding the radiation process.
- Improve the computation of the medium-induced radiation spectrum by extending the before-mentioned exact formalism beyond the static case, allowing the evolution of the medium to be described by a hydrodynamic simulation.
- Analyze the impact that initial stages may have on jet quenching. We achieve this goal by evaluating the in-medium emission spectrum for different initial stages scenarios and using the results to describe simultaneously the single-inclusive particle suppression  $R_{AA}$  and the high- $p_T$  azimuthal asymmetry  $v_2$  in every scenario.

### 1.2.2 Methodology

This thesis is focused on jet quenching, more precisely on the computation of the medium-induced radiation spectrum in the soft limit. The theory we employ to perform the calculations is the before-mentioned pQCD. Moreover, in the following section we properly explain the jet quenching techniques that will be useful throughout the whole thesis.

## 1.3 Jet quenching

As previously stated, jet quenching is an umbrella term that assembles several different processes which have in common the fact that they modify jets when passing through a strongly interacting medium. In this section we describe some basic features of jet quenching, emphasizing the most relevant ones for medium-induced gluon radiation, which is going to be the central part of this thesis. We remark that we will not provide rigorous derivations for most of the results, focusing on physical discussions rather than technical details. Proper deductions of the results that we introduce here can be found in literature, see for instance the following reviews [1, 52, 53].

## Propagation through medium. Eikonal approximation and Wilson lines

We start by describing how we treat the propagation of a high energy parton inside a medium like the QGP employing jet quenching techniques. This section will be mainly based on [1] and we refer the reader there for further details.

First of all, we need to make several assumptions to be able to analyze this problem analytically in an easy way. The most important ones refer to how we treat the medium. Essentially, we assume that the QGP can be considered like a classical background field  $A(x)$ , being  $x$  the position in coordinate space, that experiences no recoil due to the parton propagation, i.e, the medium does not feel any modification when particles travel through it. At microscopic level we suppose that the medium is composed by a collection of scattering centers which can interact with our jet.

Regarding the traveling parton, we assume that its energy is very large. In fact, we will use the eikonal limit, in which the only modification the particle can experience from interactions with the medium is a rotation of its color, but not longitudinal momentum changes. Hence, the parton will propagate in a straight trajectory, which is described by a Wilson line that we briefly derive in this section.

As eikonal is an expression employed in many different fields and formalisms to denote high energy limit but with different connotations in each one of them, let us rigorously state what we mean here by eikonal limit. In the following we are going to employ light cone coordinates, which are defined as

$$x^\pm = \frac{1}{\sqrt{2}}(x_0 \pm x_3), \quad p^\pm = \frac{1}{\sqrt{2}}(p_0 \pm p_3), \quad (1.5)$$

being  $x$  the position and  $p$  the momentum. With these variables the scalar product is defined as

$$x_\mu p^\mu \equiv x \cdot p = x^+ p^- + x^- p^+ - \mathbf{x} \cdot \mathbf{p}, \quad (1.6)$$

being the bold coordinates 2D vectors in the transverse plane. In particular this implies that in the massless case  $p^2 = 0 \rightarrow p^- = \mathbf{p}^2/(2p^+)$ , a very useful relation we will use. In these light cone coordinates the plus component of  $x$  plays the role of time, thus we will denote it by  $x^+ \equiv t$ . Moreover, the longitudinal momentum  $p^+$  plays the role of the energy, hence we use  $p^+ \equiv \omega$  from now on.

In the eikonal limit all components of the momentum can be neglected except the longitudinal one  $\omega$ . Assuming that the parton is a right mover propagating in the positive direction of  $x_3$ , its large momentum component is  $\omega \gg |\mathbf{p}|$  and  $|\mathbf{p}| \gg p^- \simeq 0$ , which can be seen employing  $p^- = \mathbf{p}^2/(2\omega)$ . Thus, in the high energy limit, the minus component of the momentum of the parton is suppressed  $p^- \approx 0$ , and hence for the high energy parton the only relevant physics happens locally at  $x^- = 0$ . This allows us to simplify the dependencies of the background field  $A_\mu^a(x) \simeq A_\mu^a(t, 0, \mathbf{x}) \equiv A_\mu^a(t, \mathbf{x})$  and write  $p^\mu A_\mu^a(x) \simeq \omega A_-^a(t, \mathbf{x})$  when dealing with field insertions in the Feynman diagrams.

Now that we have stated what eikonal limit means for us, let us go back to the original problem we want to discuss, the propagation of a high energy parton inside a strongly interacting medium. Consider that we have a trajectory like the one shown in Fig. 1.1, where a quark<sup>1</sup> travels inside a medium interacting with it an arbitrary number of times  $n$ . The parton initially had a given 4-momentum  $p$  and, after all of the scatterings, it has a different 4-momentum  $p'$ . We assume that these parton-medium interactions are mediated by soft gluons and, hence, the scatterings are elastic and changes in transverse momentum coming from them are of the order of the characteristic medium scale. We are going to describe the parton propagation by computing the resummed  $S$  matrix, that accounts for all possible gluon exchanges between medium and particle

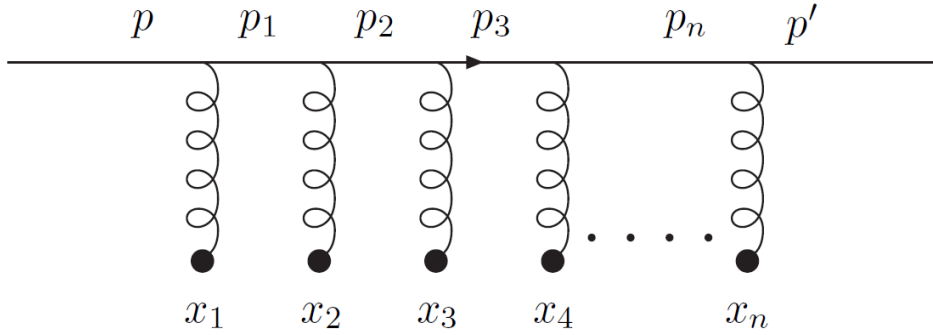


Figure 1.1: Schematic representation of eikonal propagation off a highly energetic parton travelling through a medium.  $x_1, \dots, x_n$  denote the positions of the scattering centers the parton interacts with,  $p_1, \dots, p_n$  are intermediate momenta,  $p$  is the initial momentum and  $p'$  the final one. Figure extracted from [1] under a Creative Commons license, CC-BY 4.0.

$$S = \sum_{n=0}^{\infty} S_n, \quad (1.7)$$

with  $S_n$  the  $S$  matrix that takes in account  $n$  scatterings. Given that  $n = 0$  is the vacuum scenario (no scatterings with the medium) the first non trivial contribution comes from the  $n = 1$  term. Considering only one scattering in Fig. 1.1 gives us the diagram that we need to evaluate to compute  $S_1$ . Using standard Feynman rules we can write  $S_1$  as [1, 20, 21]

$$S_1 = \int d^4x \, e^{i(p'-p) \cdot x} \bar{u}(p') i g A_\mu^a(x) t^a \gamma^\mu u(p), \quad (1.8)$$

where  $p$  and  $p'$  are, respectively, initial and final 4-momentum,  $x$  are the spacetime coordinates of the scattering center,  $u$  ( $\bar{u}$ ) is the Dirac spinor (antispinor) of the quark

<sup>1</sup>The gluon case is completely analogous, only needing to change color factors from fundamental representation to adjoint.

(antiquark),  $t^a$  is the color matrix and  $A_\mu^a$  is the classical field that encodes the medium information.

Using the eikonal limit in which all components of the momentum can be neglected except  $p^+$  we have  $p^+ \equiv \omega \simeq \omega' \equiv p'^+$ , which highly simplifies the Dirac structure of the problem  $\frac{1}{2} \sum_\lambda \bar{u}^\lambda(p') \gamma^\mu u^\lambda(p) = 2p^\mu$  where we have averaged over initial state spin configuration and summed over final state spin, as usual. Moreover, we recall that in the eikonal limit we can simplify the  $A_\mu^a(x)$  dependencies on  $x$  and write  $p^\mu A_\mu^a(x) \simeq \omega A_-^a(t, \mathbf{x})$ . With all of this we can express eq. (1.8) as

$$S_1 = 2\omega(2\pi)\delta(\omega' - \omega) \int d^2\mathbf{x} e^{-i(\mathbf{p}' - \mathbf{p}) \cdot \mathbf{x}} \int dt ig A_-^a(t, \mathbf{x}) t^a, \quad (1.9)$$

here the Dirac delta ensures energy conservation in the plus component of the momentum and we have neglected the minus components of the momentum in phases. Now let us continue with the following element of the scattering matrix  $S_2$ . Using again Feynman rules in Fig. 1.1 with 2 scatterings we have

$$S_2 = \int \frac{d^4 p_1}{(2\pi)^4} d^4 x_1 d^4 x_2 e^{i(p_1 - p) \cdot x_1} e^{i(p' - p_1) \cdot x_2} \bar{u}(p') ig A_{\mu_1}^{a_1}(x_1) t^{a_1} \gamma^{\mu_1} \frac{i \not{p}_1}{p_1^2 + i\epsilon} ig A_{\mu_2}^{a_2}(x_2) t^{a_2} \gamma^{\mu_2} u(p), \quad (1.10)$$

where we must integrate over the intermediate momentum  $p_1$ . Using the usual commutation relation of the gamma matrices and the massless Dirac equation, in the eikonal limit the Dirac structure of the problem is again very easy  $\frac{1}{2} \sum_\lambda \bar{u}^\lambda(p') \gamma^{\mu_1} \not{p}_1 \gamma^{\mu_2} u^\lambda(p) = (2p^{\mu_1})(2p^{\mu_2})$  and we only have to perform the momentum integrals. The ones over  $p_1^+ \equiv \omega_1$  and  $\mathbf{p}_1$  are trivial, as they are the definition of Dirac delta in 1 and 2 dimensions, respectively. The remaining integral over  $p_1^-$  is not immediate, but it can be computed using basic complex integration techniques

$$\int \frac{dp_1^-}{2\pi} \frac{e^{-ip_1^- (t_2 - t_1)}}{p_1^2 + i\epsilon} \simeq \int \frac{dp_1^-}{2\pi} \frac{e^{-ip_1^- (t_2 - t_1)}}{2\omega_1 p_1^- + i\epsilon} = -\frac{i}{2\omega} \theta(t_2 - t_1), \quad (1.11)$$

with  $\theta(x)$  being the step function. Finally, the integrations over the  $x^-$  components are done exactly as before in the case of  $S_1$ , and they are Dirac deltas that ensure the energy conservation. Putting all of this together and neglecting again the  $p^-$  components on phases eq. (1.10) can be written as

$$S_2 = 2\omega(2\pi)\delta(\omega' - \omega) \int d^2\mathbf{x} e^{-i\mathbf{x} \cdot (\mathbf{p}' - \mathbf{p})} \frac{1}{2!} \mathcal{P} \left[ ig \int dt A_-^a(t, \mathbf{x}) t^a \right]^2, \quad (1.12)$$

where we have used the path order operator  $\mathcal{P}$  to deal with the medium fields and the step function

$$\int dx_1 dx_2 \dots dx_n \theta(x_2 - x_1) \theta(x_3 - x_2) \dots \theta(x_n - x_{n-1}) = \frac{1}{n!} \mathcal{P} \left[ \int dx \right]^n. \quad (1.13)$$

We can compute higher order terms of the  $S$  matrix following the previous steps. The generalization to any number of scatterings is straightforward and it is obvious that, in general, the  $S$  matrix contribution corresponding to have  $n$  scatterings with the medium is given by

$$S_n = 2\omega(2\pi)\delta(\omega' - \omega) \int d^2\mathbf{x} e^{-i\mathbf{x}\cdot(\mathbf{p}'-\mathbf{p})} \frac{1}{n!} \mathcal{P} \left[ ig \int dt A_-^a(t, \mathbf{x}) t^a \right]^n, \quad (1.14)$$

hence, the fully resummed  $S$  matrix is

$$S = 2\omega(2\pi)\delta(\omega' - \omega) \int d^2\mathbf{x} e^{-i\mathbf{x}\cdot(\mathbf{p}'-\mathbf{p})} W(\mathbf{x}), \quad (1.15)$$

where we have defined the Wilson line in the fundamental representation as

$$W(\mathbf{x}) = \mathcal{P} \exp \left[ ig \int dt A_-^a(t, \mathbf{x}) t^a \right]. \quad (1.16)$$

This equation describes the propagation of a high energy quark through a medium with a given field configuration. The only effect that the medium induces in the particle is color rotation, whereas that the amount of energy that the parton loses is negligible. The path followed by the particle travelling through the medium is a straight line trajectory.

## Beyond eikonal approximation. Propagators

We have seen in the previous section that in the eikonal approximation a high energy parton propagating inside a medium follows a trajectory described by a Wilson line. However, if we want to describe processes where this highly energetic parton radiates gluons, we must relax this eikonal approximation. We can think about it in the following way: if the parent parton has huge energy then when it radiates a gluon its energy is not going to change significantly, thus, the gluon energy must be small compared with the parent particle energy and its propagation can not be described by a Wilson line. As our goal is to employ medium-induced radiation to describe energy loss, we have to go beyond the eikonal approximation and properly describe the propagation of emitted gluons through a medium. In this section we are going to briefly comment the key details about the derivation of the propagation in this situation, the final result and its consequences, as doing a formal derivation is a bit tedious and not so physically relevant for our purposes.

The main difference that we have with respect to the previous case is that now we can not neglect the subdominant element in the denominator of the integrand in eq. (1.11). Thus, we have to keep it, resulting in a phase factor after the integration is performed

$$\int \frac{dp_1^-}{2\pi} \frac{e^{-ip_1^-(t_2-t_1)}}{p_1^2 + i\epsilon} = \int \frac{dp_1^-}{2\pi} \frac{1}{2\omega_1} \frac{e^{-ip_1^-(t_2-t_1)}}{p_1^- - \frac{\mathbf{p}_1^2}{2\omega_1} + i\epsilon} = -\frac{i}{2\omega_1} \theta(t_2 - t_1) e^{-i\frac{\mathbf{p}_1^2}{2\omega_1}(t_2-t_1)}, \quad (1.17)$$

this modification implies that the integration over transverse momentum  $\mathbf{p}_1$  is also modified, now been a Gaussian instead of a delta<sup>2</sup>

$$\int_{\mathbf{p}_1} e^{-i\mathbf{p}_1 \cdot (\mathbf{x}_1 - \mathbf{x}_2)} e^{-i\frac{\mathbf{p}_1^2}{2\omega_1}(t_2 - t_1)} = \frac{\omega_1}{2\pi i(t_1 - t_2)} \exp \left\{ -i\frac{\omega_1}{2} \frac{(\mathbf{x}_1 - \mathbf{x}_2)^2}{t_1 - t_2} \right\}, \quad (1.18)$$

and this is the free Feynman propagator of a particle that goes from spacetime point  $(\mathbf{x}_1, t_1)$  to  $(\mathbf{x}_2, t_2)$  with  $\omega_1$  playing the role of mass in the usual path integral formulation of Quantum Mechanics, which we denote as  $G_0(\mathbf{x}_2, t_2; \mathbf{x}_1, t_1)$ , where the subscript 0 represents that is the vacuum propagator. As we know, Feynman propagators can be expressed using the path integral formulation, thus  $G_0$  can be written as [54]

$$G_0(\mathbf{x}_2, t_2; \mathbf{x}_1, t_1) = \int_{\mathbf{r}(t_1)=\mathbf{x}_1}^{\mathbf{r}(t_2)=\mathbf{x}_2} \mathcal{D}\mathbf{r}(t) \exp \left\{ i\frac{\omega}{2} \int_{t_1}^{t_2} dt \left[ \frac{d\mathbf{r}}{dt} \right]^2 \right\}, \quad (1.19)$$

where  $\mathbf{r}(t)$  represents all of the possible paths that connect the two endpoints of the trajectory. Therefore, now we have one of these propagators between two consecutive field insertions. In this example we have seen that in the  $S_2$  case the modification with respect to the eikonal limit is that we have eq. (1.19) connecting  $A_-(t_1, \mathbf{x}_1)$  and  $A_-(t_2, \mathbf{x}_2)$ , and this will hold for higher orders of the expansion. Consequently, when we resum all of the contributions of the scattering matrix, these propagators between intermediate steps combine, given as result the propagator that connects the endpoints of the trajectory, which essentially can be read as a modification of the Wilson line previously defined

$$S = 2\omega(2\pi)\delta(\omega' - \omega) \int d^2\mathbf{x} e^{-i\mathbf{x} \cdot (\mathbf{p}' - \mathbf{p})} \int \mathcal{D}\mathbf{r}(t) \exp \left\{ i\frac{\omega}{2} \int dt \left[ \frac{d\mathbf{r}}{dt} \right]^2 \right\} W(\mathbf{r}). \quad (1.20)$$

This equation describes the propagation of a high energy particle through a medium when it is allowed to have changes in the transverse position. Now the path followed by the parton is a straight trajectory in the longitudinal direction, described by the Wilson line, plus a Brownian, random motion in the transverse plane, described by the path integral. With this we can define the next-to-eikonal propagator of a particle with energy  $\omega$  between  $(t_1, \mathbf{x})$  and  $(t_2, \mathbf{y})$  which replaces the Wilson line when we relax the eikonal limit

$$G(\mathbf{y}, t_2; \mathbf{x}, t_1 | \omega) = \int_{\mathbf{r}(t_1)=\mathbf{x}}^{\mathbf{r}(t_2)=\mathbf{y}} \mathcal{D}\mathbf{r}(t) \exp \left\{ i\frac{\omega}{2} \int_{t_1}^{t_2} dt \left[ \frac{d\mathbf{r}}{dt} \right]^2 \right\} W(\mathbf{r}; t_2, t_1), \quad (1.21)$$

where we are now explicitly specifying the endpoints of the Wilson line. Given the path integral representation that we are using to describe this propagator, it is trivial to realize that it satisfies a simple yet useful composition property

$$G(\mathbf{y}, t_2; \mathbf{x}, t_1 | \omega) = \int d^2\mathbf{z} G(\mathbf{y}, t_2; \mathbf{z}, t | \omega) G(\mathbf{z}, t; \mathbf{x}, t_1 | \omega), \quad (1.22)$$

<sup>2</sup>Throughout this manuscript we will employ the shorthand  $\int_{\mathbf{p}} = \int d^2\mathbf{p}/(2\pi)^2$  for integrals over transverse momentum.

that in many cases allows us to split a propagator in two (or more) propagators to simplify calculations. Even though this formulation in terms of the path integral is rigorous and formally correct, it leads to difficulties when trying to evaluate formulas analytically, as path integrals are very complicated mathematical objects that can only be solved in particular cases (for instance, vacuum and harmonic oscillator) and, accordingly, we need to employ approximations to achieve results. We will see later some of the most well-known used approximations and recent frameworks that improve classical computations in this regard (for example, by solving the path integral numerically instead of analytically, as we will do in chapter 2).

Finally, we finish this section mentioning, for completeness, that the next-to-eikonal propagator  $G$  defined in eq. (1.21) is the Green's function of the Schrodinger equation

$$\left( i\partial_t + \frac{\partial_{\mathbf{y}}^2}{2\omega} + gA_-^a(t, \mathbf{y})t^a \right) G(\mathbf{y}, t; \mathbf{x}, 0) = i\delta(t)\delta^{(2)}(\mathbf{y} - \mathbf{x}), \quad (1.23)$$

where we used the notation  $\partial_t \equiv \partial/\partial t$  and  $\partial_{\mathbf{y}}^2 \equiv \partial^2/\partial^2\mathbf{y}$ . In this formulation the eikonal limit  $\omega \rightarrow \infty$  is obtained by neglecting the term that contains the information of the propagation in the transverse plane,  $\partial_{\mathbf{y}}$ . Performing this modification,  $G$  naturally becomes  $W$ .

## Medium averages

The standard procedure to compute the in-medium gluon emission spectrum consists on drawing the Feynman diagram of the process, compute its associate amplitude employing the Feynman rules and, finally, square the amplitude to obtain the spectrum. It is important to notice then that in all of the work that we did in previous sections our results are for a given field configuration of the medium  $A_-(t, \mathbf{x})$ . Therefore, when computing the squared amplitude we have to average over all possible medium configurations. These medium averages are one of the main points in the thesis, as we will discuss in detail how to improve their computation in realistic conditions.

Assuming that the medium is formed by a set of independent scattering centers described by Gaussian distribution, as commonly done, the two-point correlator function is given by (see [52] and appendix B of [55])

$$\langle A_-^a(t, \mathbf{x}) A_-^b(t', \mathbf{y}) \rangle = g^2 n(t) \delta^{a,b} \delta(t - t') \tilde{V}(\mathbf{x} - \mathbf{y}), \quad (1.24)$$

where  $n(t)$  is the linear density of the medium that has the information of how are distributed the scattering centers along the trajectory and  $\tilde{V}(\mathbf{r})$  is the elastic collision rate in configuration space which contains the microscopic details about the parton-medium interaction. Eq. (1.24) has a straightforward interpretation: the correlator between fields is local in color, time and space, having an isotropic profile in the transverse plane. It is specially relevant the fact that the interaction is instantaneous, which allows us to compute the amplitude for a given field configuration and then average over all possible medium configurations.

Regarding the potential  $\tilde{V}(\mathbf{r})$  which describes the parton-medium interactions, in general we prefer to work with it in momentum space, thus we have to employ the Fourier transform to switch the representation from configuration space to momentum

$$\tilde{V}(\mathbf{r}) = \int_{\mathbf{q}} e^{i\mathbf{q}\cdot\mathbf{r}} V(\mathbf{q}). \quad (1.25)$$

At leading order in the coupling constant this elastic collision rate behaves like  $V(\mathbf{q}) \sim 1/\mathbf{q}^4$ . This dependence on the momentum implies that for large momentum (at short distances) the collision rate must have the behavior of the Coulomb potential for any realistic interaction model. On the opposite limit, for low momentum  $\mathbf{q} \rightarrow 0$  (large distances) the divergence of the potential should be regulated by the medium, typically at its Debye mass. In this thesis we will use two different models for the collision rate. The first one, the most used by us, is the Yukawa potential, also known as Gyulassy-Wang model [56]

$$V_Y(\mathbf{q}) = \frac{8\pi\mu^2}{(\mathbf{q}^2 + \mu^2)^2}, \quad (1.26)$$

where  $\mu$  is the medium screening mass that regularizes the behavior of the potential at large distances (short momentum). It is related to the Debye mass in a thermal medium,  $\mu^2 \sim m_D^2$ . The second model we will use is the hard thermal loop (HTL) interaction [57], which provide a more precise description of thermal interactions

$$\frac{1}{2}n(t)V_H(\mathbf{q}) = \frac{g^2 N_c m_D^2 T}{\mathbf{q}^2(\mathbf{q}^2 + m_D^2)}, \quad (1.27)$$

with  $T$  the temperature of the medium.

Physical observables involve color-less objects, which are built by traces in color space. The simplest of them is the correlation of two Wilson lines

$$\frac{\text{Tr} \langle W^\dagger(\mathbf{x}) W(\mathbf{y}) \rangle}{N_c} = \frac{1}{N_c} \text{Tr} \left\langle \exp \left\{ -ig \int dt A^\dagger(t, \mathbf{x}) \right\} \exp \left\{ -ig \int dt A(t, \mathbf{y}) \right\} \right\rangle, \quad (1.28)$$

where we have omitted the color matrices for simplicity. If we had a gluon instead of a quark then the Wilson lines should be in adjoint representation and we have to change the color factor  $N_c$ , corresponding to average over initial color configurations, by  $N_c^2 - 1$ . Expanding the exponents and taking the terms which only account for one scattering center we get the following leading order contribution

$$\begin{aligned} \frac{\text{Tr} \langle W^\dagger(\mathbf{x}) W(\mathbf{y}) \rangle}{N_c} &\simeq \mathcal{P} \frac{1}{N_c} \text{Tr} \left\langle 1 + \frac{1}{2}(ig)^2 \left[ \int dt A^\dagger(t, \mathbf{x}) \right]^2 \right. \\ &\quad \left. + \frac{1}{2}(ig)^2 \left[ \int dt A(t, \mathbf{x}) \right]^2 - (ig)^2 \left[ \int dt A^\dagger(t, \mathbf{x}) \right] \left[ \int dt A(t, \mathbf{x}) \right] \right\rangle, \end{aligned} \quad (1.29)$$

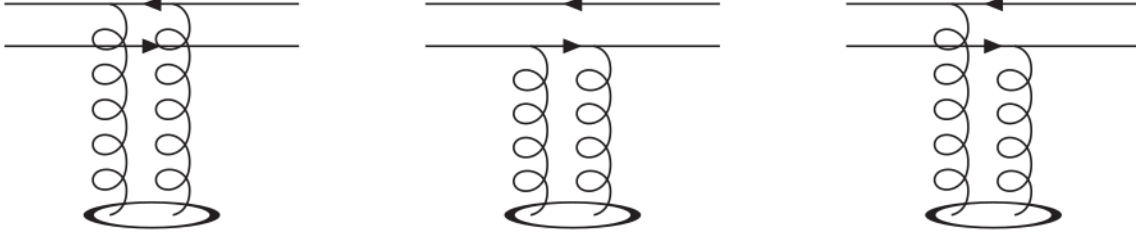


Figure 1.2: Schematic representation of the different contributions to the dipole cross section at leading contribution. Figure extracted from [1] under a Creative Commons license, CC-BY 4.0.

which is quadratic in the fields because linear terms vanished due to the color trace. These terms can be schematically interpreted as the two-scattering diagrams shown in Fig. 1.2.

The three diagrams illustrated in Fig. 1.2 represent the dipole (pair quark-antiquark, being in the amplitude and conjugated amplitude respectively) interacting twice with the medium. In the first two pictures the transverse dimension of the dipole is not resolved and they are sometimes called contact (or virtual) terms. The last diagram is able to resolve the dipole and these kind of contributions are called real terms.

Eq. (1.29) just accounts for the leading contribution in an opacity expansion of the medium to the average (1.28). Computing subleading terms it can be shown that, after some manipulations, the full expression of (1.28) is

$$\frac{\text{Tr} \langle W^\dagger(\mathbf{x}) W(\mathbf{y}) \rangle}{N_c} = \exp \left\{ -C_F \int dt \, n(t) \sigma(\mathbf{y} - \mathbf{x}) \right\}, \quad (1.30)$$

where  $C_F$  is the Casimir color factor in fundamental representation,  $4/3$  (3 for gluons in the adjoint) and we defined the dipole cross section

$$\sigma(\mathbf{r}) = \int_{\mathbf{q}} (1 - e^{-i\mathbf{q} \cdot \mathbf{r}}) V(\mathbf{q}). \quad (1.31)$$

This dipole cross section is one of the most important objects, because it allows us to go from diagram amplitudes to physical quantities like spectra. Here it is written in configuration space, but we can also use the Fourier transform to switch to momentum representation

$$\sigma(\mathbf{q}) = -V(\mathbf{q}) + (2\pi)^2 \delta^{(2)}(\mathbf{q}) \int_l V(\mathbf{l}), \quad (1.32)$$

by doing so we can see explicitly the interpretation in terms of real-virtual contributions discussed in Fig.1.2. The first term in eq. (1.32) represents real contributions to the dipole cross section, whereas that the second one accounts for virtual interactions.

If we analyze processes more complicated than the one shown in Fig. 1.1 we would have more Wilson lines or propagators (1.21). However, when computing physical objects we have to square the amplitude and, therefore, we have to evaluate quantities like eq. (1.30) or more complicated (averages of four, six... Wilson lines). As we just saw, to compute (1.30) the dipole cross section is essential, this is the reason why it is one of the most important quantities that we have. Moreover, the dipole cross section contains the details about the parton-medium interaction through the elastic collision rate (see (1.31) and (1.32)). Treating the medium averages with both multiple scatterings and perturbative tails is the main novelty in this thesis. This is accomplished by computing the radiation spectrum with a framework previously introduced in [58] which is used in the original parts of the thesis: chapters 3, 4 and 5.

## Medium-induced gluon radiation spectrum and BDMPS-Z framework

In previous sections we described how is the propagation of high energy partons travelling through a medium in the eikonal limit and next-to-eikonal case, and how to perform the medium averages. Here we are going to employ those results to compute one of the most interesting jet quenching objects for us, the gluon radiation spectrum induced by the medium. Again, we are not providing a rigorous derivation of the equation, rather we focus on some important features that we have to take in account to compute the spectrum (a more rigorous derivation can be found, for example, in [1]).

The calculation of the spectrum is done using the Baier-Dokshitzer-Mueller-Peigne-Schiff-Zakharov (BDMPS-Z) formalism [59–62]. We illustrate in Fig. 1.3 the kind of processes that we analyze. A high energy parton (a quark in this case) radiates a gluon due to interactions with the medium and then continues its propagation experiencing elastic scatterings. To be able to employ the results we derived in previous sections regarding the propagation of high energy particles through a medium we need to consider, like before, that all of the medium scatterings are mediated by soft gluons. The radiated gluon itself can also interact with the medium and has energy  $\omega$  and transverse momentum  $\mathbf{k}$ . We consider that the parent parton is eikonal, whereas that the daughter gluon is next-to-eikonal, thus it can change its transverse momentum interacting with the medium, but both of them have energies much larger than their respective transverse momentum magnitude  $\omega \gg k$ . This implies that the opening angle of the radiated gluon is small. Moreover, when computing the average of the squared amplitude we need to assume that interactions are instantaneous so we can use the medium averages we just discussed.

In this manuscript, we will primarily focus on the so-called soft emission limit. In this limit, if the parent parton possesses an initial energy  $E$  before emitting the gluon, it maintains the same energy  $E$  after the emission process occurs. This constraint leads to the radiated gluon's energy fraction, denoted as  $z$ , approaching zero asymptotically as  $z = \omega/E \rightarrow 0$ . Essentially, this implies that we are effectively treating the parent parton as having infinite energy  $E \rightarrow \infty$ , while the energy of the emitted gluon  $\omega$  remains

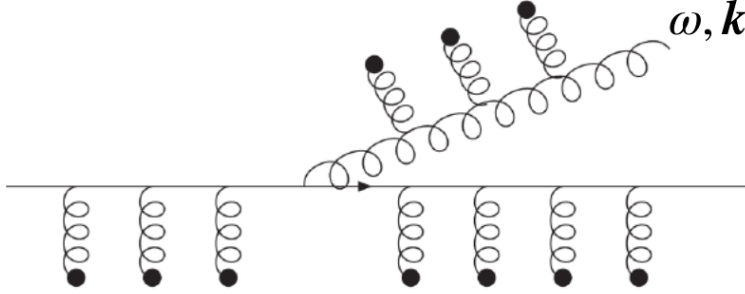


Figure 1.3: Schematic representation of a gluon emission off a high energy parton induced by the medium. The parent particle is considered to be eikonal, thus does not lose energy and only experiences color rotations due to interactions with the medium. The daughter gluon, which has energy  $\omega$  and transverse momentum  $\mathbf{k}$ , is assumed to be soft, therefore its propagation through the medium is next-to-eikonal.

finite and non-zero  $\omega = zE \neq 0$ . It is important to note that this consideration is not a universally applied assumption. Alternatively, it is possible to maintain the energy fraction  $z$  at a finite value. The key distinction between this alternative approach and our current focus lies in the necessity to incorporate the appropriate splitting function into the gluon emission vertex [63] when applying the Feynman rules.

Under these considerations we can write the amplitude of the diagram shown in Fig. 1.3 employing standard Feynman rules and the fact that we already know that the contribution of the quark line is a Wilson line (1.16) and the next-to-eikonal gluon is described by a propagator (1.21). With this the amplitude can be written as<sup>3</sup> (see [55, 64, 65] for effective Feynman rules inside a medium)

$$\mathcal{M} = -\frac{g}{\omega} \int d^2\mathbf{y} e^{-i\mathbf{k}\cdot\mathbf{y}} \int_0^\infty dt i\partial_x G^{ba}(\mathbf{y}, L; \mathbf{x}, t|\omega)|_{\mathbf{x}=0} W(\mathbf{0}; 0, t) t^a W(\mathbf{0}; t, L), \quad (1.33)$$

where  $L$  is the length of the medium under consideration, the gluon is emitted at time  $t$  with transverse position  $\mathbf{x}$  and its final position when going out of the medium is  $\mathbf{y}$ . As we are working in the soft emission limit  $z \rightarrow 0$ , we do not have to include the splitting function in the contribution of the vertex. We have written explicitly the color indices of the gluon propagator in adjoint representation and we considered, for simplicity, that the parent quark has its transverse position fixed at  $\mathbf{x} = \mathbf{0}$ . Also, we assumed that quark and medium are both created at the same time  $t = 0$ .

This last assumption is commonly employed in theory calculations and even in phenomenological studies which describe experimental data. However, this consideration is

<sup>3</sup>We are going to systematically ignore the amplitude of the hard process that creates the original high energy parton, as this contribution cancels out exactly when computing the radiation spectrum.

not realistic, because the initial high energy parton is created at the collision time, but the QGP needs some time to be formed. Roughly speaking, if we consider that the collision takes place at time zero, then the medium initialization time is between  $\tau \sim 0.2 - 1$  fm. Not considering the parton propagation before entering the medium neglects the effects of initial stages. As studying the early stages of the collision is a very important goal of this thesis, we will relax this approximation and properly describe the emitter propagation before entering the medium. We do so in the final chapter of the thesis 5.

As previously discussed, to compute the radiation spectrum induced by the medium we have to square eq. (1.33) and average over medium configurations. Under the considerations that we are taking in account where the parent parton is eikonal but the daughter particle is next-to-eikonal, we can distinguish three different regions when we compute the averages of Wilson lines and propagators. We represent in Fig. 1.4 the gluon emission in the amplitude (which takes place at time  $t$ ) along with the same process in the conjugated amplitude (where the gluon is emitted at  $t'$ ), so the different regions can be clearly seen.

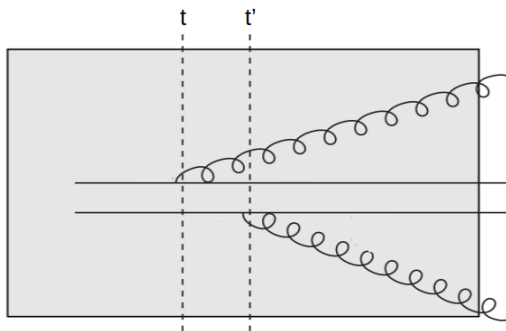


Figure 1.4: Schematic representation of the soft gluon emission in the amplitude (upper part of the picture) and in the complex conjugated amplitude (bottom part of the picture).

First, we have the case when there is no gluon emission in the amplitude or in the complex conjugated amplitude, corresponding to times smaller than  $t$ . As the quark is completely eikonal, its transverse position does not change due to interactions with the medium and the Wilson line in the conjugated amplitude is the hermitian of the one in the amplitude. Therefore, their trace is trivially the identity and this region does not contribute.

The first non-trivial term comes from the second region, comprehended between  $t$  and  $t'$ . It corresponds to the case in which the gluon emission takes place in the amplitude, but not yet in the complex conjugated amplitude. Now we have three different contributions, first the Wilson line of the quark in the amplitude, then the propagator of the radiated gluon also in the amplitude, and finally the Wilson line of the quark in the conjugated amplitude. The medium average of these three elements can be computed and after some

algebra it can be written as

$$\mathcal{K}(t', \mathbf{z}; t, \mathbf{x}) = \int_{\mathbf{r}(t)=\mathbf{x}}^{\mathbf{r}(t')=\mathbf{z}} \mathcal{D}\mathbf{r} \exp \left[ \int_t^{t'} d\xi \left( i \frac{\omega}{2} \dot{\mathbf{r}}^2 - \frac{1}{2} n(\xi) \sigma(\mathbf{r}) \right) \right], \quad (1.34)$$

this object is what we call emission kernel, because it contains all of the information about the details of how is produced the gluon emission. We have written it explicitly in coordinate space, but we will frequently use it also in momentum space

$$\mathcal{K}(t', \mathbf{z}; t, \mathbf{x}) = \int_{\mathbf{p}\mathbf{q}} e^{i(\mathbf{q}\cdot\mathbf{z}-\mathbf{p}\cdot\mathbf{x})} \tilde{\mathcal{K}}(t', \mathbf{q}; t, \mathbf{p}). \quad (1.35)$$

Finally, the last region corresponds to the case when the gluon has already been emitted both in amplitude and complex conjugated amplitude. Accordingly, we have the contribution of four elements, two quark Wilson lines and two gluon propagators. However, as quark and antiquark have already emitted the gluon and they are both eikonal, their Wilson lines are related by hermitian conjugation (notice that this is true due to the soft limit  $z \rightarrow 0$ . If this is not assumed, then the situation is much more complicated). Consequently, their contributions cancel out, as in the first region, and we only need to deal with the propagators. In this case it is also possible to work analytically with these elements to obtain an expression for the average over medium configurations. The final result for the contribution of the last region is

$$\mathcal{P}(t'', \mathbf{k}; t', \mathbf{q}) = \int d^2\mathbf{x} e^{-i(\mathbf{k}-\mathbf{q})\cdot\mathbf{x}} \exp \left\{ -\frac{1}{2} \int_{t'}^{t''} d\xi n(\xi) \sigma(\mathbf{x}) \right\}, \quad (1.36)$$

and this is what we call momentum broadening, as it describes how the transverse momentum of the radiated gluon changes since its emission until the final value  $\mathbf{k}$ . Taking all of these considerations in account it is possible to show that amplitude (1.33) leads to the following spectrum

$$\omega \frac{dI}{d\omega d^2\mathbf{k}} = \frac{2\alpha_s C_R}{(2\pi)^2 \omega^2} \text{Re} \int_0^\infty dt' \int_0^{t'} dt \int d^2\mathbf{y} e^{-i\mathbf{k}\cdot\mathbf{y}} \exp \left\{ -\frac{1}{2} \int_{t'}^\infty d\xi n(\xi) \sigma(\mathbf{y}) \right\} \times \quad (1.37)$$

$$\times \partial_{\mathbf{x}} \cdot \partial_{\mathbf{y}} \mathcal{K}(t', \mathbf{y}; t, \mathbf{x})|_{\mathbf{x}=\mathbf{0}},$$

which is the so-called BDMPS-Z result [59–62]. Here, to make it general, we introduced the color factor  $C_R$  that varies depending on the representation of the parent parton. If it is a quark, as we were considering, then  $C_R = C_F = 4/3$ , on the other hand, if it is a gluon, then  $C_R = C_A = 3$ . In eq. (1.37) we have the medium-induced radiation, however, we are also accounting for vacuum emissions that take place once the leading parton exists the medium. The standard procedure for subtracting this vacuum radiation is to replace the medium kernel with the difference between the medium kernel and the free one  $\mathcal{K} \rightarrow \mathcal{K} - \mathcal{K}_0$ , which is determined by imposing  $n(\xi) = 0$  for all  $\xi$  in eq. (1.34).

We can also write the spectrum in transverse momentum space, as we will work in this representation many times during the thesis

$$\omega \frac{dI}{d\omega d^2\mathbf{k}} = \frac{2\alpha_s C_R}{(2\pi)^2 \omega^2} \text{Re} \int_0^\infty dt' \int_0^{t'} dt \int_{pq} \mathbf{p} \cdot \mathbf{q} \tilde{\mathcal{K}}(t', \mathbf{q}; t, \mathbf{p}) \mathcal{P}(\infty, \mathbf{k}; t', \mathbf{q}). \quad (1.38)$$

Eqs. (1.37) and (1.38) describe the emission process of soft gluons radiated off a high energy particle that is travelling through a strongly interacting medium. This expression is only valid under some assumptions which we already mentioned through the derivation, but we summarize now the most important ones.

First, partons are allowed to change their transverse momentum due to interactions with the medium, but their energies are much bigger than the magnitude of their respective momentum  $\omega \gg |\mathbf{k}|$ , which implies that the opening angle of the radiation is small. Second, parton-medium interactions are mediated by soft gluons. The scatterings are elastic and changes in transverse momentum coming from them are of the order of the characteristic medium scale. Third, interactions are considered to be instantaneous. The formation time of the radiated gluons, which can be identified as the time difference between the emission in amplitude and conjugated amplitude ( $t' - t$  in Fig. 1.4), and that is usually estimated as

$$t_f \simeq \frac{2\omega}{\mathbf{k}^2}, \quad (1.39)$$

is way bigger than the time scale of any single interaction. The same thing applies to any time scales for the evolution, which are also bigger than time scales of single interactions. These considerations enable us to compute interactions for a particular medium configuration and then average over all allowed configurations. Finally, radiated gluons are assumed to be very soft  $\omega/E = z \ll 1$ , but this is not a general assumption and can be relaxed<sup>4</sup>.

## Approximations of the BDMPS-Z spectrum

The BDMPS-Z spectrum we derived in the previous section eq. (1.38) is the master formula we employ to compute the gluon radiation induced by the medium. The evaluation of in-medium emissions is the building block for calculating the energy loss in many phenomenological studies, thus computing the spectrum accurately is a decisive task. However, eq. (1.38) is an expression difficult to evaluate without further simplifications, mainly due to the presence of a path integral in the emission kernel (1.34) which is difficult to solve in a general case. Therefore, it is widely extended the use of several frameworks which rely on approximations. We mention some of them in this final section of the introduction.

Starting with the classical approaches, the multiple soft scatterings framework, also known as Harmonic Oscillator (HO), BDMPS-Z/Armesto-Salgado-Wiedemann (BDMPS-Z/ASW) or simply ASW [69–71], is one of the most used formalisms in phenomenological

<sup>4</sup>Details about the case when the daughter particle has a finite energy fraction can be found in [66–68].

works [72–74]. Its main assumption is to approximate the dipole cross section by its small distances behaviour. Consequently, it can describe soft scatterings, but not hard ones. A more detailed definition of this approach will be done chapter 3.

On the opposite limit, we have the few hard scatterings approximation, sometimes called opacity expansion. This framework performs an expansion of the emission kernel and momentum broadening using the opacity of the medium as expanding parameter. Even though it can be done systematically to any order and achieve a complete description of the emission process [75], in practice we usually employ only the leading order term, also known as first opacity or Gyulassy-Levai-Vitev (GLV) approximation [76, 77], which takes in account just one single hard scattering. We will properly present this formalism in chapter 2.

For completeness, we briefly comment two more widely known classical approximations that we are not going to employ in this thesis. The first one is the so-called Arnold-Moore-Yaffe (AMY) framework [78, 79], which considers the propagation of a parton in an infinitely large, weakly-coupled medium in thermal equilibrium that is described by the HTL collision rate. Therefore, all medium properties are determined specifying only the temperature and the baryochemical potential. This perturbative description of the thermal medium can only be applied at very high temperature  $T \gg T_c$ , being  $T_c$  the characteristic transition temperature. We highlight that this is the only formalism which does not incorporate vacuum radiation, as the medium is infinite and therefore the particle never propagates in vacuum. In fact, the AMY result can be obtained by imposing  $L \rightarrow \infty$  in the BDMPS-Z spectrum.

The final classical framework that we mention is the Higher Twist (HT) [80, 81]. In this formalism the medium is described by matrix elements of gauge field operators. Interactions between medium and parton are formulated by characterizing the medium in terms of expectation values of multiple field correlation functions. With this description, the medium properties are determined in terms of higher-twist matrix elements. This framework is valid at leading order in path length  $L$  dependence. Comparisons between these four different classical formalisms can be found, for instance, in Refs. [16, 82, 83].

In the last 2-3 years the state of the art of the computation of medium-induced radiation has deeply changed. There has been renewed efforts trying to achieve a deep understanding of single gluon emissions. Some of the most important improvements made include: describing the effects of having an inhomogeneous medium in the transverse plane, thus relaxing the high energy eikonal approximation [84–87]; resum multiple scatterings beyond the multiple soft approximation [2, 4, 58, 88–92]; incorporating a proper matching to the infrared sector [93–95]; or not considering the radiated gluon to be soft [66, 68, 96]. We shortly comment here two of the more recently developed formalisms that improve the performance of the classical ones we just described: on the numerical side we have [90, 97], where the authors compute the spectrum exactly without approximations employing a Monte Carlo implementation; and on the analytical side we have the so-called Improved Opacity Expansion (IOE) [91, 92], which performs an expansion of the emission kernel around the HO solution. By doing so this formalism accounts for multiple

soft and single hard scatterings at the same time. This last framework is introduced in detail in chapter 3.

In this thesis we employ a new way to compute the BDMPS-Z spectrum numerically exactly, without employing any of the analytical approximations explained in this section. This new method, which was originally introduced in [58], is properly presented in chapter 2. Once that we describe how to compute the in-medium radiation, we will employ this approach for several original studies. First, in chapter 3 we compare the full result with some analytical approaches, checking in that way the validity of the approximations they rely on, which allow us to gain knowledge about the physics behind the emission process. Then, in chapter 4 we extend the computation of the spectrum originally done for the static case in [2, 58] to realistically evolving media, which is crucial to allow the implementation of the framework in phenomenology. Finally, in chapter 5 we use the radiation spectrum to qualitatively analyze the effects that initial stages may have on jet quenching observables.

MARCOS GONZÁLEZ MARTÍNEZ

# Medium-induced gluon radiation with full resummation of multiple scatterings

Based on the discussion in chapter 1, the evaluation of the general BDMPS-Z spectrum (1.38) is a difficult task, so approximations are usually employed to simplify the procedure. In this chapter we discuss in detail a formalism recently developed that allows to compute the spectrum including a realistic potential with resummation over multiple scatterings. The main idea is to find a numerical solution for the differential equations satisfied by the emission kernel (1.34) and the momentum broadening (1.36) instead of directly solving them, which is a very difficult task in the general case.

This framework was originally introduced in Ref. [58]. But, as the formalism plays a crucial role in the rest of the manuscript, which represents the novel work done by the PhD student, we use this chapter to explain the approach, summarizing the most important details. Therefore, Ref. [58] is the guide we use in this chapter and we refer the interested reader there for further details about the method, numerical results of the evaluation of the spectrum, discussions and comparisons with another approaches.

When computing the gluon energy distribution without kinematical constraint, which is the integral of the  $\mathbf{k}$ -differential spectrum over all  $\mathbf{k}$  space, this formalism is analogous to the ones presented in [88,89]. The main difference is that here we are able to compute the radiation spectrum differential in  $\mathbf{k}$ , whereas in [88,89] only the energy distribution is considered. Also, this approach employs the soft limit as an assumption (to simplify the evaluation of the BDMPS-Z spectrum in the last region of Fig. 1.4), but in [88,89]

the authors computed the energy spectrum beyond this limit (which is easy to do if one only cares about the energy distribution). Recently, in [96] it was computed the full  $\mathbf{k}$ -dependent radiation spectrum beyond the soft limit, however, for the moment the implementation of the method was only achieved for the  $\gamma \rightarrow q\bar{q}$  splitting with few numerical results.

## 2.1 Difficulties of evaluating the BDMPS-Z expression

Let us start by explaining in detail which are the main complications we encounter when evaluating the BDMPS-Z medium-induced radiation spectrum (1.38). As we mentioned before, the emission kernel is given by a path integral (eqs. (1.34) and (1.35)) with imaginary potential, which is very difficult to compute in a general case. This is the main reason why, in practice, analytical approaches like the ones we mentioned in the previous section are widely used when calculating the spectrum. Moreover, achieving the solution of the emission kernel for a general medium profile  $n(t)$  is quite challenging. Therefore, it is common to employ the so-called “brick” approximation, which relies on considering that the medium is described by a collection of static scatterings centers  $n(t) = n_0$  during the whole propagation of the parton. This is an strong assumption, because nowadays we know that the QGP is an evolving media which is well-described by hydrodynamic simulations. However, this consideration simplifies the calculations a lot (sometimes it is almost impossible to achieve results without this approximation) and we can use it as a first step to gain some intuition about the most important physics. Consequently, this assumption is widely used, even though we know it is not correct. As the title of this thesis states, here we aim to describe the temporal structure of the QCD collectivity, therefore, we must surpass the brick approximation and be able to evaluate the spectrum for a generic medium profile  $n(t)$ . The framework we present here allow us to go beyond the brick and evaluate the spectrum in realistic conditions.

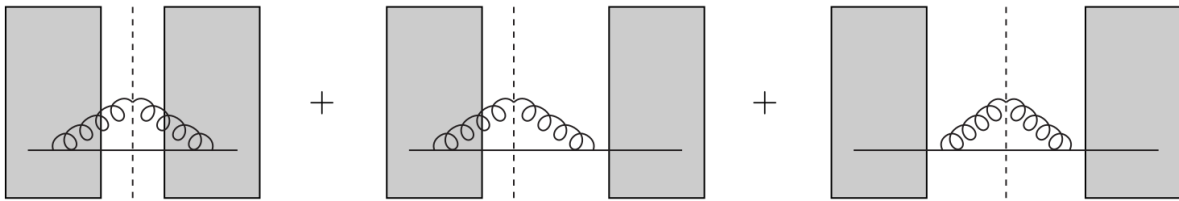


Figure 2.1: Schematic representation of the three different contributions to the medium-induced gluon radiation spectrum. On the left side of each picture it is represented the emission in the amplitude and on the right side in the complex conjugated amplitude.

The other problem we encounter when evaluating the BDMPS-Z spectrum (1.37) is the fact that the upper limit of time integrals is infinite, consequently, the emission of

the gluon can take place anywhere. From a formal perspective this is not a problem at all, everything is well formulated and the resulting spectrum is finite. However, when it comes to numerical computations, things become more intricate. The common approach consists on dividing the whole spectrum in three different contributions, as illustrated in Fig. 2.1. In each picture we are representing the diagram of the amplitude (left) and the corresponding diagram of the conjugated amplitude (right). These three contributions represent the following scenarios: first, the emission of the gluon occurring in the medium for both the amplitude and conjugated amplitude (left picture); second, emission inside the medium for the amplitude and outside the medium for the conjugated amplitude (central picture); and finally, emissions occurring outside the medium for both the amplitude and conjugated amplitude (right picture). Therefore, they are usually called in-in, in-out and out-out contributions, respectively. The out-out term, which solely accounts for vacuum radiation, is typically subtracted when analyzing emissions within the medium. This method is useful because it allows for some analytical simplifications in the in-out contribution employing the fact that the final part of the propagation of the gluon in the conjugated amplitude is through vacuum. However, even though these manipulations do not modify the spectrum, which remains finite, when we compute separately the in-in and in-out terms they are both divergent. Thus, what happens is that there are precise cancellations between these two terms, resulting in a finite, non-divergent spectrum. But these cancellations are very difficult to achieve numerically and, hence, the computation time of the spectrum is very large.

The new way to compute the spectrum originally proposed in [58] which we use in this thesis overcomes the two limitations: first, it does not need the employment of any further approximations to simplify the path integral (we can even completely ignore the brick assumption, which is done in chapter 4); and, second, the in-in and in-out contributions are merged together in a single term which has a finite integration domain over time.

## 2.2 Rearranging the spectrum

Following the original work done in [58], we consider the emission kernel  $\tilde{\mathcal{K}}$  and momentum broadening  $\mathcal{P}$  as propagators which satisfy some particular differential equation that can be solved numerically. By employing these differential equations, we will also be able to perform the time integral over  $t'$  in eq. (1.38) analytically, which facilitates the numerical implementation of the method. For convenience, from now on we will work in momentum space.

The first step is to write the Schwinger-Dyson type differential equations that describe the evolution of the emission kernel  $\tilde{\mathcal{K}}$  (1.35) and the momentum broadening  $\mathcal{P}$  (1.36)

$$\mathcal{P}(t'', \mathbf{k}; t', \mathbf{q}) = (2\pi)^2 \delta^{(2)}(\mathbf{k} - \mathbf{q}) - \frac{1}{2} \int_{t'}^{t''} ds n(s) \int_{\mathbf{k}'} \sigma(\mathbf{k}' - \mathbf{q}) \mathcal{P}(t'', \mathbf{k}; s, \mathbf{k}'), \quad (2.1)$$

$$\begin{aligned} \tilde{\mathcal{K}}(t', \mathbf{q}; t, \mathbf{p}) &= (2\pi)^2 \delta^{(2)}(\mathbf{q} - \mathbf{p}) e^{-i\frac{p^2}{2\omega}(t'-t)} - \frac{1}{2} \int_t^{t'} ds n(s) \\ &\quad \times \int_{\mathbf{k}'} \sigma(\mathbf{q} - \mathbf{k}') e^{-i\frac{q^2}{2\omega}(t'-s)} \tilde{\mathcal{K}}(s, \mathbf{k}'; t, \mathbf{p}). \end{aligned} \quad (2.2)$$

It can be easily checked that both kernel and broadening satisfy these equations just by plugging (1.35) and (1.36) into (2.2) and (2.1), respectively. Given that in these equations the time dependence on  $t'$  is explicit, we can employ them to analytically perform the integration over  $t'$  in eq. (1.38). Replacing eqs. (2.1) and (2.2) into eq. (1.38) we get

$$\begin{aligned} \omega \frac{dI}{d\omega d^2\mathbf{k}} &= \frac{2\alpha_s C_R}{(2\pi)^2 \omega^2} \text{Re} \int_0^\infty dt \int_t^\infty dt' \left[ \mathbf{k}^2 e^{-i\frac{k^2}{2\omega}(t'-t)} \right. \\ &\quad - \frac{1}{2} \int_t^{t'} ds n(s) \int_{\mathbf{p}\mathbf{k}_1} \mathbf{p} \cdot \mathbf{k} e^{-i\frac{k^2}{2\omega}(t'-s)} \sigma(\mathbf{k} - \mathbf{k}_1) \tilde{\mathcal{K}}(s, \mathbf{k}_1; t, \mathbf{p}) \\ &\quad - \frac{1}{2} \int_{t'}^\infty ds n(s) \int_{\mathbf{p}\mathbf{k}_1} \mathbf{p}^2 e^{-i\frac{p^2}{2\omega}(t'-t)} \sigma(\mathbf{k}_1 - \mathbf{p}) \mathcal{P}(\infty, \mathbf{k}; s, \mathbf{k}_1) \\ &\quad + \frac{1}{4} \int_t^{t'} ds_1 n(s_1) \int_{t'}^\infty ds_2 n(s_2) \int_{\mathbf{p}\mathbf{q}\mathbf{k}_1\mathbf{k}_2} \mathbf{p} \cdot \mathbf{q} e^{-i\frac{q^2}{2\omega}(t'-s_1)} \\ &\quad \left. \times \sigma(\mathbf{q} - \mathbf{k}_1) \sigma(\mathbf{k}_2 - \mathbf{q}) \tilde{\mathcal{K}}(s_1, \mathbf{k}_1; t, \mathbf{p}) \mathcal{P}(\infty, \mathbf{k}; s_2, \mathbf{k}_2) \right], \end{aligned} \quad (2.3)$$

the first term accounts for the vacuum contribution (is what we called out-out term before), thus we neglect it from now on, as we are only interested in analyzing medium-induced radiation. Changing the order of integration and performing the integral over  $t'$  for the rest of the terms we get

$$\begin{aligned} \omega \frac{dI}{d\omega d^2\mathbf{k}} &= \frac{2\alpha_s C_R}{(2\pi)^2 \omega} \text{Re} \int_0^\infty dt \left[ \int_t^\infty ds n(s) \int_{\mathbf{p}\mathbf{k}_1} i \frac{\mathbf{p} \cdot \mathbf{k}}{\mathbf{k}^2} \sigma(\mathbf{k} - \mathbf{k}_1) \tilde{\mathcal{K}}(s, \mathbf{k}_1; t, \mathbf{p}) \right. \\ &\quad - \int_t^\infty ds n(s) \int_{\mathbf{p}\mathbf{k}_1} i e^{-i\frac{p^2}{2\omega}(s-t)} \sigma(\mathbf{k}_1 - \mathbf{p}) \mathcal{P}(\infty, \mathbf{k}; s, \mathbf{k}_1) \\ &\quad + \frac{1}{2} \int_t^\infty ds_1 n(s_1) \int_{s_1}^\infty ds_2 n(s_2) \int_{\mathbf{p}\mathbf{q}\mathbf{k}_1\mathbf{k}_2} i \frac{\mathbf{p} \cdot \mathbf{q}}{\mathbf{q}^2} \left( e^{-i\frac{q^2}{2\omega}(s_2-s_1)} - 1 \right) \\ &\quad \left. \times \sigma(\mathbf{q} - \mathbf{k}_1) \sigma(\mathbf{k}_2 - \mathbf{q}) \tilde{\mathcal{K}}(s_1, \mathbf{k}_1; t, \mathbf{p}) \mathcal{P}(\infty, \mathbf{k}; s_2, \mathbf{k}_2) \right]. \end{aligned} \quad (2.4)$$

Here we can combine the first term and the contribution without phase of the last term employing eq. (2.1); and the part which has phase with the second term using eq. (2.2) reaching the following result for the rearranged spectrum

$$\begin{aligned} \omega \frac{dI}{d\omega d^2\mathbf{k}} &= \frac{2\alpha_s C_R}{(2\pi)^2 \omega} \text{Re} \int_0^\infty dt \int_t^\infty ds n(s) \int_{\mathbf{p}\mathbf{q}\mathbf{l}} i \mathbf{p} \cdot \left( \frac{\mathbf{l}}{l^2} - \frac{\mathbf{q}}{q^2} \right) \\ &\quad \times \sigma(\mathbf{l} - \mathbf{q}) \tilde{\mathcal{K}}(s, \mathbf{q}; t, \mathbf{p}) \mathcal{P}(\infty, \mathbf{k}; s, \mathbf{l}). \end{aligned} \quad (2.5)$$

## 2 Medium-induced gluon radiation with full resummation of multiple scatterings

Now we change the integration order in  $t$  and  $s$  and, since  $n(s)$  is non-zero only when we are inside the medium  $0 < s < L$ , the whole contribution of the spectrum vanishes for any other  $s$  value. The same thing happens in the momentum broadening (see eq. (1.36)), thus we can take  $L$  as its endpoint. The final result is then

$$\omega \frac{dI}{d\omega d^2\mathbf{k}} = \frac{2\alpha_s C_R}{(2\pi)^2 \omega} \text{Re} \int_0^L ds n(s) \int_0^s dt \int_{\mathbf{p}q\mathbf{l}} i\mathbf{p} \cdot \left( \frac{\mathbf{l}}{l^2} - \frac{\mathbf{q}}{q^2} \right) \times \sigma(\mathbf{l} - \mathbf{q}) \tilde{\mathcal{K}}(s, \mathbf{q}; t, \mathbf{p}) \mathcal{P}(L, \mathbf{k}; s, \mathbf{l}). \quad (2.6)$$

With these changes we have already solved one of the problems we had when evaluating the spectrum: now the integration domain of both  $s$  and  $t$  is finite, and as we have no separation between in-in and in-out terms we do not have to worry about keeping the interference between both contributions with huge numerical precision, which is highly costly in terms of computational power and slows down the evaluation of the spectrum.

Moreover, this new expression of the BDMPS-Z medium-induced radiation spectrum has another advantage regarding the numerical evaluation: there is one explicit  $\sigma$  factor in eq. (2.6). We mentioned in section 1.3 that, for any realistic parton-medium interaction model, the collision rate behaves like the Coulomb one at large momentum  $\mathbf{p}$ ,  $V(\mathbf{p}) \sim 1/\mathbf{p}^4$ . When we translate this in terms of the dipole cross section, the behavior is exactly the same,  $\sigma(\mathbf{p}) \sim 1/\mathbf{p}^4$  at large  $\mathbf{p}$ . This fact ensures that both integrals over  $\mathbf{q}$  and  $\mathbf{l}$  are convergent and we have no vacuum contributions. Also, when we take  $\mathcal{P}$  and  $\tilde{\mathcal{K}}$  as propagators, this feature provides an initial condition which can be evolved.

### 2.3 Employing differential equations to compute the in-medium radiation spectrum

Now that we have reorganized the spectrum in a nicer way (2.6), the next step done in [58] is to explain how to compute it exactly by solving the differential equations that satisfy  $\tilde{\mathcal{K}}$  and  $\mathcal{P}$ . We start by realizing that the momentum broadening  $\mathcal{P}$  satisfies the following differential equation (which can be easily derived by differentiating (2.1))

$$\partial_\tau \mathcal{P}(\tau, \mathbf{k}; s, \mathbf{l}) = -\frac{1}{2} n(\tau) \int_{\mathbf{k}'} \sigma(\mathbf{k} - \mathbf{k}') \mathcal{P}(\tau, \mathbf{k}'; s, \mathbf{l}), \quad (2.7)$$

with initial condition

$$\mathcal{P}(s, \mathbf{k}; s, \mathbf{l}) = (2\pi)^2 \delta^{(2)}(\mathbf{k} - \mathbf{l}). \quad (2.8)$$

For convenience, instead of solving this differential equation we define

$$\phi(\tau, \mathbf{k}; s, \mathbf{q}) = n(s) \int_{\mathbf{l}} \left( \frac{\mathbf{l}}{l^2} - \frac{\mathbf{q}}{q^2} \right) \sigma(\mathbf{l} - \mathbf{q}) \mathcal{P}(\tau, \mathbf{k}; s, \mathbf{l}), \quad (2.9)$$

thus, employing (2.7) and (2.8), it is obvious that  $\phi(\tau, \mathbf{k}; s, \mathbf{q})$  satisfies the same differential equation as  $\mathcal{P}$

$$\partial_\tau \phi(\tau, \mathbf{k}; s, \mathbf{q}) = -\frac{1}{2}n(\tau) \int_{\mathbf{k}'} \sigma(\mathbf{k} - \mathbf{k}') \phi(\tau, \mathbf{k}'; s, \mathbf{q}), \quad (2.10)$$

having as initial condition

$$\phi(s, \mathbf{k}; s, \mathbf{q}) = n(s) \left( \frac{\mathbf{k}}{\mathbf{k}^2} - \frac{\mathbf{q}}{\mathbf{q}^2} \right) \sigma(\mathbf{k} - \mathbf{q}). \quad (2.11)$$

Therefore, once that we solve this differential equation, we have a solution for the broadening factor  $\mathcal{P}$ . Moving on to the emission kernel, we realize that it satisfies the following differential equation (again, it is easy to check that the derivative of (2.2) gives this result)

$$\partial_t \tilde{\mathcal{K}}(s, \mathbf{q}; t, \mathbf{p}) = \frac{i\mathbf{p}^2}{2\omega} \tilde{\mathcal{K}}(s, \mathbf{q}; t, \mathbf{p}) + \frac{1}{2}n(t) \int_{\mathbf{k}'} \sigma(\mathbf{k}' - \mathbf{p}) \tilde{\mathcal{K}}(s, \mathbf{q}; t, \mathbf{k}'), \quad (2.12)$$

which has as initial condition

$$\tilde{\mathcal{K}}(s, \mathbf{q}; s, \mathbf{p}) = (2\pi)^2 \delta^{(2)}(\mathbf{q} - \mathbf{p}). \quad (2.13)$$

We define now

$$\psi(s, \mathbf{k}; t, \mathbf{p}) = \int_{\mathbf{q}} \phi(L, \mathbf{k}; s, \mathbf{q}) \tilde{\mathcal{K}}(s, \mathbf{q}; t, \mathbf{p}), \quad (2.14)$$

hence, employing (2.12), it is easy to see that this operator satisfies the following equation

$$\partial_t \psi(s, \mathbf{k}; t, \mathbf{p}) = \frac{i\mathbf{p}^2}{2\omega} \psi(s, \mathbf{k}; t, \mathbf{p}) + \frac{1}{2}n(t) \int_{\mathbf{k}'} \sigma(\mathbf{k}' - \mathbf{p}) \psi(s, \mathbf{k}; t, \mathbf{k}'), \quad (2.15)$$

with initial condition

$$\psi(s, \mathbf{k}; s, \mathbf{p}) = \phi(L, \mathbf{k}; s, \mathbf{p}), \quad (2.16)$$

this equation may have some problems when solving it numerically, due to the first term in the r.h.s. of eq. (2.15), which causes oscillations. Switching to interaction picture we can alleviate the computational effort needed to achieve the solution

$$\psi_I(s, \mathbf{k}; t, \mathbf{p}) = e^{\frac{i\mathbf{p}^2}{2\omega}(s-t)} \psi(s, \mathbf{k}; t, \mathbf{p}). \quad (2.17)$$

Consequently, the differential equation we have to solve is

$$\partial_t \psi_I(s, \mathbf{k}; t, \mathbf{p}) = e^{\frac{i\mathbf{p}^2}{2\omega}(s-t)} \frac{1}{2}n(t) \int_{\mathbf{k}'} \sigma(\mathbf{k}' - \mathbf{p}) e^{-\frac{i\mathbf{k}'^2}{2\omega}(s-t)} \psi_I(s, \mathbf{k}; t, \mathbf{k}'), \quad (2.18)$$

having as initial condition

$$\psi_I(s, \mathbf{k}; s, \mathbf{p}) = \phi(L, \mathbf{k}; s, \mathbf{p}). \quad (2.19)$$

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Taking all of these considerations in account, we can rewrite the full spectrum (2.6) in the following way

$$\omega \frac{dI}{d\omega d^2\mathbf{k}} = \frac{2\alpha_s C_R}{(2\pi)^2 \omega} \text{Re} \int_0^L ds \int_0^s dt \int_{\mathbf{p}} ie^{-\frac{i\mathbf{p}^2}{2\omega}(s-t)} \mathbf{p} \cdot \boldsymbol{\psi}_I(s, \mathbf{k}; t, \mathbf{p}). \quad (2.20)$$

Formally the problem is already well formulated: to evaluate the spectrum we start by computing  $\boldsymbol{\phi}$  solving its differential equation, which then is employed as initial condition for the resolution of the differential equation that allows us to get  $\boldsymbol{\psi}_I$ . Finally, once that we have computed  $\boldsymbol{\psi}_I$ , we plug the result in eq. (2.20) and perform the remaining integrations. However, some of these integrals can be evaluated analytically, thus we are going to continue manipulating the spectrum to further simplify the problem before computing it numerically.

In practice, for the majority of cases of interest we encounter the direction of momentum  $\mathbf{k}$  is not relevant, therefore, we can write everything as function of its magnitude  $k$  and integrate over its direction. This allows us to employ the rotational symmetry to perform the angular integrals analytically. Moreover, it is convenient to rescale all of the variables to make them dimensionless, as it is more suitable for numerical evaluations. We rescale time variables with the length of the medium  $L$  in the following way  $s \rightarrow Ls$  and dummy momentum variables as  $\mathbf{p} \rightarrow \sqrt{2\omega/L} \mathbf{p}$ . The transverse momentum of the emitted gluon is rescaled through the typical transverse momentum transfer  $\mu$ , which is usually set to the Debye mass of the medium, and the relevant scale to rescale the gluon energy is the characteristic gluon frequency  $\bar{\omega}_c = \mu^2 L/2$ . The procedure to perform the angular integral and rescale the variables of the spectrum does not provide any relevant information to our discussion and it was already done in [58], thus, to facilitate the reading we skip the intermediate steps and write directly the final result. We refer the interested reader to Ref. [58] to learn about the details.

The expression of the full medium-induced gluon radiation spectrum we use to compute the numerical results is

$$x \frac{dI}{dx d\kappa^2} = \frac{\alpha_s C_R}{\pi^2} \text{Re} \int_0^1 ds \int_0^s dt \int_0^\infty dp ip e^{-ip^2(s-t)} f_x(s, \kappa; t, p), \quad (2.21)$$

where we defined

$$\boldsymbol{\psi}_I(s, \mathbf{k}; t, \mathbf{p}) = \frac{\mathbf{p}}{p^2} \tilde{\psi}_I(s, \mathbf{k}; t, p), \quad (2.22)$$

$$f_x(s, \kappa; t, p) = \frac{1}{2} \mu^2 L \int_0^{2\pi} \frac{d\theta_\kappa}{2\pi} \tilde{\psi}_I(sL, \mu\boldsymbol{\kappa}; tL, p\sqrt{2\omega/L}), \quad (2.23)$$

$$g_x(\tau, l; s, q) = 2\omega \tilde{\phi}(\tau L, l\sqrt{2\omega/L}; sL, q\sqrt{2\omega/L}). \quad (2.24)$$

The function  $f_x$  obeys the following differential equation

$$\partial_t f_x(s, \kappa; t, p) = \frac{1}{2} \tilde{n}(t) L \int_0^\infty \frac{dq}{2\pi} \left[ q \tilde{V}_1(q, p; x) f_x(s, \kappa; t, p) - e^{-i(q^2 - p^2)(s-t)} p \tilde{V}_2(q, p; x) f_x(s, \kappa; t, q) \right], \quad (2.25)$$

having as initial condition

$$f_x(s, \kappa; s, p) = \frac{1}{x} g_x(1, \kappa/\sqrt{x}; s, p), \quad (2.26)$$

and the other function  $g_x$  satisfies

$$\partial_\tau g_x(\tau, l; s, p) = -\frac{1}{2} \tilde{n}(\tau) L \int_0^\infty \frac{q \, dq}{2\pi} \tilde{V}_1(l, q; x) [g_x(\tau, l; s, p) - g_x(\tau, q; s, p)], \quad (2.27)$$

with the initial condition given by

$$g_x(s, l; s, p) = \frac{1}{2} \tilde{n}(s) L \left[ \tilde{V}_1(l, p; x) - \frac{p}{l} \tilde{V}_2(l, p; x) \right]. \quad (2.28)$$

Here we have also defined the following integrals of the potential

$$V_1(k, q) \equiv V_1(|\mathbf{k}|, |\mathbf{q}|) = \int_0^{2\pi} \frac{d\theta_{kq}}{2\pi} V(\mathbf{k} - \mathbf{q}), \quad (2.29)$$

$$V_2(k, q) \equiv V_2(|\mathbf{k}|, |\mathbf{q}|) = \int_0^{2\pi} \frac{d\theta_{kq}}{2\pi} \cos \theta_{kq} V(\mathbf{k} - \mathbf{q}), \quad (2.30)$$

and the rescaled versions of  $V_i$  and  $n(t)$

$$\tilde{V}_i(l, q; x) = \frac{2\omega}{L} V_i \left( l \sqrt{\frac{2\omega}{L}}, q \sqrt{\frac{2\omega}{L}} \right), \quad \text{and} \quad \tilde{n}(t) = n(tL). \quad (2.31)$$

As we will see, for the models of the collision rate that we consider in this thesis the angular integrals in  $V_i$  can be done analytically, further simplifying the numerical labor needed. Those models, eqs. (1.26) and (1.27), are the most commonly employed in phenomenological studies.

Finally, the dimensionless variables that we will employ to evaluate and plot the spectrum are

$$\kappa^2 = \frac{k^2}{\mu^2}, \quad x = \frac{\omega}{\bar{\omega}_c} = \frac{2\omega}{\mu^2 L}. \quad (2.32)$$

## 2.4 Energy distribution

In several situations we are more interested in the medium-induced energy distribution rather than the transverse momentum dependent spectrum. It is computed simple by integrating the  $\mathbf{k}$ -differential spectrum over  $\mathbf{k}$

$$\omega \frac{dI}{d\omega} = \int_0^\omega dk \, k \int_0^{2\pi} d\theta_k \, \omega \frac{dI}{d\omega d^2\mathbf{k}}, \quad (2.33)$$

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being  $k$  the modulus of the transverse momentum and  $\theta_k$  its associated angle. It is important to realize here that we are choosing  $\omega$  as the upper limit of the integration over  $k$ . We do so because of the gluon kinematics  $(\omega, \mathbf{k}, k^-)^2 = \omega^2 - \mathbf{k}^2 - k^{-2} = 0 \rightarrow \omega \geq k$ . However, sometimes it is considered the extreme case of integrating over all of the transverse plane, taking  $\omega \rightarrow \infty$  as integration limit

$$\omega \frac{dI}{d\omega} = \int_0^\infty dk \, k \int_0^{2\pi} d\theta_k \, \omega \frac{dI}{d\omega d^2\mathbf{k}}, \quad (2.34)$$

which is the energy distribution with no kinematical constraint. This non-physical case is sometimes studied because it is easier to compute, therefore it can be used to gain basic knowledge of the main physics before performing more difficult calculations. However, we have to keep in mind that the absence of a kinematic cut-off leads to non-physical results in the low energy region, being the spectrum divergent there. But this region has the smallest contribution to phenomenology, thus sometimes this spectrum is even employed in studies that describe experimental data. We will analyze the spectrum without kinematical cut-off, because it is widely known in the community, but when computing observables we will employ the physical spectrum with kinetic constraint (2.33).

Evaluating the energy distribution with finite kinematic constraint  $k \leq \omega$  in the approach we are employing yields

$$x \frac{dI}{dx} = \int_0^{\omega^2/\mu^2} d\kappa^2 \, x \frac{dI}{dx d\kappa^2} = \frac{\alpha_s C_R}{\pi^2} \text{Re} \int_0^1 ds \int_0^s dt \int_0^\infty dp \, ip e^{-ip^2(s-t)} F_x(s, t; p), \quad (2.35)$$

where we defined the integrated version of  $f_x$

$$F_x(s, t; p) = \int_0^{\bar{R}x^2/2} d\kappa^2 \, f_x(s, \kappa; t, p), \quad (2.36)$$

being  $\bar{R} = \bar{\omega}_c L = \frac{1}{2}\mu^2 L^2$  the kinematical constraint parameter, and satisfying  $F_x$  the same differential equation as  $f_x$  eq. (2.25), but with a different initial condition

$$F_x(s, s; p) = \int_0^{\bar{R}x^2/2} dl^2 \, g_x(1, l; s, p), \quad (2.37)$$

and  $g_x$  is obtained exactly as before through eqs. (2.27) and (2.28).

The procedure to remove the kinematical constraint on the energy distribution consists on taking  $\bar{R} \rightarrow \infty$  while keeping  $\bar{\omega}_c$  fixed and finite, thus it corresponds to integrate eq. (2.37) up to infinite. By integrating over transverse momentum  $l$  from 0 to  $\infty$  in eq. (2.27) the r.h.s. naturally vanishes and, therefore,  $g_x$  is constant and equal to the initial condition (2.28) for all times. The physical interpretation of this result is that the broadening does not play a role anymore. In fact, as  $\mathcal{P}$  is the probability of having the radiated gluon in the final state with transverse momentum  $\mathbf{k}$ , integrating over all possible values of  $\mathbf{k}$  gives trivially 1. With this consideration we can still employ eq. (2.35) to evaluate the energy distribution with no kinematical constraint.

We emphasize that removing the kinematical constraint goes in contradiction with one of the basic assumptions of the derivation of the BDMPS-Z spectrum: the gluon transverse momentum must be very small compared to its energy  $k \ll \omega$ , thus, integrating over all transverse momentum plane taking  $k \rightarrow \infty$  as upper limit of the integral is not compatible. That is the reason why the spectrum diverges for low gluon energies given non-physical results. However, when keeping  $\bar{R}$  finite the spectrum vanishes at small energies, as it should for any realistic situation. Nevertheless, the high energy behavior of the  $\bar{R} \rightarrow \infty$  spectrum is the correct one.

## 2.5 Numerical evaluation of the full resummed spectrum

We finalize this chapter by showing the expressions we have to numerically solve to compute the BDMPS-Z spectrum with full resummation over multiple scatterings for the two collision rates models we will employ in the rest of the thesis. For illustration purposes, in all of the evaluations we will do in this thesis the value of the strong coupling constant is set to  $\alpha_s = 0.3$  and we consider that the parent parton is a quark, thus we have to take  $C_R = C_F = 4/3$ . We do not present any numerical result in this section though, as this work was originally done in [58] and it is not an original contribution of the current thesis. The expressions shown here are for the brick and they correspond to the ones derived in [58]. The numerical evaluation of the spectrum with the brick approximation  $n(t) = n_0$  will be done in chapter 3, and the more general case  $n(t) \neq n_0$  will be discussed in chapter 4.

The models we chose for the parton-medium interaction are the Yukawa-type (1.26) and the HTL (1.27) potentials. The selection was made due to them been ones of the most used collision rates in phenomenology, as they have the correct UV physics, but the formalism described here is not limited to this selection and can be applied to any model. Furthermore, both these potentials have the same high momentum behavior,  $V(\mathbf{q}) \sim 1/\mathbf{q}^4$ , which guarantees that all momentum integrations approach 0 for momentum high enough and, therefore, we can evaluate the integrals employing a finite upper cut-off as integration limit. It was meticulously checked that the results do not change when varying the cut-off if it is sufficiently large.

### 2.5.1 Yukawa parton-medium interaction model

To evaluate the spectrum numerically we need to specify the integrations of the potential  $\tilde{V}_i$  which appear in the differential equations. With this particular collision rate the  $\tilde{V}_i$  terms can be evaluated analytically. We start by writing the rescaled Yukawa potential

$$\tilde{V}(\mathbf{q}, x) = \frac{8\pi}{x(\mathbf{q}^2 + 1/x)^2}. \quad (2.38)$$

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Plugging it in the rescaled versions of  $V_1$  (2.29) and  $V_2$  (2.30) (see eq. (2.31)) the angular integrals can be computed analytically employing contour integration techniques, yielding

$$\tilde{V}_1(q, p; x) = \frac{4}{x} \int_0^{2\pi} \frac{d\theta}{(p^2 + q^2 - 2pq \cos \theta + 1/x)^2} = \frac{8\pi(p^2 + q^2 + 1/x)}{x[(p^2 + q^2 + 1/x)^2 - 4p^2q^2]^{3/2}}, \quad (2.39)$$

$$\tilde{V}_2(q, p; x) = \frac{4}{x} \int_0^{2\pi} \frac{d\theta \cos \theta}{(p^2 + q^2 - 2pq \cos \theta + 1/x)^2} = \frac{16\pi pq}{x[(p^2 + q^2 + 1/x)^2 - 4p^2q^2]^{3/2}}. \quad (2.40)$$

Moreover, it is important to remark the normalization of the collision rate used here

$$\int_0^\infty \frac{dq}{2\pi} q \tilde{V}_1(q, p) = \int_{\mathbf{q}} \tilde{V}(\mathbf{q} - \mathbf{p}) = 2, \quad (2.41)$$

which guarantees that the correct parameter in the opacity expansion is  $n_0 L$ . With these integrals already performed we can give the final expressions of the spectra fully resummed over multiple scatterings.

Starting with the  $k$ -differential spectrum, plugging (2.39) and (2.40) in eqs. (2.21)-(2.28) the complete BDMPS-Z spectrum is then given by eq. (2.21) where  $f_x(s, \kappa; t, p)$  satisfies the following differential equation and initial condition

$$\partial_t f_x(s, \kappa; t, p) = n_0 L \left[ f_x(s, \kappa; t, p) - \frac{4p^2}{x} \int_0^\infty dq \frac{q e^{-i(q^2 - p^2)(s-t)}}{[(p^2 + q^2 + 1/x)^2 - 4p^2q^2]^{3/2}} f_x(s, \kappa; t, q) \right], \quad (2.42)$$

$$f_x(s, \kappa; s, p) = \frac{1}{x} g_x(1, \kappa/\sqrt{x}; s, p), \quad (2.43)$$

and  $g_x$  is obtained by solving

$$\partial_\tau g_x(\tau, l; s, p) = -n_0 L \left[ g_x(\tau, l; s, p) - \frac{2}{x} \int_0^\infty dq \frac{q(q^2 + l^2 + 1/x)}{[(q^2 + l^2 + 1/x)^2 - 4q^2l^2]^{3/2}} g_x(\tau, q; s, p) \right], \quad (2.44)$$

$$g_x(s, l; s, p) = \frac{4\pi n_0 L(l^2 - p^2 + 1/x)}{x[(l^2 + p^2 + 1/x)^2 - 4l^2p^2]^{3/2}}. \quad (2.45)$$

On the other hand, the energy distribution is given by eq. (2.35) with  $F_x(s, t; p)$

satisfying the same differential equation as  $f_x(s, \kappa; t, p)$  but with different initial condition

$$\partial_t F_x(s, t; p) = n_0 L \left[ F_x(s, t; p) - \frac{4p^2}{x} \int_0^\infty dq \frac{q e^{-i(q^2 - p^2)(s-t)}}{[(p^2 + q^2 + 1/x)^2 - 4p^2 q^2]^{3/2}} F_x(s, t; q) \right], \quad (2.46)$$

$$F_x(s, s; p) = \int_0^{\bar{R}x/2} dl^2 g_x(1, l; s, p), \quad (2.47)$$

and  $g_x(s, l; s, p)$  is computed exactly as before through eqs. (2.44) and (2.45). Finally, the energy spectrum without kinematic constraint is obtained by taking  $\bar{R} \rightarrow \infty$  with fixed  $\bar{\omega}_c$  in the previous equations. These sets of equations are already suitable for being solved by numerical algorithms and are the ones we will use to compute the full spectrum.

### 2.5.2 GLV limit

When plotting the all-order full resummed spectrum with frequency we will compare it with one of the analytical approaches introduced in section 1.3, to quantify the impact of properly describing the emission process. The chosen framework is the GLV limit, which we describe in more detail in this section.

As we mentioned before, the GLV result corresponds to taking the leading order term in the opacity expansion. The opacity expansion considers that  $\sigma(\mathbf{r})$  is described by the full dipole cross section in the Yukawa case eq. (1.26) and uses this assumption to perform an expansion of the emission kernel and momentum broadening in powers of the opacity  $(n(t)\sigma(\mathbf{r}))^N$ . Taking just the first opacity  $N = 1$  is the so-called GLV approximation, which correctly describes the high energy tail of the full spectrum.

This statement can be explained in terms of the gluon formation time  $t_f$  (1.39). The formation time is proportional to the gluon energy and to the inverse of its transverse momentum squared. Thus, high energies correspond to large gluon formation times and, for energies above the characteristic energy of the medium  $\bar{\omega}_c \equiv \mu^2 L/2$ , the formation time is of the order of the total length of the medium  $L$ . Therefore, the only way in which is compatible to have energies larger than  $\bar{\omega}_c$  with the gluon being emitted inside the medium  $t_f \leq L$  is to have a large transverse momentum  $k^2 \gg \mu^2$ . Large momentum transfers with the medium are a consequence of having a rare, single hard scattering, which is precisely the physical interpretation of the GLV result.

The  $N = 1$  opacity result can be obtained straightforwardly from the full spectrum eq. (2.6) by replacing  $\tilde{\mathcal{K}}$  and  $\mathcal{P}$  by their vacuum versions, which can be easily derived from eqs. (2.1) and (2.2) neglecting the medium density  $n(s) = 0$

$$\mathcal{P}_0(t'', \mathbf{k}; t', \mathbf{q}) = (2\pi)^2 \delta^{(2)}(\mathbf{k} - \mathbf{q}), \quad (2.48)$$

$$\tilde{\mathcal{K}}_0(t', \mathbf{q}; t, \mathbf{p}) = (2\pi)^2 \delta^{(2)}(\mathbf{q} - \mathbf{p}) e^{-i\frac{p^2}{2\omega}(t'-t)}, \quad (2.49)$$

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where the subscript 0 stands for vacuum scenario. Replacing eqs. (2.48) and (2.49) into eq. (2.6) we get

$$\begin{aligned} \omega \frac{dI^{\text{GLV}}}{d\omega d^2\mathbf{k}} &= \frac{2\alpha_s C_R}{(2\pi)^2 \omega} \text{Re} \int_0^L ds n(s) \int_0^s dt \int_{pql} i\mathbf{p} \cdot \left( \frac{\mathbf{l}}{l^2} - \frac{\mathbf{q}}{q^2} \right) \sigma(\mathbf{l} - \mathbf{q}) (2\pi)^2 \delta^{(2)}(\mathbf{q} - \mathbf{p}) \\ &\quad \times e^{-i\frac{p^2}{2\omega}(s-t)} (2\pi)^2 \delta^{(2)}(\mathbf{k} - \mathbf{l}) \\ &= \frac{2\alpha_s C_R}{(2\pi)^2 \omega} \text{Re} \int_0^L ds n(s) \int_0^s dt \int_p i\mathbf{p} \cdot \left( \frac{\mathbf{k}}{k^2} - \frac{\mathbf{p}}{p^2} \right) \sigma(\mathbf{k} - \mathbf{p}) e^{-i\frac{p^2}{2\omega}(s-t)} = \\ &= \frac{2\alpha_s C_R}{(2\pi)^2 \omega} \int_p \mathbf{p} \cdot \left( \frac{\mathbf{k}}{k^2} - \frac{\mathbf{p}}{p^2} \right) \sigma(\mathbf{k} - \mathbf{p}) \text{Re} \left\{ \frac{2\omega}{p^2} \int_0^L ds n(s) \left( e^{-i\frac{p^2}{2\omega}s} - 1 \right) \right\}, \end{aligned} \quad (2.50)$$

which is the  $\mathbf{k}$ -dependent GLV spectrum for a general parton-medium interaction model  $\sigma(\mathbf{k} - \mathbf{p})$  and with an arbitrary medium evolution profile  $n(s)$ .

Performing the angular integral and rescaling variables as we did for the full spectrum yields

$$\begin{aligned} x \frac{dI}{dx d\kappa^2} &= \frac{\alpha_s C_R}{2\pi^2} \frac{L}{x} \text{Re} \int_0^1 ds \tilde{n}(s) \int_0^s dt \int_0^\infty dp ip e^{-ip^2(s-t)} \\ &\quad \times \left[ \tilde{V}_1(\kappa/\sqrt{x}, p; x) - \frac{\sqrt{x}p}{\kappa} \tilde{V}_2(\kappa/\sqrt{x}, p; x) \right], \end{aligned} \quad (2.51)$$

where all of the quantities are previously defined. In this simple case it is trivial to perform the time integral over  $t$ , as all of the dependence on this variable is explicitly contained in the exponential. Hence, this equation can be simplified as

$$\begin{aligned} x \frac{dI}{dx d\kappa^2} &= \frac{\alpha_s C_R}{2\pi^2} \frac{L}{x} \text{Re} \int_0^1 ds \tilde{n}(s) \int_0^\infty \frac{dp}{p} \left( 1 - e^{-ip^2s} \right) \\ &\quad \times \left[ \tilde{V}_1(\kappa/\sqrt{x}, p; x) - \frac{\sqrt{x}p}{\kappa} \tilde{V}_2(\kappa/\sqrt{x}, p; x) \right], \end{aligned} \quad (2.52)$$

and this is the first opacity  $N = 1$  spectrum for a generic medium profile  $n(s)$ . We have to make some assumption on this variable to be able to perform the remaining time integral over  $s$ . For illustration purposes, we employ the brick approximation as before. Therefore, if we consider an uniform medium  $n(s) = n_0 \theta(L - s)$  in eq. (2.52), we can easily compute the final time integral

$$x \frac{dI}{dx d\kappa^2} = \frac{\alpha_s C_R}{2\pi^2} \frac{n_0 L}{x} \int_0^\infty dp \frac{p^2 - \sin p^2}{p^3} \left[ \tilde{V}_1(\kappa/\sqrt{x}, p; x) - \frac{\sqrt{x}p}{\kappa} \tilde{V}_2(\kappa/\sqrt{x}, p; x) \right], \quad (2.53)$$

where we discarded the full imaginary terms when taking the real part. Finally, particularizing for the Yukawa-type model for the parton medium interaction (1.26) we simply have to plug eqs. (2.39) and (2.40) in eq. (2.53), which yields

$$x \frac{dI}{dx d\kappa^2} = \frac{4\alpha_s C_R}{\pi} n_0 L \int_0^\infty dp \frac{p^2 - \sin p^2}{p^3} \frac{\kappa^2 - xp^2 + 1}{[(\kappa^2 + xp^2 + 1)^2 - 4x\kappa^2 p^2]^{3/2}}, \quad (2.54)$$

and this is the final expression of the GLV  $k$ -dependent spectrum. This spectrum shall be compared to the full result (2.21).

Regarding the energy distribution, we simply have to integrate (2.54) over the transverse momentum variable  $\kappa$ . Keeping the kinematical constraint is done by imposing  $\kappa < \bar{R}x^2/2$  as integration limit (see eq. (2.36)). By doing so the energy distribution for the first opacity in a brick is given by

$$x \frac{dI}{dx} = \frac{4\alpha_s C_R}{\pi} n_0 L \int_0^\infty dp \frac{p^2 - \sin p^2}{p^3} \left[ \frac{1}{xp^2 + 1} - \frac{1}{\sqrt{(\frac{1}{2}\bar{R}x^2 + xp^2 + 1)^2 - 2\bar{R}x^3 p^2}} \right], \quad (2.55)$$

which shall be compared to (2.35). Removing the kinetic cut-off in this case gives a very simple semi-analytical expression. We just set  $\bar{R} \rightarrow \infty$  in the previous equation, which cancels out the last term

$$x \frac{dI}{dx} = \frac{4\alpha_s C_R}{\pi} n_0 L \int_0^\infty dp \frac{p^2 - \sin p^2}{p^3} \frac{1}{xp^2 + 1}, \quad (2.56)$$

and this spectrum is the GLV energy distribution without kinetic constraint.

Finally, we briefly discuss now the dependencies of the spectrum. The spectrum computed with both formalisms, GLV and full resummation, in general depends on three medium parameters  $n_0$ ,  $L$  and  $\mu^2$  and two gluon variables  $k^2$  and  $\omega$ . However, as we decided to rescale all of the quantities in the spectrum to make them dimensionless, we choose as parameters the following combination

$$n_0 L, \quad \bar{\omega}_c = \frac{1}{2}\mu^2 L, \quad \text{and} \quad \bar{R} = \bar{\omega}_c L. \quad (2.57)$$

We also remind here that we have rescaled the transverse momentum and energy variables eq. (2.32)  $x = \omega/\bar{\omega}_c$  and  $\kappa^2 = k^2/\mu^2$ .

### 2.5.3 HTL collision rate

We conclude this chapter by explaining how to compute the fully resummed in-medium radiation spectrum employing a different elastic collision rate model for the parton-medium interaction, the HTL potential (1.27). It is not our intention to provide appropriate comparisons between different models, those can be found in [58]. To not extend the manuscript for too long we will just provide the outcome of the angular integrals of the potential that enter the computation of the spectrum. Plugging the HTL potential eq. (1.27) in eqs. (2.29) and (2.30) yields

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$$\frac{1}{2}n V_1(q, p) = g^2 N_c T \left( \frac{1}{|p^2 - q^2|} - \frac{1}{\sqrt{(p^2 + q^2 + m_D^2)^2 - 4p^2 q^2}} \right), \quad (2.58)$$

$$\frac{1}{2}n V_2(q, p) = \frac{g^2 N_c T}{2pq} \left( \frac{p^2 + q^2}{|p^2 - q^2|} - \frac{p^2 + q^2 + m_D^2}{\sqrt{(p^2 + q^2 + m_D^2)^2 - 4p^2 q^2}} \right), \quad (2.59)$$

where we have computed the angular integrations through usual contour techniques. Equivalently, their rescaled versions (2.31) are given by

$$\frac{1}{2}\tilde{n} \tilde{V}_1(q, p; x) = g^2 N_c T \left( \frac{1}{|p^2 - q^2|} - \frac{1}{\sqrt{(p^2 + q^2 + 1/x)^2 - 4p^2 q^2}} \right), \quad (2.60)$$

$$\frac{1}{2}\tilde{n} \tilde{V}_2(q, p; x) = \frac{g^2 N_c T}{2pq} \left( \frac{p^2 + q^2}{|p^2 - q^2|} - \frac{p^2 + q^2 + 1/x}{\sqrt{(p^2 + q^2 + 1/x)^2 - 4p^2 q^2}} \right). \quad (2.61)$$

We notice here that the outcome of the integration of the HTL collision rate yields divergent results for  $p = q$ . However, when solving the differential equations and initial conditions the divergences always cancel, being the final result finite. Even so, we must take special care in numerical calculations to not evaluate the momentum space points with divergences and discontinuities. Let us consider, for instance, the initial condition of the  $g_x$  function. Introducing (2.60) and (2.61) in (2.28) we get

$$g_x(s, l; s, p) = \frac{g_s^2 N_c T L}{2l^2} \left[ \text{sgn}(l - p) + \frac{-l^2 + p^2 + 1/x}{\sqrt{(l^2 + p^2 + 1/x)^2 - 4l^2 p^2}} \right], \quad (2.62)$$

which has an evident discontinuity for  $p = l$ . Nevertheless, the discontinuity does not generate singularities in further steps of the evaluation, and in actual numerical computations we set the discontinuous part to 0. This initial condition is integrated over  $l^2$  between  $l = 0$  and  $l \rightarrow \infty$ . A careless look at eq. (2.62) would tell us that  $g_x(s, l; s, p)$  behaves like  $1/l^2$  in those endpoints, but if we analyze it in more detail we realize that for those two particular points the term inside brackets goes to 0. Then, even though the integration of  $g_x(s, l; s, p)$  seems to be problematic, the final result is actually convergent.

After computing the initial condition for  $g_x$  the next step is to evolve it through the correspondent differential equation (2.27). Similar to our previous encounter, we face a related issue in this equation as  $\tilde{V}_1(l, q; x)$  exhibits a divergence when  $l = q$ . However, it is worth noting that this singularity is not problematic because, at this point, the term enclosed in brackets vanishes. Consequently, the integrand becomes discontinuous at  $l = q$ , and in practice we set its value to the average between the left-handed and right-handed limits. Finally,  $g_x$  is used as initial condition in the differential equation that describes  $f_x$  (2.25), which also presents divergences at  $p = q$  for both  $\tilde{V}_1$  and  $\tilde{V}_2$ . Nevertheless, when considering the sum inside the brackets, an exact cancellation occurs

between the divergent terms. Naturally, this result introduces a new discontinuity at that specific point, which we address in the same manner as we did for  $g_x$ .

Regarding the parameter dependence of the full spectrum employing the HTL potential, we can see in eq. (1.27) that the collision rate depends on the temperature of the thermalized medium  $T$  and the Debye screening mass  $m_D$ , which takes the role of  $\mu$ . Therefore, it may seem that we have one more parameter than before, as with the Yukawa model we had dependencies on  $n$ ,  $L$  and  $\mu$  (2.57). However, the number of parameters is the same for both approaches, as when introducing the HTL collision rate in the spectrum the dependence on the medium density  $n$  is replaced by  $T$  (2.60)-(2.61). Following (2.57) we define the next quantities as the free parameters we will use for numerical evaluations

$$TL, \quad \bar{\omega}_c^H = \frac{1}{2}m_D^2L, \quad \text{and} \quad \bar{R}^H = \bar{\omega}_c^H L. \quad (2.63)$$

# 3

## **Full spectrum vs. analytical approximations: the role of multiple scatterings**

To comply with the regulations for doctoral studies of the University of Santiago de Compostela we include here the full information of the article on which this chapter is based:

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| From soft to hard radiation: the role of multiple scatterings in medium-induced gluon emissions, <i>J. High Energ. Phys.</i> <b>03</b> (2021) 102                                                                                                                                                                                                                                                                                                                                                                                                                                       |
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| <p><b>PhD Student Contribution</b></p> <p>Active participation in all the calculations presented in the paper, as well as in the discussions, meetings and writing of the paper. All of the authors have the same contribution to the publication.</p> <p><b>Chapter of the thesis in which it is used:</b> 3</p>                                                                                                                                                                                                                                                                       |
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In this chapter, we study the different regimes of the gluon energy distribution employing numerical results of the spectrum with full resummation over multiple scatterings in distinct approximations (namely, Yukawa and HTL). The evaluation of the all-order spectrum will be compared with several analytical approximations in the regions where they can be applied. This comparative analysis allows us to uncover the underlying physics governing the radiation process, as different approximations rely on distinct assumptions, thus the closest one to the full result will be the framework which better encodes the physical description of the gluon emission. The findings of this chapter are published in [2].

Essentially, we can identify three different regions in the spectrum regarding the energy of the radiated gluon: low, medium and high energy regimes. Our results will corroborate that the large energy regime is well described by the single hard scattering

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approximation, as it was already checked in [58]. In the intermediate energy region we will show explicitly that accounting for multiple scatterings is essential, as the GLV result overpredicts our full spectrum and the IOE approach, whose leading order term is the HO approximation, describes it with high precision. Finally, we will derive the correct  $\omega \rightarrow 0$  limit of the BDMPS-Z spectrum. With this result we are able to find, for the first time, the effect of multiple scatterings in the ultra soft region of the spectrum (also called Bethe-Heitler regime), which results in a multiplicative suppression factor with respect to the single radiation.

As our goal here is just to compare different theoretical frameworks without describing experimental data, for convenience, we decide to perform the comparisons in the simplest scenario, the energy distribution without kinematical constraint  $\bar{R} \rightarrow \infty$ . The computation of this spectrum employing the GLV approximation and the full resummation over multiple scatterings is done following the procedure explained in the previous chapter.

## 3.1 Energy regimes and energy scales

We are going to use this short section to explain in detail the different energy regions of the energy distribution and which are the scales that separates them. The number of scatterings a radiated gluon is expected to encounter in a medium characterized by a length  $L$  and a mean free path  $\lambda$  is of the order of  $L/\lambda$ . For every scattering the gluon may acquire a transverse momentum of the order of the typical momentum transfer  $\tilde{\mu}$  coming from the scattering center, being larger momentum transfers unusual. Then, gluons which have not undergone an uncommon hard scattering with the medium have a transverse momentum between

$$\tilde{\mu}^2 \lesssim \mathbf{k}^2 \lesssim \frac{L\tilde{\mu}^2}{\lambda}. \quad (3.1)$$

It is also reasonable to assume that medium length represents an upper limit for the value of the formation time of the gluons (1.39)  $t_f \sim 2\omega/\mathbf{k}^2$ . Therefore, gluons with an energy

$$\omega \gtrsim \frac{\tilde{\mu}^2 L^2}{2\lambda} \equiv \omega_c, \quad (3.2)$$

must have experienced one or more momentum transfers much bigger than the typical one  $\tilde{\mu}$ . Consequently, during the gluon emission at least one hard scattering occurred. This single hard scattering dominates the whole emission process, as is well illustrated, for instance, in Fig. 2 of Ref. [98]. Hence, in the high energy limit of the spectrum the opacity expansion at first order, which only encodes the information of one single hard scattering, gives a good description of the spectrum with full resummation over multiple scatterings, as it was already checked in [58].

Considering the opposite scenario, if the gluon formation time is smaller than the mean free path  $\lambda$ , then it is impossible to undergo two or more scatterings during the

emission process. Thus, gluons which possess an energy

$$\omega \lesssim \frac{1}{2} \tilde{\mu}^2 \lambda \equiv \omega_{\text{BH}} , \quad (3.3)$$

are also expected to be well described considering just one scattering. This region of the spectrum is called Bethe-Heitler (BH) regime [99].

We are left then with the intermediate energy region of the spectrum  $\omega_{\text{BH}} < \omega < \omega_c$  where multiple soft scatterings are expected to dominate the gluon propagation. Therefore, in this regime the HO approximation and extensions like the IOE should provide a good description of the full medium-induced radiation spectrum.

We can relate the parameters from this section with the ones we defined in chapter 2 in the following way by introducing a constant factor of order 1  $C$ :  $\tilde{\mu}^2 = C\mu^2$  and  $\lambda = C/n_0$ . With this choice we can now establish a correspondence between the previous fundamental parameters of the spectrum  $n_0 L$  and  $\bar{\omega}_c$ , and the two energy scales we defined in this section employing the following relations

$$\bar{\omega}_c^2 = \frac{\omega_{\text{BH}} \omega_c}{C^2} , \quad \text{and} \quad (n_0 L)^2 = C^2 \frac{\omega_c}{\omega_{\text{BH}}} . \quad (3.4)$$

For a general case the energy distribution is perfectly described just by using two dimensionless combinations of the energies  $\omega$ ,  $\omega_c$  and  $\omega_{\text{BH}}$ , as we showed in chapter 2 (in the  $\bar{R} \rightarrow \infty$  limit the kinematic constraint is not a parameter anymore, thus we only need two of the three quantities defined in eq. (2.57) to determine the spectrum). This important feature will be employed in section 3.3, first defining the energy scales in terms of the parameters used in the HO approach and then setting a relation between the parameters employed in both formalisms, HO approximation and fully resummed spectrum.

## 3.2 Single scattering regimes

In this section we will analyze the asymptotic energy limits of the spectrum, focusing on the regimes where one scattering should provide a good description of the full result. We already know that the high energy region is well described by the GLV approach, thus, here we will pay special attention to the BH regime. Nonetheless, the derivation we follow here can be applied to both limits, hence we are able to provide analytical proof, not just comparing numerical results, that the  $N = 1$  order in opacity expansion corresponds to the high energy limit of the spectrum with full resummation over multiple scatterings.

As our goal is to check if one single scattering is enough or not to describe the emission process in the low and large energy limits, the approach we follow is to derive the first opacity contributions  $N = 1$  and  $N = 2$ , and see explicitly if the former dominates the latter in the asymptotic limits of the energy distribution. Different terms in the opacity expansion are easily build by replacing iteratively the recursive equation of the emission kernel (2.2) into the complete spectrum (2.6) (the  $\bar{R} \rightarrow \infty$  limit is easily obtained

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by integrating (2.6) over all  $\mathbf{k}$  space. By doing so, the contribution of the momentum broadening vanishes, as we discussed in chapter 2. We also notice that, since  $\tilde{\mathcal{K}}$  does not depend on  $l$ , the integral over  $l$  of the second term in brackets is zero). With this method the first two contributions  $N = 1, 2$  yield

$$\omega \frac{dI^{\text{med}}}{d\omega} \Big|_{N=1} = \frac{2\alpha_s C_R}{\omega} n_0 \text{Re} \int_0^L ds \int_0^s dt \int_{pq} i \frac{\mathbf{p} \cdot \mathbf{q}}{q^2} \sigma(\mathbf{q} - \mathbf{p}) e^{-i \frac{p^2}{2\omega}(s-t)}, \quad (3.5)$$

$$\begin{aligned} \omega \frac{dI^{\text{med}}}{d\omega} \Big|_{N=2} &= \frac{2\alpha_s C_R}{\omega} n_0 \text{Re} \int_0^L ds \int_0^s dt \int_{pql} i \frac{\mathbf{p} \cdot \mathbf{q}}{q^2} \sigma(\mathbf{q} - \mathbf{l}) e^{-i \frac{l^2}{2\omega}(s-t)} \\ &\times \left[ -\frac{n_0}{2} \int_t^s ds' \sigma(\mathbf{l} - \mathbf{p}) e^{-i \frac{p^2 - l^2}{2\omega}(s'-t)} \right]. \end{aligned} \quad (3.6)$$

Now let us focus on the integration over transverse momentum  $\mathbf{p}$  in the second term  $N = 2$ . Replacing the dipole cross section by the collision rate through eq. (1.32) we get the following expression for the integral

$$\int_{\mathbf{p}} \mathbf{p} \sigma(\mathbf{l} - \mathbf{p}) e^{-i \frac{p^2 - l^2}{2\omega}(s'-t)} = \int_{\mathbf{p}} V(\mathbf{l} - \mathbf{p}) \left[ \mathbf{l} - \mathbf{p} e^{-i \frac{p^2 - l^2}{2\omega}(s'-t)} \right]. \quad (3.7)$$

The r.h.s. phase factor becomes trivially 1 for  $\mathbf{p} - \mathbf{l}$  small enough, therefore, we are remaining with the integral over  $\mathbf{p}$  of  $(\mathbf{l} - \mathbf{p})V(\mathbf{l} - \mathbf{p})$ , which vanishes due to symmetry. On the other hand, if  $\mathbf{p}$  is very big the phase factor oscillates rapidly and its contribution to the integration is negligible, thus the integral is also 0 in this case. Hence, in the asymptotic limits we are interested in the whole contribution of the phase factor can be considered as imposing a lower cut-off in the momentum integration and approximate

$$\int_{\mathbf{p}} \mathbf{p} \sigma(\mathbf{l} - \mathbf{p}) e^{-i \frac{p^2 - l^2}{2\omega}(s'-t)} \simeq \mathbf{l} \int_{q^2 > M^2} V(\mathbf{q}) \equiv \mathbf{l} \Sigma(M^2), \quad (3.8)$$

where  $M^2$  is the undetermined IR cut-off. With this consideration the second opacity contribution is then given by

$$\begin{aligned} \omega \frac{dI^{\text{med}}}{d\omega} \Big|_{N=2} &\simeq \frac{2\alpha_s C_R}{\omega} n_0 \text{Re} \int_0^L ds \int_0^s dt \int_{ql} i \frac{\mathbf{l} \cdot \mathbf{q}}{q^2} \sigma(\mathbf{q} - \mathbf{l}) e^{-i \frac{l^2}{2\omega}(s-t)} \\ &\times \left[ -\frac{n_0}{2} \int_t^s ds' \Sigma(M^2) \right]. \end{aligned} \quad (3.9)$$

In the large energy limit  $\omega$  is huge, then the phase factor in the  $\mathbf{p}$  integral (3.7) will be one unless  $\mathbf{p}$  is also very large. This sets an approximate value for the cut-off  $M^2 \sim 2\omega/L$ , as if  $p^2$  is much smaller than that factor the integration is zero by symmetry. In this case the  $\Sigma$  factor we defined (3.8) only has access to the high momentum tail of the potential. We discussed already several times that in this limit the collision rate must

behave like  $V(\mathbf{q}) \sim \mu^2/q^4$  for any realistic parton-medium interaction model. Thus, for this asymptotic case we have

$$\Sigma = \int_{q^2 > M^2} V(\mathbf{q}) \sim \int_{M^2}^{\infty} 2q \, dq \, \frac{\mu^2}{q^4} \sim 2 \frac{\mu^2}{M^2} = \frac{\mu^2 L}{\omega}, \quad (3.10)$$

where we ignored most of constant numerical factors for simplicity. Therefore, we have explicitly shown that the second term in the opacity expansion has an additional factor  $1/\omega$  compared to the  $N = 1$  contribution, which suppress it for large energies. Higher order terms in the expansion would be suppressed by additional powers of the energy  $\omega$ , resulting in the GLV approximation  $N = 1$  first opacity result being the asymptotic limit of the full spectrum for large gluon energies  $\omega$ .

On the other hand, in the small energy limit the phase factor we have in (3.7) has rapid oscillations unless  $(\mathbf{p} - \mathbf{l})^2$  is also very small, resulting in a negligible contribution to the integral. Therefore, the contribution of this factor is to cut some possible divergence when we have no-momentum transfer. As we have to keep the phase in the first line of (3.9), now the IR cut-off is set to  $M^2 \sim l^2$ . If there is no divergence in the IR sector, then  $M^2$  is no longer necessary and the cut-off can be ignored  $M^2 = 0$ , in which case  $\Sigma$  is just the total cross section of the collision.

The interpretation of these results can be made in terms of real-virtual cancellations. We remind that the two terms of the dipole cross section (1.32) represent the contribution of real and virtual terms to the scattering, respectively. For the high energy limit the cancellation between both contributions is almost exact, resulting in single hard scatterings controlling the radiation probability, whereas that subsequent soft scatterings do not play a role in the emission process and just contribute to momentum broadening. In the opposite limit, for small gluon energies the cancellation between both contributions does not occur. The restriction we discussed earlier based on formation time arguments, which limits the number of scatterings the gluon can undergo, only affects the real terms. In contrast, virtual contributions remain unconstrained. This results in real scatterings being limited just to small transverse momentum transfers, but virtual contributions are not restricted at all. As it is common in these kind of calculations, it is possible to formally resumme higher order contributions, resulting in an exponentiation of the virtual terms. Hence, it can be interpreted as a probability factor which encodes the probability of not experiencing further scatterings.

Taking in account the exponentiation of the virtual contributions of higher order terms in the opacity expansion  $N = 3, 4 \dots$  the fully resummed spectrum for low energies is given by

$$\omega \frac{dI^{\text{med}}}{d\omega} \Big|_{\omega \rightarrow 0} = \frac{2\alpha_s C_R}{\omega} \text{Re} \int_0^L ds \, n_0 \int_0^s dt \int_{pq} i \frac{\mathbf{p} \cdot \mathbf{q}}{q^2} \sigma(\mathbf{q} - \mathbf{p}) e^{-\left(i \frac{p^2}{2\omega} + \frac{1}{2} n_0 \Sigma(p^2)\right)(s-t)}, \quad (3.11)$$

comparing this equation with eq. (3.5), it is evident that it is simply the  $N = 1$  result multiplied by a no-scattering probability factor.

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These probability factors that suppress the spectrum for low energies were already present in sections 5 and 6 of Ref. [75]. That reference aims to compute arbitrary orders in the opacity expansion rather than analyzing different energy regimes, but it can be shown that the “incoherent limit” presented there, characterized by  $L \rightarrow \infty$  and  $n_0 L$  fixed, can be resummed giving an expression similar to our low energy limit (3.11). Taking the  $x \rightarrow 0$  limit, with  $x$  given by (2.32), it can be proved that both cases are equivalent.

To conclude our study of the small- $\omega$  limit of the spectrum, we provide now numerical comparisons between the recently derived eq. (3.11) and the spectrum with full resummation over multiple scatterings, evaluated as we discussed in chapter 2. We start by simplifying the low energy result, performing all of the analytical integrals we can before computing it numerically. Given the dependence of (3.11) on  $\mathbf{q}$ ,  $s$  and  $t$ , we can perform independently the integration over transverse momentum  $\mathbf{q}$  and time variables  $s$  and  $t$ . Starting with  $\mathbf{q}$

$$S(p^2) = \int_{\mathbf{q}} \frac{\mathbf{p} \cdot \mathbf{q}}{q^2} \sigma(\mathbf{q} - \mathbf{p}) = \int_{\mathbf{q}} \left(1 - \frac{\mathbf{p} \cdot \mathbf{q}}{q^2}\right) V(\mathbf{q} - \mathbf{p}), \quad (3.12)$$

here we employed again eq. (1.32) to express the dipole cross section in terms of  $V$ . As the collision rate  $V(\mathbf{q})$  is only function of  $\mathbf{q}^2$  (see (1.26) and (1.27)),  $V(\mathbf{q}) = \hat{V}(q^2)$ , we can compute the angular integral

$$\begin{aligned} S(p^2) &= \frac{1}{(2\pi)^2} \int_0^{2\pi} d\theta \int_0^\infty dq (q - p \cos \theta) \hat{V}(p^2 + q^2 - 2pq \cos \theta) \\ &= \frac{1}{4\pi} \int_{p^2}^\infty du \hat{V}(u) = \Sigma(p^2), \end{aligned} \quad (3.13)$$

where, to ensure the convergence of the remaining integral over transverse momentum at large  $p$ ,  $\Sigma$  must be taken by its full expression, even if there is no IR divergence in the potential. Here we used the previous definition of  $\Sigma$

$$\Sigma(M^2) = \int_{q^2 > M^2} V(\mathbf{q}) = \frac{1}{(2\pi)^2} \int_0^{2\pi} d\theta \int_{M^2}^\infty dq q \hat{V}(q^2) = \frac{1}{4\pi} \int_{M^2}^\infty du \hat{V}(u). \quad (3.14)$$

Regarding the time integrals, for the brick approximation we are using they can be easily performed, as we just have to integrate exponentials

$$\begin{aligned} T(p^2) &= \int_0^L ds \int_0^s dt i e^{-\left(i \frac{p^2}{2\omega} + \frac{1}{2} n_0 \Sigma(p^2)\right)(s-t)} \\ &= \frac{L}{\frac{p^2}{2\omega} - \frac{i}{2} n_0 \Sigma(p^2)} - \frac{i}{\left[\frac{p^2}{2\omega} - \frac{i}{2} n_0 \Sigma(p^2)\right]^2} \left[ e^{-\left(i \frac{p^2}{2\omega} + \frac{1}{2} n_0 \Sigma(p^2)\right)L} - 1 \right], \end{aligned} \quad (3.15)$$

we should notice here that in the  $\omega \rightarrow 0$  limit the second term is subdominant compared with the first one (it has an extra  $\omega$  power), hence we can just neglect it.

Plugging these results together in the equation of the spectrum (3.11) yields

$$\omega \frac{dI^{\text{med}}}{d\omega} \Big|_{\omega \rightarrow 0} = \frac{2\alpha_s C_R}{\omega} \text{Re } n_0 \int_p i \Sigma(p^2) T(p^2), \quad (3.16)$$

where we just need to perform numerically the last remaining integral over  $p$ . To continue the evaluation we have to specify a collision rate model, so we can compute  $\Sigma$ . As in the previous chapter, we are going to employ the same models for the potential, Yukawa-type and HTL.

Regarding the Yukawa collision rate (1.26), it is IR safe for zero momentum transfer  $\mathbf{q} = \mathbf{0}$ . Integrating it to obtain  $\Sigma$  gives

$$\Sigma_Y(M^2) = \frac{2\mu^2}{M^2 + \mu^2}, \quad (3.17)$$

and we clearly see we can take  $M = 0$ , as there is no divergence, and employ  $\Sigma_Y = 2$  in the asymptotically small energy limit.

With respect to the HTL case (1.27), it presents a divergence in the IR sector. The integration of the potential yields

$$\frac{1}{2} n_0 \Sigma_H(M^2) = \alpha_s N_c T \ln \left( 1 + \frac{m_D^2}{M^2} \right), \quad (3.18)$$

being now the low momentum cut-off  $M^2$  essential to achieve a finite result.

We remind that, for illustration purposes, in numerical evaluations the strong coupling constant value is taken to  $\alpha_s = 0.3$  and the parent parton is assumed to be a quark  $C_R = C_F = 4/3$ .

We compare in Fig. 3.1 results obtained for the full resummed energy spectrum (solid lines), the asymptotically small energy limit (3.16) (dotted lines) and the GLV first opacity result (dashed dotted lines) plotted as function of the rescaled gluon energy  $x = \omega/\bar{\omega}_c$ . On the left we have spectra employing the Yukawa potential for several values of  $n_0 L$  and on the right using the HTL model with  $TL$  as parameter (following the discussion done in chapter 2). The values of  $n_0 L$  and  $TL$  were chosen to be equivalent when employing the correspondence relations shown in [58].

It is evident that for all cases the low energy limit coincide with the full result if  $\omega$  is small enough, thus corroborating that we derived the correct small energy limit of the full BDMPS-Z spectrum. Comparing left and right plots in Fig. 3.1 we see that the shape of the spectrum for small energies strongly depends on the details of the collision rate, but the full spectrum and the low- $\omega$  limit curves do not experience additional discrepancies depending on the chosen model. Therefore, the physical picture we described here holds unalterable for any collision rate employed, independently of the IR problems it may have. Furthermore, comparing solid curves (full spectrum) and dashed-dotted ones (GLV) we explicitly see that one single scattering is not enough to properly describe the emission process in this energy regime, as GLV curves largely overestimate the correct result. Thus,

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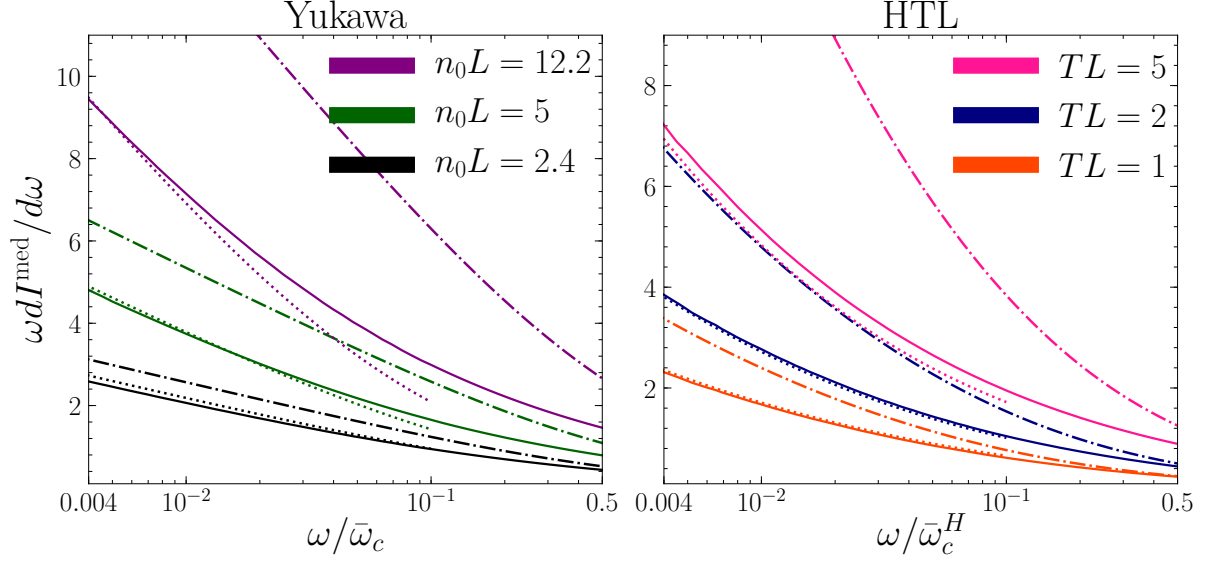


Figure 3.1: Left: full resummed in-medium energy spectrum (solid lines), its asymptotically small energy limit (3.16) (dotted lines) and the GLV first opacity result (dashed dotted lines) for the Yukawa potential plotted as a function of the rescaled gluon energy  $\omega/\bar{\omega}_c = 2\omega/(\mu^2 L)$  for several values of  $n_0 L$ . Right: same as left panel but employing the HTL collision rate and plotting the spectra as a function of  $\omega/\bar{\omega}_c^H = 2\omega/(m_D^2 L)$  for different  $TL$  values. Figure extracted from [2] under a Creative Commons license, CC-BY 4.0.

the no-scattering factor derived here coming from the resummation of virtual contributions is essential to correctly explain the radiation process at small energies.

Nevertheless, we want to emphasise that the BH region where we have described the in-medium radiation spectrum is below the energy regime where the assumptions employed to derive the BDMPS-Z spectrum (1.38) are expected to be satisfied and, therefore, the emission process may be completely different in this region. For instance, here we are not accounting for the effects coming from the total gluon energy not being equal to the longitudinal momentum, multiple emissions acting coherently and thermal masses in the propagators. Thus, our findings hold as long as the medium-induced gluon emission spectrum can be described by (1.38).

### 3.3 Multiple scattering regime

Having described the small and large energy regimes of the spectrum, we now shift our focus to addressing the intermediate energy area  $\omega_{\text{BH}} < \omega < \omega_c$ , where a single scattering is not enough to describe the emission process. This is the energy region where it is usually assumed that approximations like the HO, which resums multiple soft scatterings, provide

a good description of the energy distribution.

### 3.3.1 Medium-induced energy distribution

The multiple soft scattering approximation, which was briefly mentioned in section 1.3, is explained here in more detail. Within the HO approach the dipole cross section in coordinate space is given by its small distances behavior<sup>1</sup>

$$n(s)\sigma(\mathbf{r}) \equiv n_0 \int_{\mathbf{q}} e^{i\mathbf{q}\mathbf{r}} \sigma(\mathbf{q}) \approx \frac{1}{2} \hat{q} \mathbf{r}^2 + \mathcal{O}(\mathbf{r}^2 \ln \mathbf{r}^2), \quad (3.19)$$

with  $\hat{q}$  the transport coefficient which describes the average transverse momentum squared per length unit transferred to the initial hard parton by the background medium, commonly known as jet quenching parameter. The employment of this approximation provides a simple expression for the energy distribution with no kinematical constraint, even though the spectra with kinetic cut-off and differential in transverse momentum are not so simple, as we will see in chapter 5.

The importance of this approach is that it describes multiple soft scatterings, in contrast to GLV that only takes in account a single hard one. Accounting for multiple scatterings ends up in a successful description of the QCD Landau-Pomeranchuk-Migdal (LPM) effect [100–102], which GLV can not describe. The LPM effect is the name it was given to the fact that, if we have multiple successive scatterings, then each scattering is not independent, rather many scatterings centers act coherently to produce a single emission. This coherent behavior of multiple scattering centers translates in suppressing the spectrum at low energies, as the effective number of scatterings which can produce an emission reduces. This is the reason why GLV is able to describe the full spectrum only for high energies, as we checked in section 3.2.

We would like to compare our formalism with the HO result, as it is expected to describe the overall behavior of the radiation process in the multiple scattering regime we are discussing now. Nevertheless, one should notice that the jet quenching parameter  $\hat{q}$  depends on an integration cut-off for any realistic collision rate which has the Coulomb behaviour at large momentum  $V(\mathbf{q}) \sim 1/\mathbf{q}^4$ . Writing explicitly the jet quenching parameter taking the leading order contribution of the dipole cross section yields

$$\hat{q}L \sim (n_0L)\mu^2 \ln \sqrt{\frac{q_{max}}{\mu}}, \quad (3.20)$$

therefore, performing a one-to-one correlation between the parameters of the HO approximation and our approach or GLV is impossible, as we have a dependence on an arbitrary cut-off that regularizes a divergent integral. The selection of this cut-off is not trivial and there have been several discussions about which should be the one employed. The common approach followed in phenomenology is to consider the cut-off as constant, since

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<sup>1</sup>Recall that we are working in the brick approximation in this chapter.

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$\hat{q}$  is taken as a local property of the medium which does not depend on the parent parton or daughter gluon properties. However, this assumption translates in reducing the free parameters of the formalisms by one<sup>2</sup> and, hence, the only comparisons we can make between HO and full spectrum are qualitative, as done in [58]. This is not the only approach we can follow for selecting the cut-off. For instance, in [91] the cut-off is chosen to be dependent on the gluon energy, setting  $\sigma(\mathbf{r})$  to its leading logarithmic approximation

$$\sigma(\mathbf{r}) = \frac{\hat{q}_0}{4n_0} \mathbf{r}^2 \ln \left( \frac{1}{\mathbf{r}^2 \mu^{*2}} \right), \quad (3.21)$$

where  $\mu^{*2}$  is related to  $\mu^2$ . By doing so, quantitative comparisons can be achieved with the HO approach. Employing the properties of the logarithms we can decompose  $\sigma(\mathbf{r})$  in the following way

$$\sigma(\mathbf{r}) = \frac{\hat{q}_0}{4n_0} \mathbf{r}^2 \left( \ln \left( \frac{Q^2}{\mu^{*2}} \right) - \ln(\mathbf{r}^2 Q^2) \right) = \sigma_{\text{HO}}(\mathbf{r}) + \sigma_{\text{pert}}(\mathbf{r}), \quad (3.22)$$

with  $\sigma_{\text{HO}}(\mathbf{r})$  being the dipole cross section in the HO framework (3.19),  $\sigma_{\text{pert}}(\mathbf{r})$  some perturbation of the HO approximation and  $Q^2$  a matching scale that plays the same role as the previous cut-off  $q_{\text{max}}$ . The choice made in [91] for  $Q^2$  is to take it of the order of  $Q_c^2$

$$Q_c^2 = \sqrt{2\omega \hat{q}_0 \ln \left( \frac{Q_c^2}{\mu^{*2}} \right)}, \quad (3.23)$$

thus, here we follow this prescription and we take  $Q^2 = aQ_c^2$ , with  $a$  a constant parameter of order 1.

This approach allows us to employ perturbation theory techniques to compute the radiation spectrum at any order

$$\omega \frac{dI}{d\omega} = \omega \frac{dI^{\text{HO=LO}}}{d\omega} + \omega \frac{dI^{\text{NLO}}}{d\omega} + \dots = \omega \frac{dI^{\text{LO}}}{d\omega} + \sum_{m=1}^{\infty} \omega \frac{dI^{\text{N}^m \text{LO}}}{d\omega}, \quad (3.24)$$

where the leading order (LO) term is the HO result by construction (3.22). Hence, if we set  $\sigma(\mathbf{r}) = \sigma_{\text{HO}}(\mathbf{r})$  we get for the LO contribution

$$\omega \frac{dI^{\text{HO}}}{d\omega} = \frac{2\alpha_s C_R}{\pi} \ln |\cos(\Omega L)|, \quad (3.25)$$

with  $\Omega$  defined as

$$\Omega = \frac{1-i}{2} \sqrt{\frac{\hat{q}}{\omega}}, \quad \text{and} \quad \hat{q} = \hat{q}_0 \ln \left( \frac{Q^2}{\mu^{*2}} \right). \quad (3.26)$$

<sup>2</sup>As we discussed in chapter 2, full spectrum results depend on the parameters defined in (2.57),  $n_0 L$ ,  $\bar{\omega}_c$  and  $\bar{R}$ . In contrast, the HO spectrum just depends on a characteristic frequency  $\omega_c = \hat{q}^2 L/2$  and a kinetic constraint  $R = \omega_c L$ .

It is important to notice that, even after employing (3.23) to fix the matching scale, the energy distribution depends on the arbitrary factor  $a$ . This is something quite obvious if we take in consideration that we introduced a matching scale to decompose the dipole cross section (3.22) so we could treat part of it as a perturbation, but now we are only considering the non-perturbative part by truncating the series to the lowest contribution, thus the result should depend on the chosen scale. Also, the HO approximation does not contain information about few hard scatterings, just multiple soft ones, hence it does not have the correct behavior for large gluon energies. Both these problems can be solved by computing the next-to-leading (NLO) term of the series [91, 92]

$$\omega \frac{dI^{\text{NLO}}}{d\omega} = \frac{\alpha_s C_R}{2\pi} \hat{q}_0 \text{Re} \left\{ \int_0^L ds \frac{-1}{k^2(s)} \left[ \ln \left( -\frac{k^2(s)}{Q^2} \right) + \gamma_E \right] \right\}, \quad (3.27)$$

with  $\gamma_E$  the Euler-Mascheroni constant and  $k^2(s)$  given by

$$k^2(s) = \frac{i\omega}{2} \Omega [\cot \Omega s - \tan \Omega(L - s)], \quad (3.28)$$

please notice that in [91] the NLO term has an incorrect sign, being the correct result the one presented in [92]. These two terms (3.25) and (3.27) are the leading order contributions of the IOE.

Right now the energy distribution depends on three different parameters,  $\mu^*$ ,  $L$  and  $\hat{q}_0$ . However, as we argued in section 3.1, the spectrum can be written in terms of just two independent variables if we define the following energy scales

$$\omega_c = \frac{\hat{q}_0 L^2}{2}, \quad \text{and} \quad \omega_{\text{BH}} = \frac{\mu^{*4}}{2\hat{q}_0}, \quad (3.29)$$

for that purpose we will employ the following variables

$$\chi^2 \equiv \frac{\omega_c}{\omega_{\text{BH}}} = \left( \frac{\hat{q}_0 L}{\mu^{*2}} \right)^2, \quad \text{and} \quad \hat{x} \equiv \frac{\omega}{\omega_{\text{BH}}}. \quad (3.30)$$

Clearly,  $\chi$  is a measurement of the size of the multiple scattering region of the spectrum, thus if  $\chi$  is big the multiple scattering approximation can be applied in a wider range of energies. At the same time,  $\chi$  gives an estimation of the number of scatterings we have during the propagation, i.e., it represents the opacity of the medium. Hence, if  $\chi$  is large the multiple scatterings approach should provide results in agreement with the full spectrum, and on the opposite scenario, if  $\chi$  is low the GLV approximation is expected to describe well the radiation process. This will be clearly seen in our numerical results, in Fig. 3.4 for instance.

First we deal with the LO contribution (3.25), rewriting it in terms of these new variables (3.30). The only part we need to focus on is inside the cosine

$$\Omega L = \frac{1-i}{2} \sqrt{\frac{\hat{q} L^2}{\omega}} = \frac{1-i}{2} \sqrt{\frac{\hat{q}_0 L^2}{\omega} \ln \frac{a Q_c^2}{\mu^{*2}}} = (1-i) \sqrt{\frac{\omega_c \ln \frac{a Q_c^2}{\mu^{*2}}}{2\omega}} = (1-i) \chi \sqrt{\frac{\Lambda + \ln a}{2\hat{x}}}, \quad (3.31)$$

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where we have defined  $\Lambda \equiv \ln(Q_c^2/\mu^{*2})$ , which satisfies a similar transcendental equation as the one followed by  $Q_c^2$  (3.23)

$$\Lambda = \frac{1}{2} \ln \left( \frac{2\omega\hat{q}_0}{\mu^{*4}} \Lambda \right) = \frac{1}{2} \ln (\Lambda \hat{x}), \quad (3.32)$$

which can be solved iteratively employing as initial condition

$$\Lambda_0 = \frac{1}{2} \ln \hat{x}, \quad (3.33)$$

therefore, from (3.32) and (3.33) it is obvious that  $\Lambda$  only depends on  $\hat{x}$ , thus,  $\Omega L$  is just function of  $\chi$  and  $\hat{x}$ , and the same applies to the HO spectrum

$$\hat{x} \frac{dI^{\text{HO}}}{d\hat{x}} = \frac{2\alpha_s C_R}{\pi} \ln \left| \cos \left( (1-i)\chi \sqrt{\frac{\Lambda + \ln a}{2\hat{x}}} \right) \right|. \quad (3.34)$$

It is very important to remark that both transcendental equations (3.23) and (3.32) can be solved just for  $\hat{x} > 2e$ , otherwise no solution can be achieved. Hence, this condition sets the minimum gluon energy for which the IOE approach is valid.

Now let us focus on the NLO contribution (3.27). We start by rescaling the time variable  $s$  as usual  $s \rightarrow sL$ . With this, the trigonometric functions inside  $k(s)$  (3.28) are already function of  $\hat{x}$  and  $\chi$ , as they only depend on  $\Omega L$  (3.31). This function  $k(s)$  appears inside the logarithm along with  $Q^2$  as a fraction  $k^2(s)/Q^2$ . The factor that multiplies the trigonometric functions in this term is then

$$\frac{k^2(s)}{Q^2} \propto \frac{i\omega}{2} \frac{\Omega}{Q^2} = \frac{i\Omega L}{2a} \sqrt{\frac{\omega}{2\hat{q}_0 L^2 \Lambda}} = \frac{(1+i)}{4\sqrt{2}a} \sqrt{\frac{\Lambda + \ln a}{\Lambda}}, \quad (3.35)$$

which is already written as a function of  $\hat{x}$  only. The last remaining term is the overall factor which multiplies the whole spectrum. Ignoring the constant normalization  $\alpha_s C_R/(2\pi)$ , we just have to put together  $\hat{q}_0$ , the  $L$  coming from the rescaling of the dummy integration variable  $s$  and the  $i\omega\Omega/2$  factor of  $k(s)$

$$\frac{2\hat{q}_0 L}{i\omega\Omega} = -\frac{4i\omega_c}{\omega} \frac{1}{1-i} \sqrt{\frac{2\omega}{\omega_c(\Lambda + \ln a)}} = 2(1-i)\chi \sqrt{\frac{2}{\hat{x}(\Lambda + \ln a)}}. \quad (3.36)$$

Plugging all of this inside the NLO spectrum (3.27) we reach the following expression

$$\hat{x} \frac{dI^{\text{NLO}}}{d\hat{x}} = \frac{\sqrt{2}\alpha_s C_R}{\pi} \text{Re} \left\{ \frac{(i-1)\chi}{\sqrt{\hat{x}(\Lambda + \ln a)}} \int_0^1 \frac{ds}{K^2(s)} \left[ \ln \left( -\frac{(1+i)}{4a\sqrt{2}} \sqrt{\frac{\Lambda + \ln a}{\Lambda}} K^2(s) \right) + \gamma_E \right] \right\}, \quad (3.37)$$

here we have defined a new function  $K(s)$  analogous to the previous  $k(s)$  and given by

$$K^2(s) = \cot \left( (1-i)\chi \sqrt{\frac{\Lambda + \ln a}{2\hat{x}}} s \right) - \tan \left( (1-i)\chi \sqrt{\frac{\Lambda + \ln a}{2\hat{x}}} (1-s) \right). \quad (3.38)$$

With this it is explicitly shown that both contributions of the spectrum, HO and NLO, are function of  $\chi$  and  $\hat{x}$  only. Notice however that until now we have been neglecting the dependence on the parameter  $a$ , which establishes the exact value of the matching scale  $Q^2 = aQ_c^2$ . We did so because in [92] it was discussed that when considering together both contributions of the spectrum, HO+NLO, the sum has a weak dependence on  $Q^2$ , thus, if we take (3.34) and (3.37) at the same time, then the final result should be (almost) independent of  $a$ , leaving as physical parameters  $\hat{x}$  and  $\chi$  only, as we have been considering.

We check the validity of this statement in Fig. 3.2, where we show the HO term (dotted lines) and the two first contributions of the whole spectrum HO+NLO (dotted lines) for several values of  $\chi$  and  $a$ , plotted as a function of the rescaled gluon energy  $\hat{x}$ . From left to right we have increasing values of  $\chi$  (or, equivalently,  $n_0L$ ) and different colors represent different  $a$ 's. We see that, even though the LO term is very sensitive to changes in  $a$  for any  $\chi$ , when considering the sum of both contributions the results are more robust for  $\chi$  values big enough. We have found that this situation holds for  $\chi \sim 14$  or larger. Given that from now on we are never going to work with any of the contributions independently, always considering their sum, we set the parameter  $a$  to  $a = 1$ , being careful to not take in consideration results for low  $\chi$  values.

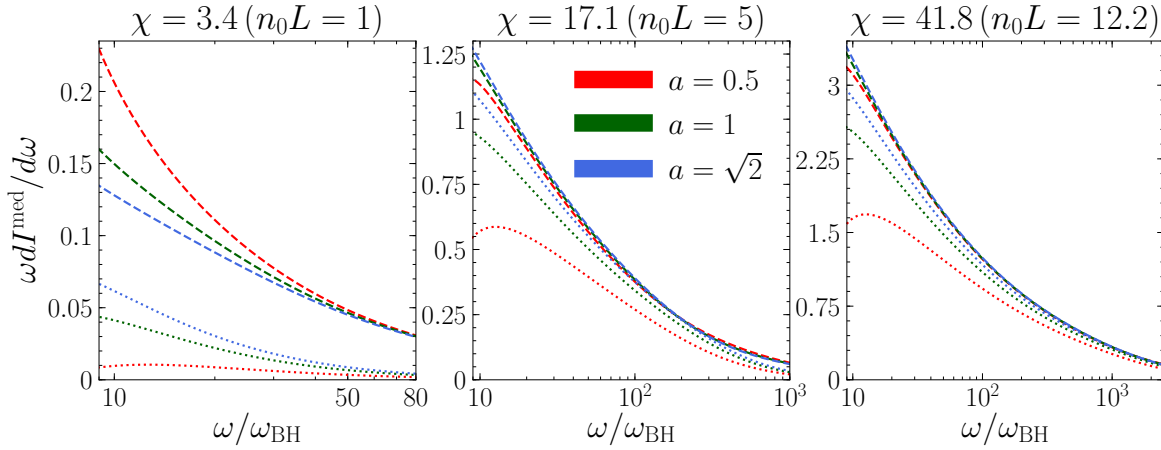


Figure 3.2: Left plot: HO (dotted lines) and HO+NLO (dashed lines) contributions to the energy spectrum as a function of the rescaled gluon energy  $\hat{x} = \omega/\omega_{\text{BH}}$  employing  $\chi = 3.4$  (or  $n_0L = 1$ ) and for several values of the parameter  $a$  which sets the matching scale  $Q^2 = aQ_c^2$ . Central plots: same as previous case using  $\chi = 17.1$  (or  $n_0L = 5$ ). Right plot: same as previous cases with  $\chi = 41.8$  (or  $n_0L = 12.2$ ). Figure extracted from [2] under a Creative Commons license, CC-BY 4.0.

It is important to comment that, as we mentioned before, there are higher order

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contributions to the spectrum which would provide a more accurate description of the gluon emission. These terms were computed in [92] but here we are not going to consider them for two reasons. First, they are given by more complex formulae, which require longer computation time, thus going against the aim of using approximations in order to get simple equations that are easy to evaluate instead of computing the full solution. Second, and most importantly, in [92] the authors discuss in detail the relative contribution of the NNLO correction compared with respect to HO+NLO, claiming that its effect is small. Therefore, the HO+NLO spectrum should provide a good approximation of the full result, which we will explicitly check by comparing both spectra.

Finally, we also remark that the introduction of  $Q_c^2$  was done to regularize and UV divergent integration (it plays the role of  $q_{max}$  in (3.20)), thus, it has to be big enough to fulfill this duty. This feature imposes an even more stringent limitation on the minimum energy value  $\hat{x}$  where we can employ this formalism compared to the constraint arising from the inability to solve (3.23) and (3.32) for  $\hat{x} < 2e$ . We have found that results can only be trusted for energies larger than  $\hat{x} > 10$ , which we will employ as lower limit in all of the numerical results that we will provide.

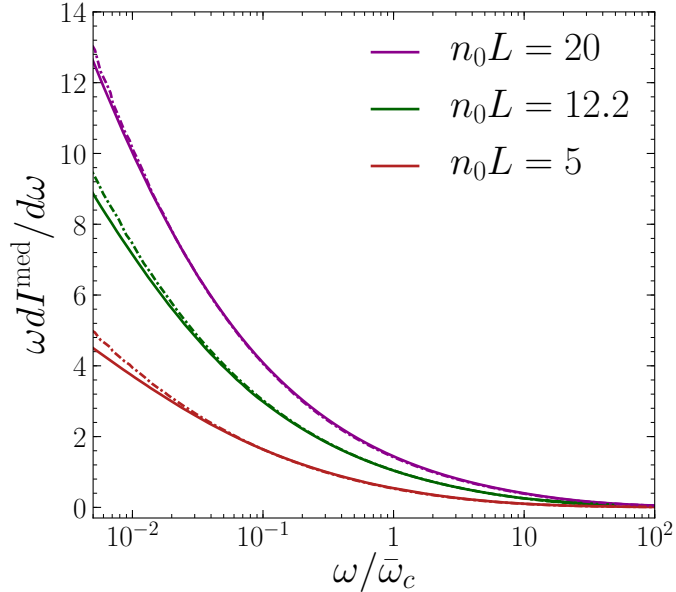


Figure 3.3: All-order in-medium radiation energy spectrum for the Yukawa (solid lines) and HTL (dashed dotted) potentials plotted as a function of the rescaled energy  $x = \omega/\bar{\omega}_c$ . Different colors represent different values for  $n_0 L$ : magenta corresponds to  $n_0 L = 20$  (equivalent to  $TL = 8$ ), green corresponds to  $n_0 L = 12.2$  (or  $TL = 5$ ), and red corresponds  $n_0 L = 5$  (or  $TL = 2$ ). Figure extracted from [2] under a Creative Commons license, CC-BY 4.0.

To establish comparisons between both formalisms, IOE at HO+NLO accuracy and the spectrum fully resummed over multiple scatterings, it is mandatory to provide a set

of relations between the parameters of each framework. The parameters of the spectrum depend on the particular collision rate model employed. In section 3.2 we used both HTL and Yukawa-type interaction potentials to discuss the low energy regime of the spectrum. However, in the intermediate energy region we are working now, the outcome of both models is very similar, as the biggest differences in their behavior are located for small momentum transfers. We show this explicitly in Fig. 3.3, where we illustrate comparison between spectra computed with the Yukawa and HTL collision rates for different values of  $n_0L$  and  $TL$ . The mapping between parameters of both models was done employing the leading-logarithmic mapping derived in [92] for small dipole sizes, which was also used in [58]

$$m_D^2 = e\mu^2, \quad TL = \frac{n_0L}{e\alpha_s N_c}, \quad \bar{\omega}_c^H = e\bar{\omega}_c, \quad \text{and} \quad x_H = \frac{x}{e}. \quad (3.39)$$

We clearly see in Fig. 3.3 that for moderate and high energies  $\omega \geq 2 \cdot 10^{-2}\bar{\omega}_c$  the agreement between spectra computed using the Yukawa and HTL collision rates is excellent. Therefore, in the region of interest for this section we can just employ one of them and the results and conclusions will hold for the other as well. We chose, for simplicity, to use only the Yukawa interaction model in numerical evaluations.

Then, we have to provide a relation between the parameters employed in the full numerical evaluation of the spectrum and the IOE at HO+NLO accuracy for the Yukawa collision rate. We follow again Ref. [92], which tells us that we should identify

$$\hat{q}_0 = n_0\mu^2, \quad \text{and} \quad \mu^{*2} = C\mu^2, \quad (3.40)$$

with

$$C = \frac{e^{-1+2\gamma_E}}{4} \simeq 0.29. \quad (3.41)$$

These relations allow us to establish the following correspondence between the variables employed to evaluate the full spectrum,  $n_0L$  and  $x = \omega/\bar{\omega}_c$  (2.57), and those entering the computation of the HO+NLO approximation,  $\chi$  and  $\hat{x}$  (3.30)

$$\hat{x} = \frac{n_0L}{C^2} x, \quad \text{and} \quad \chi = \frac{n_0L}{C}. \quad (3.42)$$

Unlike the HO approximation, with this approach we are able to employ this one-to-one mapping of the parameters (3.42) to establish quantitative comparisons between different formalisms.

In Fig. 3.4 we illustrate comparison between our all-order fully resummed approach (magenta solid lines), the IOE at HO+NLO precision (green dashed lines) and the first opacity  $N = 1$  approximation (blue dashed dotted lines), computed through eq. (2.56). From left to right we present increasing values of  $n_0L$  (or  $\chi$ ), being the smallest value just above the border for which the HO+NLO result is independent of the chosen matching scale (see Fig. 3.2) and the biggest value is set to coincide with the value of the parameters

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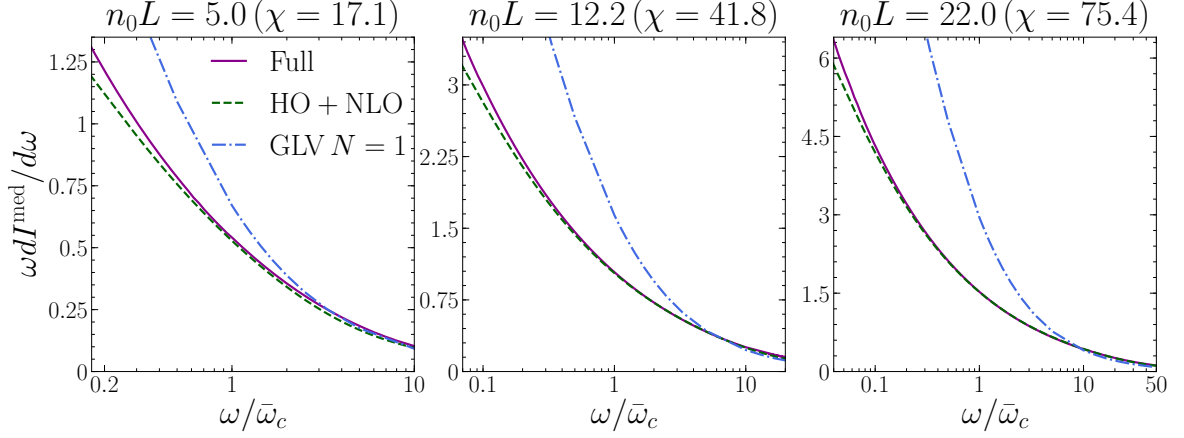


Figure 3.4: Medium-induced energy spectrum with all-order resummation of multiple scatterings for the Yukawa collision rate (magenta solid lines) compared to the IOE at HO+NLO accuracy (green dashed) and GLV  $N = 1$  approximation (blue dashed dotted) shown as a function of the rescaled gluon energy  $x = \omega/\bar{\omega}_c$  employing different values of the opacity  $n_0 L$  (or  $\chi$ ). Figure extracted from [2] under a Creative Commons license, CC-BY 4.0.

employed in Ref. [92]. The energy range of every panel was selected to show the region where the IOE framework can be employed ( $\hat{x} > 10$ ) and multiple scatterings are thought to be essential for the emission process. We see in all of the three plots that in the high energy region there is a perfect agreement between all formalisms, as it should. This is especially relevant because the IOE encodes the correct high energy behaviour of the spectrum, which other approximations that account for multiple scatterings, like the HO, can not reproduce. In [58] it is performed a comparison between the HO and full result where this feature is well illustrated.

Focusing now on the intermediate energy regime that we are studying in this section, we see in Fig. 3.4 that for all cases the HO+NLO approach is a good approximation of the all-order full resummed spectrum, whereas that the first opacity result highly overestimates it. Rising the value of  $n_0 L$  increases the range of applicability of the IOE approximation (remember that  $n_0 L$  and  $\chi$  are essentially the same (3.42) and  $\chi$  directly measures how big the multiple scattering region is (3.30)) at the same time that reduces differences between the HO+NLO and full result. These results not only check the validity of the IOE approach, but also expose the crucial importance of correctly accounting for multiple scatterings below the high energy region of the spectrum to achieve a precise description of the gluon emission process. Our numerical results explicitly illustrate the key role that multiple scatterings play in in-medium gluon radiation.

### 3.3.2 In-medium emission rates

We conclude the study of the multiple scattering region by adopting an inverted perspective compared to our previous approach. Rather than fixing the opacity ( $n_0L$  or  $\chi$ ) while varying the energy ( $x$  or  $\hat{x}$ ), we now set  $\hat{x}$  as constant and treat  $\chi$  as the variable of interest. Equivalently, we are interested in studying the spectrum as function of the medium length  $L$ , which is easier to achieve by employing the emission rates instead of the spectrum itself. These rates are commonly defined as the derivative of the spectrum with respect to  $L$

$$\omega \frac{d^2 I}{d\omega dL}. \quad (3.43)$$

Instantaneous emission rates inside the medium can be interpreted as the probability that a soft gluon is radiated off a hard parton at a given time. However, as we mentioned before, gluons have an extended formation time that is not zero (1.39). Therefore, their emission can not be attributed to any given time coordinate [89]. Thus, this particular interpretation of the emission rates is only valid for formation times much smaller than the medium length, which is an assumption employed in the AMY formalism, as we discussed in section 1.3. Using this framework it is possible to include multiple emissions through rate equations, as it is done, for example, in MARTINI [103], a Monte Carlo event generator. Interpreting emission rates as the probability of radiating gluons at concrete times would be possible in the low energy region of the spectrum too, as in this regime the formation times are very small. Nevertheless, this is the part of the spectrum that has the smallest contribution in phenomenological studies, so it has not been considered physically interesting until now.

Medium-induced emission rates provide us with very interesting information about the importance of multiple scatterings. In the case of short media, where a single scattering should work well, the first opacity result predicts a linear growth of the emission rates, whereas that for the opposite limit, in the asymptotic case of an infinitely large medium  $L \rightarrow \infty$  the rates saturate to the AMY result computed in [104]. These two properties of the emission rates have already been confirmed in [89]. Here we are going to compute the rates of the all-order fully resummed spectrum, which are equivalent to the ones evaluated in [89], and compare them to the rates obtained from the IOE at HO+NLO accuracy.

Employing the full resummed result explained in chapter 2 the computation of the emission rates is very simple. As in eq. (2.35) the medium length only appears in the upper limit of the  $s$  integration (remember that the  $s$  variable is rescaled by  $L$ ), performing the derivative of this expression with respect to  $L$  corresponds to just not computing the integral over the position of the last scattering center  $s$ .

For the IOE we only need to derive the analytical expressions of the LO and NLO contributions. There are two equivalent ways of proceeding: either deriving with respect to  $L$  the spectrum before we rescale it (eqs. (3.25) and (3.27)) or taking the rescaled spectrum (eqs. (3.34) and (3.37)) and deriving it with respect to  $\chi$ . If expressed in terms of the same variables the outcome of both methods is exactly the same. We follow the

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latter approach, as it is further easy and quick than the former. Deriving (3.34) with respect to  $\chi$  yields for the HO contribution (recall that we are working with  $a = 1$ )

$$\hat{x} \frac{d^2 I^{\text{HO}}}{d\hat{x} d\chi} = \frac{2\alpha_s C_R}{\pi} \text{Re} \left\{ (i-1) \sqrt{\frac{\Lambda}{2\hat{x}}} \tan \left( (1-i)\chi \sqrt{\frac{\Lambda}{2\hat{x}}} \right) \right\}, \quad (3.44)$$

whereas that considering the derivative of (3.37) leads to the following expression for the NLO term

$$\hat{x} \frac{d^2 I^{\text{NLO}}}{d\hat{x} d\chi} = \frac{\sqrt{2}\alpha_s C_R}{\pi} \text{Re} \left\{ \frac{i-1}{\sqrt{\hat{x}\Lambda}} \int_0^1 \frac{ds}{K^2(s)} \left[ \ln \left( -\frac{(1+i)}{4\sqrt{2}} K^2(s) \right) + \gamma_E \right. \right. \\ \left. \left. + \frac{\chi}{K^2(s)} \frac{dK^2(s)}{d\chi} \left( 1 - \ln \left( -\frac{(1+i)}{4\sqrt{2}} K^2(s) \right) - \gamma_E \right) \right] \right\}, \quad (3.45)$$

with  $K^2(s)$  already defined in eq. (3.38) and its corresponding derivative with respect to the variable  $\chi$  can be written as

$$\frac{dK^2(s)}{d\chi} = -(1-i) \sqrt{\frac{\Lambda}{2\hat{x}}} \left[ \frac{s}{\sin^2 \left( (1-i)\chi \sqrt{\frac{\Lambda}{2\hat{x}}} s \right)} + \frac{1-s}{\cos^2 \left( (1-i)\chi \sqrt{\frac{\Lambda}{2\hat{x}}} (1-s) \right)} \right]. \quad (3.46)$$

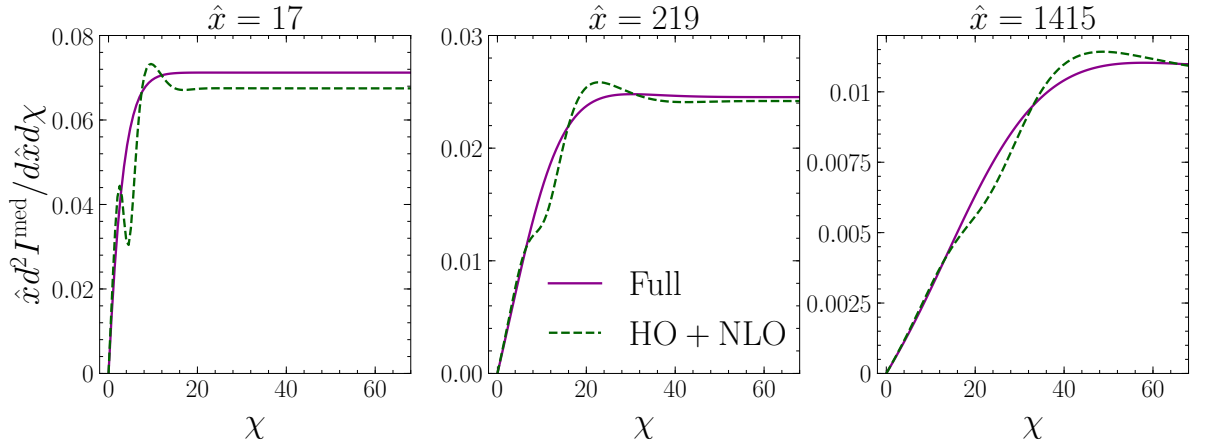


Figure 3.5: In-medium emission rates for the full resummed numerical evaluation of the spectrum (magenta solid lines) and the IOE at HO+NLO approximation (green dashed) as a function of  $\chi$  for several values of the rescaled gluon energy  $\hat{x} = \omega/\omega_{\text{BH}}$ :  $\hat{x} = 17$  (left panel),  $\hat{x} = 219$  (central panel) and  $\hat{x} = 1415$  (right panel). Figure extracted from [2] under a Creative Commons license, CC-BY 4.0.

We show in Fig. (3.5) the results of the evaluation of the emission rates for both formalisms, full resummed spectrum (magenta solid lines) and HO+NLO approximation

(green dashed lines) as a function of  $\chi$  and for several gluon rescaled energies (from left to right increasing values of  $\hat{x}$ ). We confirm in these plots that the rates have the expected behavior: they start growing linearly for small values of  $\chi$  (dilute/small media) and saturate to a constant value for large  $\chi$  (dense/large media). However, the performance of the IOE is more limited compared to the full result, exhibiting oscillations in the transition region between the low and large  $\chi$  limits. The amplitude of these oscillations diminishes as the energy increases, as expected. For large energies, the IOE converges towards the first opacity result, which lacks such oscillations. These results are in agreement with Fig. 3.4, where in the left panel we saw that for low energies and small  $\chi$  value the IOE starts to break. The second important discrepancy we can see in Fig. 3.5 is that, even though both approaches reach saturation at (more or less) the same time, they saturate to different values. The difference between both asymptotic values of the emission rates is bigger for smaller rescaled energies, corroborating again that IOE results in the low energy regime should not be trusted. The agreement between the two frameworks is much better for larger energies, however, in this case the asymptotic value of the rates is reached for very large  $\chi$  values, as it is illustrated in the right plot.

We can compute the asymptotic value of the emission rates in both formalisms and compare them. We start with the rates of the all-order fully resummed result. Following the discussion done in chapter 2, the full BDMPS-Z medium-induced energy distribution without kinetic cut-off ( $\bar{R} \rightarrow \infty$ ) for a brick, eqs. (2.35)-(2.37), can be written in a more compact way as (see [2] for details)

$$x \frac{dI^{\text{med}}}{dx} = 4\alpha_s C_R \text{Re} \int_0^1 ds \int_0^s dt \int_{\mathbf{p}} i\mathbf{p} \cdot \mathbf{F}_x(t; \mathbf{p}), \quad (3.47)$$

with  $\mathbf{F}_x(t; \mathbf{p})$  a 2-dimensional function satisfying the following differential equation

$$\partial_t \mathbf{F}_x(t; \mathbf{p}) = -ip^2 \mathbf{F}_x(t; \mathbf{p}) - \frac{1}{2} n_0 L \int_{\mathbf{p}'} \tilde{\sigma}(\mathbf{p}' - \mathbf{p}; x) \mathbf{F}_x(t; \mathbf{p}'), \quad (3.48)$$

having as initial condition

$$\mathbf{F}_x(0; \mathbf{p}) = n_0 L \int_{\mathbf{q}} \frac{\mathbf{q}}{q^2} \tilde{\sigma}(\mathbf{q} - \mathbf{p}; x), \quad (3.49)$$

where  $\tilde{\sigma}(\mathbf{q}; x)$  is the rescaled version of the dipole cross section

$$\tilde{\sigma}(\mathbf{q}; x) = x\mu^2 \sigma(\mathbf{q} \sqrt{x\mu^2}), \quad (3.50)$$

and with  $x$  denoting the previously defined rescaled gluon energy  $x = \omega/\bar{\omega}_c$ . Performing the derivative of eq. (3.47) with respect to  $L$  and employing our dictionary (3.42) we see that the rates of the full resummed spectrum in terms of  $\chi$  and  $\hat{x}$  are given by

$$\hat{x} \frac{d^2 I}{d\hat{x} d\chi} = 4\alpha_s C_R \text{Re} \int_0^1 dt \int_{\mathbf{p}} i\mathbf{p} \cdot \mathbf{F}_{\hat{x}}(t; \mathbf{p}) \frac{1}{C}, \quad (3.51)$$

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satisfying  $\mathbf{F}_{\hat{x}}$  the following differential equation

$$\partial_t \mathbf{F}_{\hat{x}}(t; \mathbf{p}) = -ip^2 \mathbf{F}_{\hat{x}}(t; \mathbf{p}) - \frac{1}{2} C^2 \int_{\mathbf{p}'} \tilde{\sigma}(\mathbf{p}' - \mathbf{p}; \hat{x}) \mathbf{F}_{\hat{x}}(t; \mathbf{p}'), \quad (3.52)$$

with initial condition

$$\mathbf{F}_{\hat{x}}(0; \mathbf{p}) = C^2 \int_{\mathbf{q}} \frac{\mathbf{q}}{q^2} \tilde{\sigma}(\mathbf{q} - \mathbf{p}; \hat{x}), \quad (3.53)$$

and  $\tilde{\sigma}(\mathbf{q}; \hat{x})$  defined as

$$\tilde{\sigma}(\mathbf{q}; \hat{x}) = \hat{x} \mu^2 \sigma(\mathbf{q} \sqrt{\hat{x} \mu^2}). \quad (3.54)$$

To compute the  $L \rightarrow \infty$  asymptotic limit of the rates we just need to perform the  $t$  integration from 0 to  $\infty$  (remember that  $t$  is rescaled as  $tL$ ). The only dependence on  $t$  is contained in  $\mathbf{F}_{\hat{x}}(t; \mathbf{p})$ , so we simply integrate its differential equation (3.52)

$$\mathbf{F}_{\hat{x}}(\infty; \mathbf{p}) - \mathbf{F}_{\hat{x}}(0; \mathbf{p}) = -ip^2 \int_0^\infty dt \mathbf{F}_{\hat{x}}(t; \mathbf{p}) - \frac{1}{2} C^2 \int_{\mathbf{p}'} \tilde{\sigma}(\mathbf{p}' - \mathbf{p}; \hat{x}) \int_0^\infty dt \mathbf{F}_{\hat{x}}(t; \mathbf{p}'), \quad (3.55)$$

employing  $\mathbf{F}_{\hat{x}}(\infty; \mathbf{p}) = 0$ , eq. (3.53) and defining

$$\mathbf{G}_{\hat{x}}(\mathbf{p}) = \int_0^\infty dt \mathbf{F}_{\hat{x}}(t; \mathbf{p}) \frac{1}{C}, \quad (3.56)$$

the asymptotic limit of the full in-medium emission rates is given by

$$\lim_{\chi \rightarrow \infty} \hat{x} \frac{d^2 I}{d\hat{x} d\chi} = 4\alpha_s C_R \operatorname{Re} \int_{\mathbf{p}} i\mathbf{p} \cdot \mathbf{G}_{\hat{x}}(\mathbf{p}), \quad (3.57)$$

with  $\mathbf{G}_{\hat{x}}(t; \mathbf{p})$  satisfying the following integral equation

$$\int_{\mathbf{q}} \frac{\mathbf{q}}{q^2} \tilde{\sigma}(\mathbf{q} - \mathbf{p}; \hat{x}) = \frac{ip^2}{C} \mathbf{G}_{\hat{x}}(\mathbf{p}) + \frac{1}{2} C \int_{\mathbf{p}'} \tilde{\sigma}(\mathbf{p}' - \mathbf{p}; \hat{x}) \mathbf{G}_{\hat{x}}(\mathbf{p}). \quad (3.58)$$

This equation can be computed with numerical methods and it is in agreement with the soft limit of the AMY result, which can be seen comparing with eqs. 6 and 7 of Ref. [104].

On the other hand, for the analytical expansion we just need to take the  $\chi \rightarrow \infty$  limit of the previously calculated rates (3.44) and (3.45). Regarding the LO contribution, the  $\chi \rightarrow \infty$  limit of the tan function yields  $-i$ , thus, after taking the real part we get

$$\lim_{\chi \rightarrow \infty} \hat{x} \frac{d^2 I^{\text{HO}}}{d\hat{x} d\chi} = \frac{\alpha_s C_R}{\pi} \sqrt{\frac{2\Lambda}{\hat{x}}}. \quad (3.59)$$

Now focusing on the NLO term, all of the dependence on  $\chi$  is contained in the  $K^2(s)$  function (3.38). Taking the  $\chi \rightarrow \infty$  limit of this expression we get

$$\lim_{\chi \rightarrow \infty} K^2(s) = 2i, \quad (3.60)$$

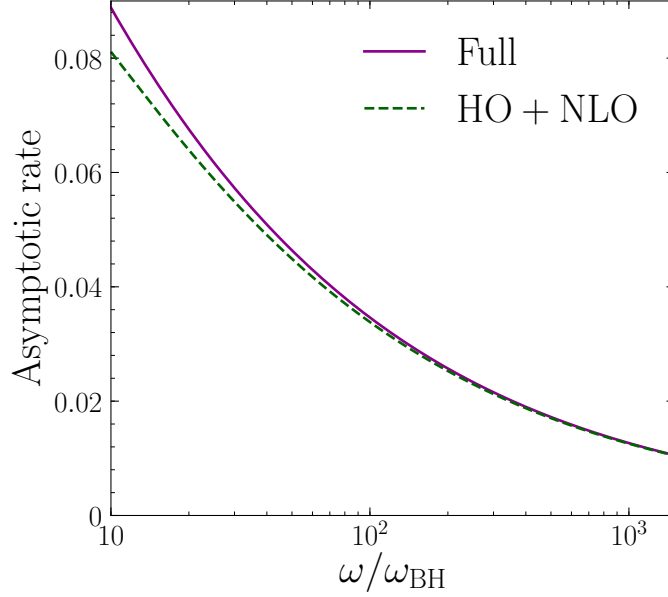


Figure 3.6: Asymptotic  $\chi \rightarrow \infty$  limit of the emission rates for the fully resummed evaluation (magenta solid line) and the IOE at HO+NLO accuracy (green dashed line) as a function of the energy  $\hat{x} = \omega/\omega_{\text{BH}}$ . Figure extracted from [2] under a Creative Commons license, CC-BY 4.0.

and, therefore, the second line of (3.45) vanishes, as  $K^2(s)$  is constant, and we only have the contribution of the first line. The integral over  $s$  is trivially 1 and we take the real part of the logarithm, yielding the final result

$$\lim_{\chi \rightarrow \infty} \hat{x} \frac{d^2 I^{\text{NLO}}}{d\hat{x} d\chi} = \frac{\alpha_s C_R}{\pi} \frac{1}{\sqrt{2\Lambda\hat{x}}} \left[ \gamma_E - \ln 2 + \frac{\pi}{4} \right]. \quad (3.61)$$

We plot in Fig. 3.6 results for the asymptotic  $\chi \rightarrow \infty$  limit value of the rates for both approaches, all-order evaluation (magenta solid line) and HO+NLO approximation (green dashed line) as a function of the gluon rescaled energy  $\hat{x} = \omega/\omega_{\text{BH}}$ . These results are in agreement with our previous findings: the analytical expansion of the spectrum agrees with the full result for large gluon energies ( $\hat{x} > 200$ ). The situation corresponding to this high energy regime was shown in center and right plots of Fig. 3.5, where we see that the agreement between both frameworks is reached at very large values of  $\chi$  (or  $n_0 L$ ).

### 3.4 Full picture of the the emission process

The final section of this chapter serves as a summary of the results presented in previous sections 3.2 and 3.3. Whereas that in these sections we focused on analyzing every energy

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region separately, here we provide the complete description that we have nowadays of the emission process across the different energy regimes.

Starting with the high energy region of the spectrum ( $\omega > \omega_c$ ), we showed explicitly in an analytical way in section 3.2 that the first opacity result is indeed the high energy limit of the full BDMPS-Z spectrum, as it was already numerically confirmed in [58]. One single hard scattering is enough to properly describe the whole emission process. In this regime the agreement between the GLV spectrum and full solution is excellent, and we can simply employ the former as it is really fast to compute.

Moving on to lower energies, in the intermediate energy regime ( $\omega_{\text{BH}} < \omega < \omega_c$ ) the common assumption used to be that the LPM effect would gain importance, resulting in a suppression of the spectrum and, therefore, accounting for multiple scatterings would be essential in this region. We checked this statement by comparing the full spectrum with the IOE in section 3.3, resulting in a great agreement between both frameworks (on the range of applicability of the IOE:  $\omega > 10\omega_{\text{BH}}$  and  $\chi > 14$ ). This is very important because, until now, we only could perform qualitative comparisons with formalisms that take in account multiple scatterings (see [58]), but here we compared both approaches in a quantitative way.

Finally, for the small energy BH regime ( $\omega < \omega_{\text{BH}}$ ) we derived for the first time the correct low energy limit of the complete spectrum. Comparisons between our newly derived result (3.11) and the full solution were performed in section 3.2 employing different parton-medium interaction models with different small momentum behaviours, resulting in an excellent agreement for energies low enough. The physical interpretation of the radiation process in this region is that we have the first opacity  $N = 1$  spectrum, i.e., just one scattering, multiplied by a no-more scattering probability factor coming from the resummation of the contribution of virtual terms, which suppresses the GLV limit.

We show in Fig. 3.7 a summary of all of these results. Here we plot the full medium-induced gluon radiation spectrum (magenta solid line) as a function of the rescaled gluon energy  $\omega/\omega_{\text{BH}}$  for a given value of  $n_0L$  (or  $\chi$ ) compared with the GLV  $N = 1$  result (blue dashed dotted line), the HO+NLO approximation (green dashed line) and the low energy limit of the spectrum (black dotted line) for a Yukawa-type interaction model. We clearly see the features we just mentioned: the correct description of the spectrum employing one single hard scattering in the large energy limit, the importance of accounting for multiple scatterings for intermediate energies and the good performance achieved by our new result in the small energy region.

We notice, however, that the analytical understanding of the whole spectrum is not complete yet, there is a small region among  $\omega \sim 3\omega_{\text{BH}}$  and  $\omega \sim 10\omega_{\text{BH}}$  where the IOE can not be applied due to fundamental problems and the efficiency of our small energy result is not good enough. This problem was partially solved in [105] where the authors derived a better expression of the spectrum for low  $\omega$ , which they call Resummed Opacity Expansion (ROE). The first term in this expansion coincides with our result presented here (3.11) and higher orders correct its behaviour for not-so-low energies. Our results are in agreement with findings in Ref. [105], as it can be seen by comparing Fig. 3.7 with Figure

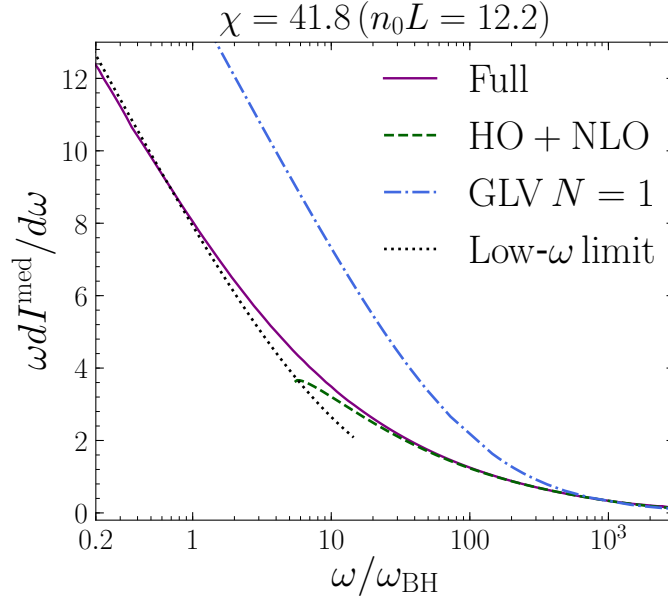


Figure 3.7: Fully resummed in-medium gluon energy distribution (magenta solid line) compared to the analytical expansion at HO+NLO approximation (green dashed line), the first opacity  $N = 1$  GLV spectrum (blue dashed dotted line), and the low energy limit of the all-order evaluation described in section 3.2 (black dotted line) as a function of the gluon rescaled energy  $\hat{x} = \omega/\omega_{\text{BH}}$ . Figure extracted from [2] under a Creative Commons license, CC-BY 4.0.

7 of [105]. However, there are some details one needs to be careful with. First, in [105] for dealing with the problematic border between the small and intermediate energy regions the authors extrapolate the results of the IOE below its range of applicability, where it is not well defined, to be able to match the result for the small  $\omega$  regime. In numerical evaluations they also employ only the first order in the ROE, which is just our low- $\omega$  limit (3.11), instead of trying to achieve a better agreement with the full spectrum. This results in having discrepancies with respect to the all-order solution, as it can be seen in the ratio plots of Figure 7 [105].

# 4

## Beyond the brick: full spectrum in evolving media

To comply with the regulations for doctoral studies of the University of Santiago de Compostela we include here the full information of the article on which this chapter is based:

In-medium gluon radiation spectrum with all-order resummation of multiple scatterings in longitudinally evolving media, *arXiv* 2307.06226 (2023)

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Active participation in all the calculations presented in the paper, as well as in the discussions, meetings and writing of the paper. All of the authors have the same contribution to the publication.

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All of the results derived so far in chapters 2 and 3 were obtained within a non-physical approximation, the consideration that the medium is a static collection of scattering centers, which is usually called the brick. But, this approximation is too simplistic, as we know that hydrodynamical simulations describe accurately the behavior of the QGP. Therefore, to perform high precision phenomenological studies able to reproduce the increasingly precise experimental results we need to keep improving the framework and overcome the brick approximation, computing the spectrum for general temporal dependencies of the medium density  $n(t) \neq n_0\theta(L-t)$  and screening mass  $\mu(t) \neq \mu_0\theta(L-t)$ . As the title of the current thesis states, studying this temporal structure of the QCD

collectivity is one of its main goals. Thus, in this chapter we will discuss in detail how to evaluate the fully resummed spectrum for longitudinally evolving media described by realistic hydrodynamic models. The findings of this chapter are published in [3].

Computing the radiation spectrum with full resummation over multiple scatterings is straightforward and it can be accomplished following the discussion presented in chapter 2. Nevertheless, this approach requires a fair amount of time to provide results for the evaluation of the spectrum, even for the brick. Furthermore, including more general temporal dependencies on the medium parameters is expected to increase the computational power needed, as in the static case there are some symmetries that can be exploited to simplify the differential equations we have to solve numerically, and, consequently, the framework is inefficient for practical implementations. Hence, we dedicate this chapter to find a procedure for evaluating efficiently the spectrum in general realistic conditions.

The first part of the chapter discusses in more detail this problem and explains the method we will follow to overcome this limitation: the use of scaling laws, simple relations between the parameters of the spectrum which allow us to identify different scenarios. With these scaling laws we can pre-tabulate the spectrum in a reference case and identify the results with the spectrum computed in general conditions. Then, we evaluate numerically the fully resummed spectrum including a variable medium density  $n(t)$  coming from a hydrodynamical model and employing different approximations (Yukawa and HTL). This is done for two key reasons. Firstly, it demonstrates the applicability of our approach in these kind of realistic conditions. Secondly, it establishes a baseline against which we can compare the outcomes derived from the scaling laws, allowing us to assess their performance. Analyzing different options for scaling laws we find one which accurately approximates the full solution of the spectrum in all energy regimes and even in extreme scenarios, enabling practical implementations of the framework. Furthermore, we also study the effect of fluctuations event-by-event in the hydrodynamical simulation, concluding that their importance is minimal. With all of these results, the formalism is ready to be implemented in phenomenology.

## 4.1 Motivation to the use of scaling laws

The next step after the emergence of new theoretical achievements is their inclusion in phenomenological studies. For that purpose, it is necessary to establish a correspondence between the parameters used to describe the properties of the medium ( $n, \mu, \hat{q}...$ ) and the outcomes of hydrodynamic models (temperature, energy density...). However, the establishment of this relationship between parameters introduces further complications when trying to compute the medium-induced radiation spectrum, which are especially relevant for frameworks that resum multiple scatterings. The main problem is that in the static case there are some symmetries that can be exploited to simplify the numerical computation of the time integrals of the spectrum (1.38), but when we have time dependencies  $n(t), \mu(t)$  those symmetries can not be applied and we have to perform the calculation for

every time step. Consequently, the computational time required to reach the evaluation of the spectrum highly increases.

This is the main reason why many theoretical calculations are first carried out in the brick approximation before being implemented for dynamic media. Actually, analytical results for the emission spectrum beyond the brick approximation which include multiple scatterings have only been achieved for the HO approach for specific functional forms of  $\hat{q}$  [106–109]. As we discussed in chapter 3, the IOE takes as input the HO approach, thus some of these findings remain applicable in the context of the IOE framework [91, 92]. However, going beyond these approximations with evolving media is a tough task which requires large computational times to achieve accurate results, as we have to numerically perform the calculations for every time step. This feature is especially disadvantageous for phenomenological studies that require the evaluation of the medium-induced radiation spectrum for a great number of different trajectories of the emitter through the QGP.

The solution of this problem is not obvious at all. It would be very convenient to have the spectrum pre-computed for a wide range of the values of the parameters, thus every time we need to evaluate the spectrum for some particular conditions we just read the correspondent pre-evaluated result. Nonetheless, the spectrum’s dependence on the parameters’ values at each point of an arbitrary trajectory of the emitter results in the impossibility of pre-tabulating the spectrum for a finite number of trajectories which can be used to evaluate a general one. In order to overcome this limitation, in [70, 110] it was proposed the employment of a set of scaling laws which establish a correspondence between a generic trajectory in a medium and an equivalent static scenario. The performance of these scaling laws was tried out against the limited available analytical approaches at the time, which essentially include the HO for multiple soft scatterings and the first opacity result for the single scattering regime.

Here we aim to evaluate our spectrum fully resummed over multiple scatterings which was explained in chapters 2 and 3 in a realistic hydrodynamical description of the medium, so it can be employed in phenomenological analysis. The first objective is easy to achieve, as the method is straightforward to extend from static evaluation to an expanding medium, and we show it explicitly in the next section. However, the employment in phenomenology would be very limited, as the computational time needed to compute the spectrum in many different trajectories would be large. Therefore, we will explore the use of scaling laws in subsequent sections that would allow us to pre-compute the full spectrum in a reference scenario, thus relieving the computational power needed and enabling the implementation in phenomenological studies.

## 4.2 All-order full resummed medium-induced radiation spectrum in longitudinally evolving media

The starting point of our discussion is the in-medium gluon radiation spectrum differential in transverse momentum presented in section 2.2, eq. (2.6)

$$\omega \frac{dI^{\text{med}}}{d\omega d^2\mathbf{k}} = \frac{\alpha_s C_R}{2\pi^2 \omega} \text{Re} \int_0^L ds n(s) \int_0^s dt \int_{pql} i\mathbf{p} \cdot \left( \frac{\mathbf{l}}{l^2} - \frac{\mathbf{q}}{q^2} \right) \times \sigma(s; \mathbf{l} - \mathbf{q}) \tilde{\mathcal{K}}(s, \mathbf{q}; t, \mathbf{p}) \mathcal{P}(L, \mathbf{k}; s, \mathbf{l}), \quad (4.1)$$

where we have written explicitly the dependence of the dipole cross section on the time variable  $s$ . As we know, the dipole cross section can be written in terms of the elastic collision rate through eq. (1.32)

$$\sigma(s; \mathbf{k}) = -V(s; \mathbf{k}) + (2\pi)^2 \delta^{(2)}(\mathbf{k}) \int_{\mathbf{q}} V(s; \mathbf{q}), \quad (4.2)$$

which is also the time dependent version of the previously introduced equation. Now we have to specify some model for the parton-medium interaction. As we have done throughout the development of the thesis, we are going to use as primary model the Yukawa-type collision rate (1.26)

$$V(s; \mathbf{q}) = \frac{8\pi\mu^2(s)}{(\mathbf{q}^2 + \mu^2(s))^2}, \quad (4.3)$$

where, again, we are writing explicitly the dependence on time variables. Results employing the HTL potential (1.27) are also presented in appendix 4.B to illustrate the flexibility of our approach and to not hinder the development of the main manuscript, as the conclusions we will extract from both interaction models are exactly the same.

The broadening  $\mathcal{P}$  and emission kernel  $\tilde{\mathcal{K}}$  in (4.1) follow the same differential equations with the same initial conditions as before (2.7)-(2.8) and (2.12)-(2.13), respectively, and they are also computed following the same procedure described in chapter 2.

The energy distribution is obtained by integrating the transverse momentum dependent spectrum (4.1) over  $\mathbf{k}$

$$\omega \frac{dI^{\text{med}}}{d\omega} = \int_0^\omega k dk \int_0^{2\pi} d\theta_k \omega \frac{dI^{\text{med}}}{d\omega d^2\mathbf{k}}, \quad (4.4)$$

here we have kept the kinematical constraint finite  $k \leq \omega$ , but as in previous sections, we also explore the  $\bar{R} \rightarrow \infty$  case due to its theoretical relevance, even though it presents non-physical divergences for small energies and should not be employed in phenomenology. For this particular case where the momentum integration is performed over all transverse

momentum space the broadening vanishes, obtaining the following expression for the energy spectrum

$$\omega \frac{dI^{\text{med}}}{d\omega} = \frac{2\alpha_s C_R}{\omega} \text{Re} \int_0^L ds n(s) \int_0^s dt \int_{pql} i\mathbf{p} \cdot \left( \frac{\mathbf{l}}{l^2} - \frac{\mathbf{q}}{q^2} \right) \sigma(s; \mathbf{l} - \mathbf{q}) \tilde{\mathcal{K}}(s, \mathbf{q}; t, \mathbf{p}) . \quad (4.5)$$

Now we realize that the emission kernel  $\tilde{\mathcal{K}}$  does not depend on the dummy momentum variable  $l$ , so the integration over  $l$  of the second term in brackets vanishes, yielding

$$\omega \frac{dI^{\text{med}}}{d\omega} = \frac{2\alpha_s C_R}{\omega} \text{Re} \int_0^L ds n(s) \int_0^s dt \int_{pql} i \frac{\mathbf{p} \cdot \mathbf{l}}{l^2} \sigma(s; \mathbf{l} - \mathbf{q}) \tilde{\mathcal{K}}(s, \mathbf{q}; t, \mathbf{p}) . \quad (4.6)$$

All of these spectra have already been computed in chapters 2 and 3 for a medium described by a brick of length  $L$  with constant linear density of scattering centers  $n(s) = n_0 \theta(L - s)$ , and constant screening mass  $\mu^2(s) = \mu_0^2 \theta(L - s)$ . In this simple scenario the static energy distribution depends only on  $n_0$ ,  $L$  and  $\mu_0^2$ , but for convenience we prefer to employ the following set of parameters (2.57)

$$\chi \equiv n_0 L, \quad \bar{\omega}_c \equiv \frac{1}{2} \mu_0^2 L, \quad \text{and} \quad \bar{R} \equiv \bar{\omega}_c L, \quad (4.7)$$

with  $\chi$  representing the opacity,  $\bar{\omega}_c$  the characteristic gluon frequency and  $\bar{R}$  the dimensionless kinematic constraint. The last parameter is not necessary for the energy spectrum with no kinematical cut-off, as this scenario corresponds to  $\bar{R} \rightarrow \infty$  for  $\bar{\omega}_c$  fixed.

The evaluation of the spectrum for a longitudinally evolving medium requires to solve the differential equations (2.7) and (2.12) for a given evolution of  $n(s)$  and  $\mu^2(s)$  along a specific trajectory, which is feasible within our framework. However, in phenomenology it is necessary to evaluate the spectrum for many different trajectories, each one with a different evolution of the medium parameters and, therefore, the evaluation time required to compute the spectra makes this approach unpractical. This feature is not specific to our formalism, but a common difficulty that arises in any framework which includes resummation over all multiple scatterings [110].

With the aim of overcoming this limitation and make the HO approximation appropriated for phenomenological analyses, in [110] it was proposed a correspondence relation between the in-medium radiation spectrum for a realistic evolving medium and an equivalent static scenario, known as scaling law. With the employment of this scaling law it was possible to pre-tabulate the static spectrum for a wide range of the values of its parameters, which served as an estimation of the spectrum along arbitrary trajectories through the QGP in phenomenological studies. The original scaling law was proposed for soft gluons and more recent publications explore different versions of this scaling to go beyond this approximation [107, 108]. Nonetheless, all of these approaches employ as building block the HO approximation, thus not correctly describing the high energy limit of the full spectrum [58].

The next sections will address the issue of using scaling laws on our all-order full resummed spectrum over multiple scatterings introduced in chapter 2 for a Yukawa-type parton-medium interaction model, with the goal of making our framework suitable for phenomenological studies. We will analyze different matching schemes, finding the one that provides the more accurate description of the spectrum for arbitrary conditions. As always in this theses, for illustration purposes we set  $\alpha_s = 0.3$  and assume the initial hard parton to be a highly energetic quark  $C_R = C_F = 4/3$ .

### 4.3 First attempt of finding scaling laws: static case with average values

First of all, we start by explaining how to implement a hydrodynamical simulation of the medium within our numerical approach. As mentioned in previous sections, we need to relate the parameters  $n(t)$  and  $\mu^2(t)$  with local properties of the medium, as the temperature, along the trajectory followed by the initial hard parton  $\xi(t)$ . We employ the common procedure used on the field and establish the following relationship

$$n_{\text{hydro}}(t) = k_1 T_{\text{hydro}}(\xi(t)), \quad \text{and} \quad \mu_{\text{hydro}}^2(t) = k_2 T_{\text{hydro}}^2(\xi(t)), \quad (4.8)$$

being  $T_{\text{hydro}}$  the temperature of the medium along the path described by the emitter, which is taken from a hydrodynamic model. The first equation in (4.8) is based on the Stefan-Boltzmann limit of the QCD equation-of-state, which establishes the medium volume density to be proportional to  $T^3$ . The second relation is motivated by the leading order result of the hard thermal loop perturbative calculation of the thermal mass of the medium [57]. The values of the proportionality parameters  $k_1$  and  $k_2$  are determined in phenomenological studies by fitting the experimental data for a particular centrality class and, hence, to achieve consistent results here we will assume that for a given centrality class the numerical values of  $k_1$  and  $k_2$  remain constant for every trajectory.

The hydrodynamic simulation we employ to extract the local temperature profiles  $T_{\text{hydro}}(\xi(t))$  is the smooth-averaged 2+1 viscous hydrodynamic model described in Refs. [111, 112]. The starting point of this hydrodynamic model is at proper time  $\tau_{\text{hydro}} = 1$  fm, its initial condition is an energy density proportional to the density of binary collisions and fixes the ratio of shear viscosity to entropy density  $\eta/s$  to  $\eta/s = 0.08$ , being constant for the whole evolution. The simulation considers the medium to be in chemical equilibrium described by an equation of state inspired by Lattice QCD calculations and it sets a freeze-out temperature at  $T_f = 140$  MeV. Further details concerning the description of the hydrodynamic model can be found in [111, 112]<sup>1</sup>. The extraction of the temperature profile

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<sup>1</sup>We notice that in these references it is also provided another hydrodynamical model which employs  $\eta/s = 0.16$  and the initial condition is based on a Color-Glass-Condensate model. We have checked that using this different model does not affect the results we present here. Also, this model will be used in a phenomenological study in chapter 5.

employed to evaluate the medium-induced emission spectrum will be done by considering different representative straight lines for various centrality classes at  $\sqrt{s_{\text{NN}}} = 5.02$  TeV Pb-Pb collisions [113,114]. Moreover, in appendix 4.A we provide results for the radiation spectrum computed along trajectories extracted considering straight lines from the EKRT event-by-event (EbyE) hydrodynamic simulations in [115].

The static radiation spectrum has been computed in a wide variability of conditions, therefore it is the first natural candidate to be taken as reference scenario to find some relation between the values of the parameters of the spectrum that allow us to approximate the full result in an arbitrary case. The employment of the average values of the medium parameters along the path followed by the hard parton was the first option taken in account for this purpose, and it is used in several phenomenological studies. We consider just the  $\bar{R} \rightarrow \infty$  case (4.6) for simplicity, which needs only two parameters to be evaluated (4.7). Taking average values corresponds to identifying

$$\chi = \int_{\tau_{\text{hydro}}}^{L+\tau_{\text{hydro}}} dt n_{\text{hydro}}(t), \quad (4.9)$$

$$\bar{\omega}_c = \frac{1}{2} \int_{\tau_{\text{hydro}}}^{L+\tau_{\text{hydro}}} dt \mu_{\text{hydro}}^2(t), \quad (4.10)$$

with  $L$  the length of the path and  $\tau_{\text{hydro}}$  the initial proper time of the hydrodynamical model.

We show in Fig. 4.1 results for the full spectrum evaluated along a representative trajectory (whose temperature profile is showcased in the inset figure) with a central production point sampled over the 0 – 10% centrality class in  $\sqrt{s_{\text{NN}}} = 5.02$  TeV Pb-Pb collisions at the LHC (solid purple curve) and the correspondent equivalent static scenario (blue dashed curve) computed employing (4.9) and (4.10). On top panels it is represented the spectrum as a function of the rescaled gluon energy  $\omega/\bar{\omega}_c$ , with  $\bar{\omega}_c$  computed by (4.10), and on bottom panels it is plotted the ratio of the equivalent static scenario with respect to the spectrum computed along the actual path shown in the inset figure. The difference between left and right plots is the value of  $k_1$  employed, which is  $k_1 = 0.5$  for left and  $k = 1.0$  for right. Employing (4.9) for the selected trajectory, they correspond, respectively, to  $\chi = 5$  and  $\chi = 10$  in terms of the variables of the static spectrum. Our results clearly show that the equivalent static scenario computed using average values for the parameters of the medium (4.9) and (4.10) overestimates the spectrum evaluated along the trajectory. One could argue that we are studying the spectrum without kinematical constraint  $\bar{R} \rightarrow \infty$  and, therefore, in the physical energy distribution the results would be better. However, the greatest discrepancies in Fig. 4.1 correspond to the high energy region of the spectrum, which remains unmodified after the kinematic cut-off is imposed.

The analysis of different trajectories sampled with other production points and for different centrality classes in  $\sqrt{s_{\text{NN}}} = 5.02$  TeV Pb-Pb collisions at the LHC for many different values of the medium parameters gave as result outcomes in agreement with those

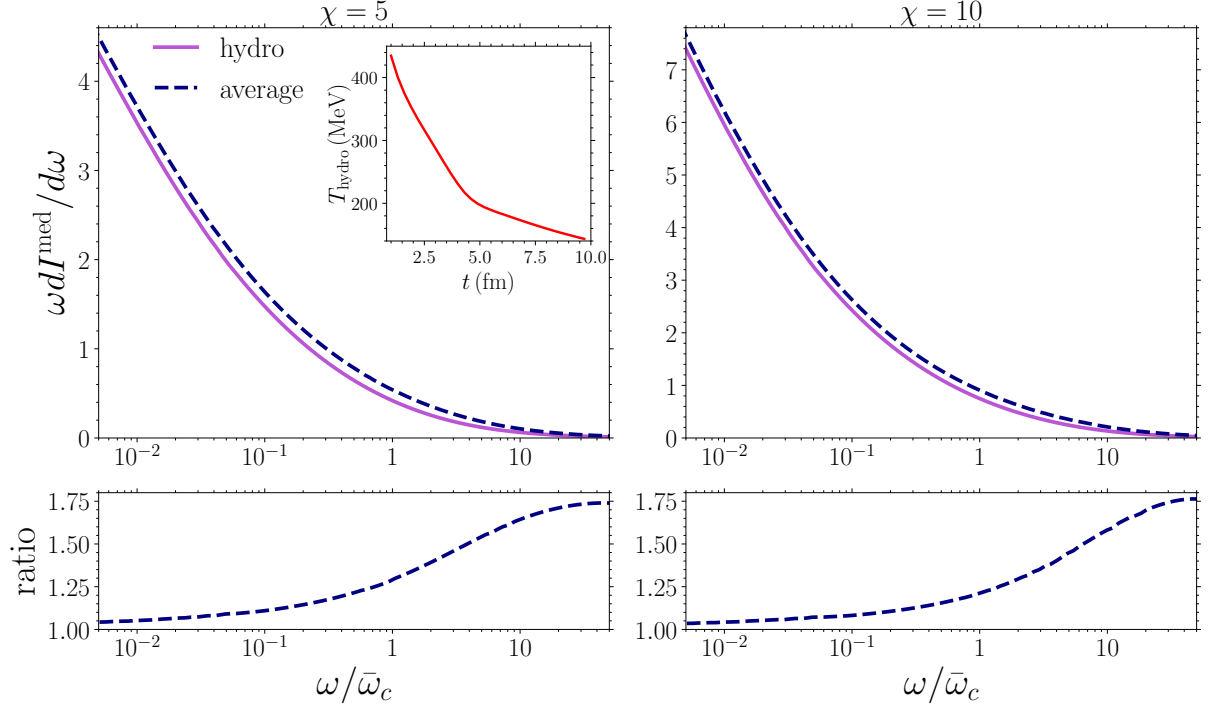


Figure 4.1: Top panels: all order in-medium energy spectrum for a Yukawa-type collision rate with  $\bar{R} \rightarrow \infty$ , and  $\chi = 5$  (left) or  $\chi = 10$  (right) as a function of the rescaled gluon energy  $\omega/\bar{\omega}_c$ . Purple curves correspond to the spectra computed along the trajectory whose temperature profile is shown in the inset figure, which was sampled with a central production point over the 0 – 10% centrality class in  $\sqrt{s_{\text{NN}}} = 5.02$  TeV Pb-Pb collisions at the LHC. Blue dashed curves correspond to the static spectrum evaluated employing average values for the medium parameters (see eqs. (4.9) and (4.10)). Bottom panels: ratio of the static spectrum w.r.t. the spectrum computed with the hydrodynamic model. Figure extracted from [3] under a Creative Commons license, CC-BY 4.0.

shown in Fig. 4.1, thus including them here would hinder the discussion unnecessarily. Therefore, the conclusion is evident: the employment of average values with the static spectrum does not correctly reproduce the spectrum computed along the real path followed by the emitter, being the largest disagreements located at high gluon energies, which was already noticed for the HO approach in [70]. We discuss in the following sections how to properly match the high energy tails of spectra computed in different medium profiles.

## 4.4 Matching the high energy tail of the spectrum

Focusing now on the high energy regime of the energy distribution, as in this region the kinematic condition is not relevant we can use the simpler expression (4.6). Moreover, we showed explicitly in section 3.2 that the large energy limit of the full spectrum corresponds

to the first order in opacity expansion result. Therefore, we can substitute the full in-medium emission kernel in (4.6) by the vacuum kernel

$$\tilde{\mathcal{K}}(s, \mathbf{q}; t, \mathbf{p}) \approx (2\pi)^2 \delta^{(2)}(\mathbf{q} - \mathbf{p}) e^{-i \frac{\mathbf{p}^2}{2\omega} (s-t)}, \quad (4.11)$$

yielding

$$\omega \frac{dI^{\text{med}}}{d\omega} \approx \frac{2\alpha_s C_R}{\omega} \text{Re} \int_0^L ds n(s) \int_{\mathbf{p}l} \frac{2\omega}{\mathbf{p}^2} \left(1 - e^{-i \frac{\mathbf{p}^2}{2\omega} s}\right) \frac{\mathbf{p} \cdot \mathbf{l}}{l^2} \sigma(s; \mathbf{l} - \mathbf{p}). \quad (4.12)$$

The  $\mathbf{l}$ -integral can be done in the same way as we did in section 3.2

$$\int_l \frac{\mathbf{p} \cdot \mathbf{l}}{l^2} \sigma(s; \mathbf{l} - \mathbf{p}) = \int_l V(s; \mathbf{l}) \theta(l^2 - \mathbf{p}^2) \equiv \Sigma(s; \mathbf{p}^2). \quad (4.13)$$

Plugging this into the spectrum and rescaling the last dummy momentum variable we get

$$\begin{aligned} \omega \frac{dI^{\text{med}}}{d\omega} &\approx \frac{2\alpha_s C_R}{\omega} \text{Re} \int_0^L ds n(s) \int_{\mathbf{p}} \frac{2\omega}{\mathbf{p}^2} \left(1 - e^{-i \frac{\mathbf{p}^2}{2\omega} s}\right) \Sigma(s; \mathbf{p}^2) \\ &= \frac{\alpha_s C_R}{\pi} \text{Re} \int_0^L ds n(s) \int_0^\infty \frac{dr}{r} (1 - e^{-ir}) \Sigma(s; 2\omega r/s). \end{aligned} \quad (4.14)$$

$\Sigma$  can be analytically computed for some collision rate models, as for the Yukawa interaction that we are employing now, obtaining (3.17)

$$\Sigma(s; \mathbf{p}^2) = \frac{2\mu^2(s)}{\mathbf{p}^2 + \mu^2(s)}. \quad (4.15)$$

Thus, for the large energy  $\omega$  limit that we are studying now we get

$$\Sigma(s; 2\omega r/s) \xrightarrow{\omega \rightarrow \infty} \frac{s \mu^2(s)}{\omega r}, \quad (4.16)$$

hence, the last two remaining integrals in (4.14) factorize and we can write

$$\begin{aligned} \omega \frac{dI^{\text{med}}}{d\omega} &\approx \frac{\alpha_s C_R}{\pi \omega} \text{Re} \int_0^L ds s n(s) \mu^2(s) \int_0^\infty \frac{dr}{r^2} (1 - e^{-ir}) \\ &= \frac{\alpha_s C_R}{2\omega} \int_0^L ds s n(s) \mu^2(s). \end{aligned} \quad (4.17)$$

With this result we clearly see that the spectrum for a general case where the medium is not static but described by an arbitrary density  $n(s)$  and with a generic thermal screening mass  $\mu^2(s)$  behaves as  $1/\omega$  in the large energy limit, which coincides with the behavior of the static result as we discussed in section 3.2 and which can be written as

$$\omega \frac{dI^{\text{med}}}{d\omega} \Big|_{\text{static}} \approx \frac{\alpha_s C_R}{2} \frac{\chi \bar{\omega}_c}{\omega}. \quad (4.18)$$

Analyzing (4.17) we realize that the dependence on the time variable  $s$  of the parameters of the medium is just through a particular integration, which gives us the form of the scaling law we must employ to match the high energy tails of spectra computed for different medium configurations. Moreover, here we employed the Yukawa-type interaction model, but this result is more general. In fact, as long as the collision rate presents the large momentum functional form  $V(s; \mathbf{p}) \propto \mu^2(s)/\mathbf{p}^4$ , like the HTL model, then the same derivation can be done, because  $\Sigma$  only catches the high momentum tail behavior of the potential for large argument variables.

## 4.5 Scaling laws using as reference the static scenario

We revisit the use of the static result as the reference scenario for the scaling laws, but employing this time an enhanced version compared to the average values case. The improvement lies in the ability to accurately match the high energy tail of the spectrum, which was the worst described region in Fig. 4.1. The comparison between (4.17) and (4.18) indicates that the high energy tails will match if we define the following combination of parameters as scaling law

$$\chi \bar{\omega}_c = \int_{\tau_{\text{hydro}}}^{L+\tau_{\text{hydro}}} dt t n_{\text{hydro}}(t) \mu_{\text{hydro}}^2(t), \quad (4.19)$$

here we shifted the limits of the integral by  $\tau_{\text{hydro}}$ , because this is the initialization time of the hydrodynamic simulation and for earlier times we have no results for  $n_{\text{hydro}}(t)$  or  $\mu_{\text{hydro}}^2(t)$ . This scaling law was already proposed for the HO approximation with  $\hat{q}(t) \sim n(t)\mu^2(t)$  and for the GLV  $N = 1$  opacity result with constant screening mass in Ref. [70].

A well-defined set of scaling laws needs to establish enough relationships to determine values for all of the medium parameters used to evaluate the radiation spectrum. For the static result we have dependence on three parameters (4.7), thus we have to include two more scaling laws to establish a proper matching between the spectrum computed for a general trajectory and the static scenario. Using [70, 116] as a guide we choose to set

$$\chi = \int_{\tau_{\text{hydro}}}^{L+\tau_{\text{hydro}}} dt n_{\text{hydro}}(t), \quad (4.20)$$

$$\chi \bar{R} = \frac{3}{2} \int_{\tau_{\text{hydro}}}^{L+\tau_{\text{hydro}}} dt t^2 n_{\text{hydro}}(t) \mu_{\text{hydro}}^2(t). \quad (4.21)$$

If we are interested in computing the no-kinematic constraint energy distribution  $\bar{R} \rightarrow \infty$ , then the last equation is not needed, as it happened in the previous section where we just defined two scaling laws.

We illustrate in Fig. 4.2 the performance of the static spectrum as reference scenario (blue dashed lines) when trying to estimate the spectrum computed along a trajectory described by a hydrodynamic simulation (purple solid lines). Here we plot the spectra as

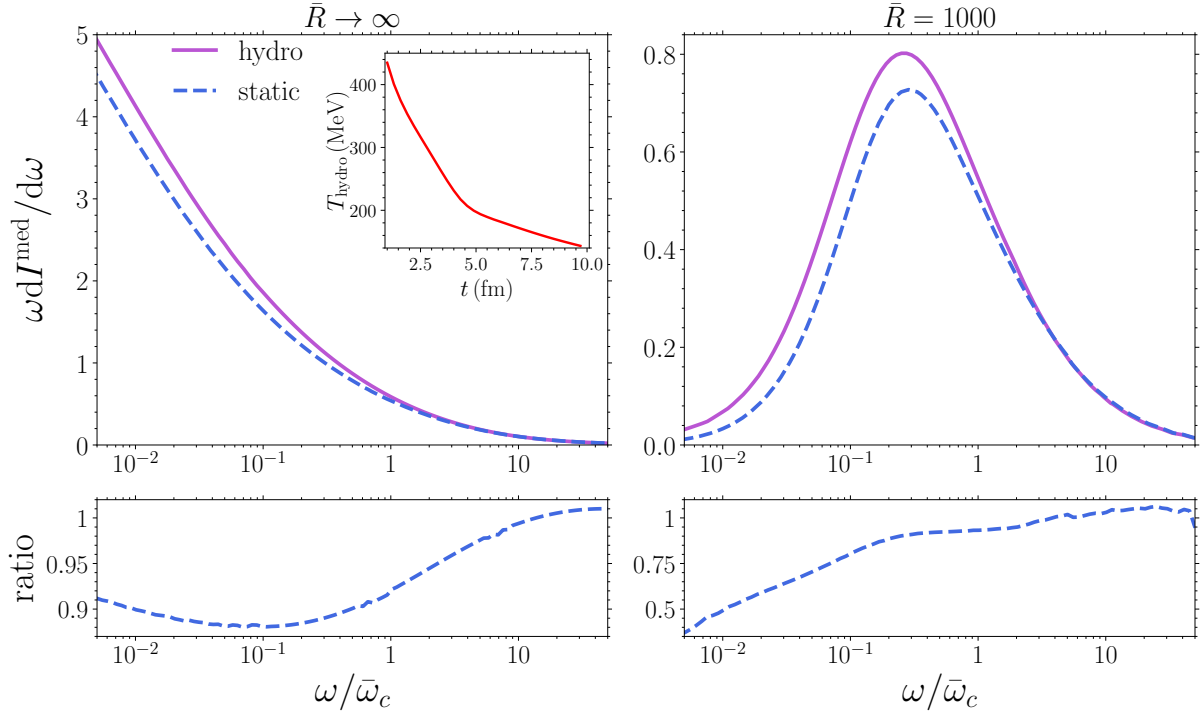


Figure 4.2: Top panels: all order in-medium energy spectrum for the Yukawa collision rate for  $\chi = 5$ , and  $\bar{R} \rightarrow \infty$  (left panel) or  $\bar{R} = 1000$  (right panel) as a function of the rescaled gluon energy  $\omega/\bar{\omega}_c$ . Purple curves correspond to the spectra computed along the trajectory whose temperature profile is shown in the inset figure, which was sampled with a central production point over the 0 – 10% centrality class in  $\sqrt{s_{\text{NN}}} = 5.02$  TeV Pb-Pb collisions at the LHC. Blue dashed curves correspond to the static spectrum evaluated employing the scaling laws (4.19)-(4.21). Bottom panels: ratio of the static scenario w.r.t. the spectrum computed with the hydrodynamic model along the selected trajectory. Figure extracted from [3] under a Creative Commons license, CC-BY 4.0.

function of the rescaled gluon energy  $\omega/\bar{\omega}_c$  with  $\bar{\omega}_c$  defined through (4.19) and the static case is computed following the scaling laws (4.19)-(4.21). The trajectory employed is the same one used in Fig. 4.1 and its temperature profile is showcased in the inset figure. Top panels show the spectra and bottom plots the ratio of the static case w.r.t. the scenario computed along the actual path. Left panels represent the energy distribution without kinematic condition  $\bar{R} \rightarrow \infty$  and right panels  $\bar{R} = 1000$  (corresponding to  $k_2 = 45$ ), and in both cases we employ  $\chi = 5$  (corresponding to  $k_1 = 0.5$ ). As expected, results in Fig. 4.2 using the new scaling laws correctly describe the high energy tail of the spectrum, in contrast to Fig. 4.1 where we employed average values of the medium parameters. Nevertheless, for smaller gluon energies discrepancies between the spectra appear and for the case with finite kinematical cut-off (right panel) the scaling breaks down, resulting in disagreements of more than 50%.

Fig. 4.2 correctly represents the general performance of the static scaling laws (4.19)-(4.21). In the next section we propose another matching scheme which the aim of improving the performance of the static scaling in the low energy region, providing results for a wide variety of conditions and always showing the static scenario as well. Thus, displaying now more results for the static scaling laws is pointless.

## 4.6 Scaling laws using as reference a power-law expansion

Scaling laws are mainly employed because they allow us to pre-tabulate the medium-induced radiation spectrum for a large range of the values of the parameters, thus we are able to estimate the spectrum for almost any realistic description of the QGP. In the case of the static scenario we just described, we could pre-evaluate the spectrum for different combinations of the free parameters (4.7) and in phenomenological studies use the relations (4.19)-(4.21) to determine the values of the parameters along every selected trajectory, instead of having to compute the spectrum for each path.

Nonetheless, as we illustrated in Fig. 4.2, using the static spectrum as reference scenario for the scaling laws yields significant errors when attempting to describe the all-order fully resummed spectrum along a realistic trajectory sampled from a hydrodynamical simulation. Therefore, here we propose the employment of another scenario as reference which allow us to describe the full spectrum more accurately for any combination of the values of the medium parameters: a power-law expanding medium, which can be used under the same conditions as the static case for pre-tabulating the spectrum.

With our newly proposed reference scenario we assume the following functional forms for the medium linear density of scattering centers  $n(t)$  and the collision rate screening mass  $\mu^2(t)$

$$n(t) = \frac{\bar{n}_0}{(t + \bar{t}_0)^\alpha}, \quad \text{and} \quad \mu^2(t) = \frac{\bar{\mu}_0^2}{(t + \bar{t}_0)^{2\alpha}}, \quad (4.22)$$

being the constants  $\bar{n}_0$  and  $\bar{\mu}_0$  proportional to the maximum values of the density and screening mass at the initial time  $\bar{t}_0$ , respectively. If we set the power  $\alpha$  to be  $\alpha = 1$  then we recover the well-known Bjorken expansion, which describes an evolving medium with free streaming along the longitudinal direction.

The scaling laws analogous to (4.19)-(4.21) for this scenario can be written as

$$\int_0^{\bar{L}} dt n(t) = \int_{\tau_{\text{hydro}}}^{L+\tau_{\text{hydro}}} dt n_{\text{hydro}}(t), \quad (4.23)$$

$$\int_0^{\bar{L}} dt t n(t) \mu^2(t) = \int_{\tau_{\text{hydro}}}^{L+\tau_{\text{hydro}}} dt t n_{\text{hydro}}(t) \mu_{\text{hydro}}^2(t), \quad (4.24)$$

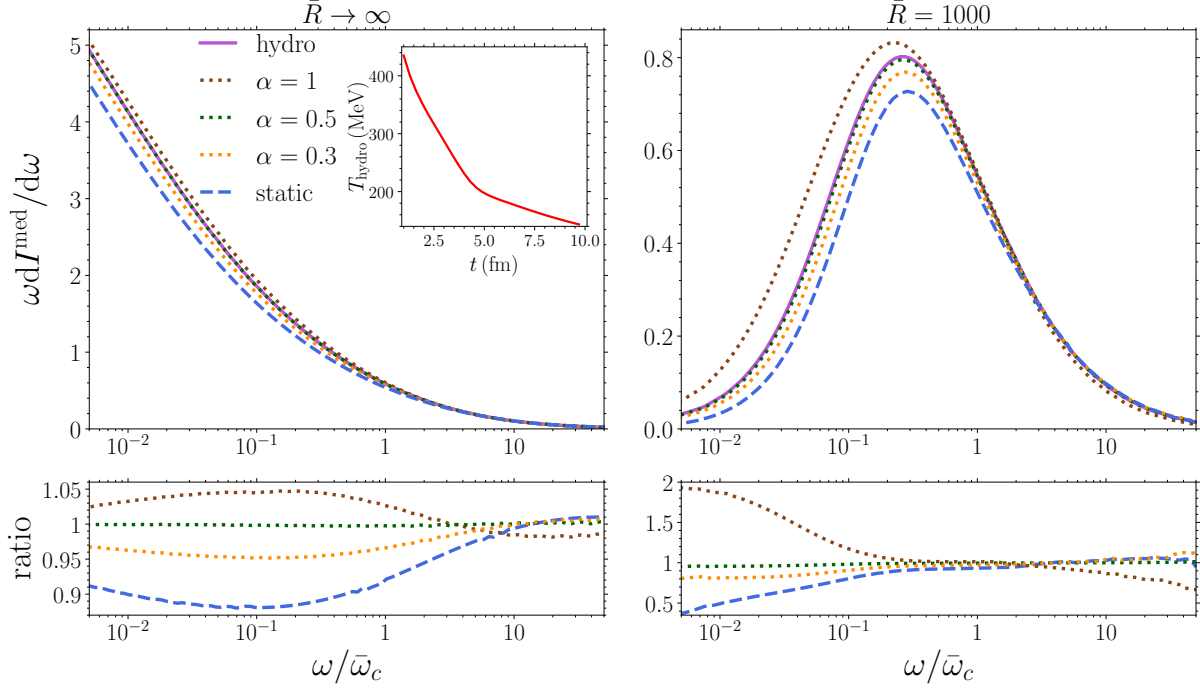


Figure 4.3: Top panels: all order in-medium energy spectrum for the Yukawa collision rate for  $\chi = 5$ , and  $\bar{R} \rightarrow \infty$  (left panel) or  $\bar{R} = 1000$  (right panel) as a function of the rescaled gluon energy  $\omega/\bar{\omega}_c$ . Purple solid lines correspond to the spectra computed along the trajectory whose temperature profile is shown in the inset figure, which was sampled with a central production point over the 0 – 10% centrality class in  $\sqrt{s_{\text{NN}}} = 5.02$  TeV Pb-Pb collisions at the LHC. Blue dashed lines correspond to the spectrum evaluated using the static scenario (4.19)-(4.21). The dotted curves correspond to the power-law scaling laws (4.23)-(4.25) for  $t_0 = 0.1$  and several values of  $\alpha$ :  $\alpha = 0.3$  (orange),  $\alpha = 0.5$  (green), and  $\alpha = 1.0$  (brown). Bottom panels: ratio of the power-law and static spectra w.r.t. the spectrum evaluated along the path with temperature profile showcased in the inset figure. Figure extracted from [3] under a Creative Commons license, CC-BY 4.0.

$$\int_0^{\bar{L}} dt t^2 n(t) \mu^2(t) = \int_{\tau_{\text{hydro}}}^{L+\tau_{\text{hydro}}} dt t^2 n_{\text{hydro}}(t) \mu_{\text{hydro}}^2(t), \quad (4.25)$$

with  $\bar{L}$  the endpoint of the trajectory along the power-law case. In this set of equations (4.24) plays the same role as (4.19) before, ensuring the correct matching of the high energy tails of both spectra following the discussion of section 4.4.

With the scaling formulated in its current configuration the number of parameters we need to specify is larger than for the static scaling, as now the density and screening mass profiles (4.22) depend on two additional parameters:  $\alpha$  and  $t_0$ . One could simply treat them as free parameters and pre-tabulate the spectrum varying them. However, this assumption would complicate our scaling laws, which we want to keep as simple as

possible. Following that spirit, we analyzed the performance of the power-law scaling for several values of  $\alpha$  and  $t_0$  in different trajectories, to check if there is some fixed value of the parameters which provides good results in any conditions. We remember that all of the results we are going to show are computed with a Yukawa collision rate, being the HTL case treated in appendix 4.B.

We start by comparing in Fig. 4.3 the energy distribution computed along the same typical trajectory sampled over the 0 – 10% centrality class in  $\sqrt{s_{NN}} = 5.02$  TeV Pb-Pb collisions at the LHC that we used in Figs. 4.1 and 4.2, whose temperature profile is showcased in the inset figure, and the results arising from the employment of the power-law scaling laws (4.23)-(4.25) for several values of  $\alpha$  with  $t_0$  fixed to  $t_0 = 0.1$ . The spectra is represented as a function of  $\omega/\bar{\omega}_c$ , with  $\bar{\omega}_c$  defined through (4.19). Here we employ the same parameter values as in previous section:  $k_1 = 0.5$  and  $k_2 = 45$ , which corresponds to  $\chi = 5$  and  $\bar{R} = 1000$ , respectively (for the spectrum with kinematic constraint shown in the right panels). The performance of the power-law scaling compared with respect to the static scenario will be analyzed later, after we have discussed the role of  $\alpha$  and  $t_0$ . In Fig. 4.3 we clearly see that the best approximation of the spectrum computed along the trajectory coming from the hydrodynamic simulation (purple solid curves) is achieved by the power-law expansion with  $\alpha = 0.5$  (green dotted lines) for both spectra, either with (left) or without (right) kinetic condition imposed. We have computed these spectra for different values of the fitting parameters  $k_1$  and  $k_2$  for several sampled trajectories and, overall, the results shown in Fig. 4.3 constitute a good representation of the results we obtained, being the  $\alpha = 0.5$  scenario the one which always provides the best description of the spectrum along the actual path. Thus, we decided to fix  $\alpha$  to  $\alpha = 0.5$  for the rest of the manuscript instead of being considered as a free parameter.

Moving on to the opposite scenario, we illustrate in Fig. 4.4 the effect on the scaling laws of varying  $t_0$  for fixed  $\alpha$ . We still represent the spectra as a function of  $\omega/\bar{\omega}_c$ , with  $\bar{\omega}_c$  defined through (4.19), and use the same trajectory as in previous cases with the same values for the free parameters:  $k_1 = 0.5$  and  $k_2 = 45$ . We clearly see that changing the value of  $t_0$  produces minimal effects in the outcomes of the scaling, being the best description of the spectrum computed along the actual trajectory (purple solid lines) reached by the power-law scenario with  $t_0 = 0.1$ . Therefore, we choose to set  $t_0 = 0.1$  as a fixed value instead of treating it as a free parameter, thus reducing in one the total number of parameters required to determine the spectrum with the approach we propose at the same time that the precision of the results remains almost unchanged.

It is important to notice that by setting both  $\alpha$  and  $t_0$  to fixed values the total number of parameters we need to specify with the power-law scaling is exactly the same as for the static scenario, three, which can be determined through the three scaling laws we already established (4.23)-(4.25). With this the complexity of the power-law scaling reduces significantly while maintaining the same levels of accuracy in the description of the spectra along realistic hydrodynamic trajectories, which will greatly help its implementation in future phenomenological works.

Once that we have selected fixed values for  $\alpha$  and  $t_0$  we proceed to analyse in detail

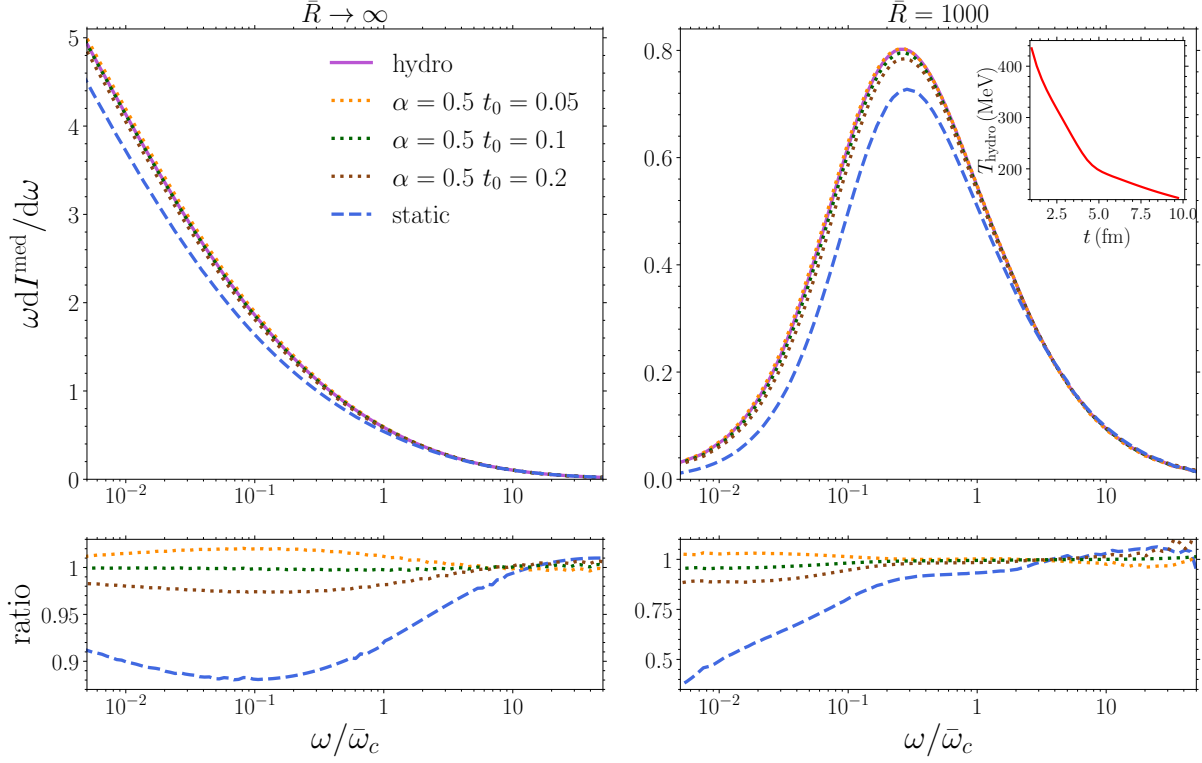


Figure 4.4: Top panels: all order in-medium energy spectrum for the Yukawa collision rate for  $\chi = 5$ , and  $\bar{R} \rightarrow \infty$  (left panel) or  $\bar{R} = 1000$  (right panel) as a function of the rescaled gluon energy  $\omega/\bar{\omega}_c$ . Purple solid lines correspond to the spectra computed along the trajectory whose temperature profile is shown in the inset figure, which was sampled with a central production point over the 0 – 10% centrality class in  $\sqrt{s_{\text{NN}}} = 5.02$  TeV Pb-Pb collisions at the LHC. Blue dashed lines correspond to the spectrum evaluated using the static scenario (4.19)-(4.21). The dotted curves correspond to the power-law scaling laws (4.23)-(4.25) for  $\alpha = 0.5$  and several values of  $t_0$ :  $t_0 = 0.05$  (orange),  $t_0 = 0.1$  (green), and  $t_0 = 0.2$  (brown). Bottom panels: ratio of the power-law and static spectra w.r.t. the spectrum evaluated along the path with temperature profile showcased in the inset figure. Figure extracted from [3] under a Creative Commons license, CC-BY 4.0.

the performance of our proposed power-law scaling in comparison with the classical static scenario. We show in Fig. 4.5 the full resummed medium-induced energy spectrum evaluated along the same path as in previous figures (purple solid line), whose temperature profile is showcased in the inset figure, and for the same values of the parameters  $k_1 = 0.5$  and  $k_2 = 45$ , which correspond, respectively, to  $\chi = 5$  and  $\bar{R} = 1000$ , when the kinematic constraint is imposed. The spectrum is plotted as a function of  $\omega/\bar{\omega}_c$ , with  $\bar{\omega}_c$  defined through (4.19), and it is compared to the spectrum resulting from the application of the static scaling laws (4.19)-(4.21) (blue dashed line) and the power-law scenario scaling computed through (4.23)-(4.25) (green dotted curve). We clearly see that the performance of

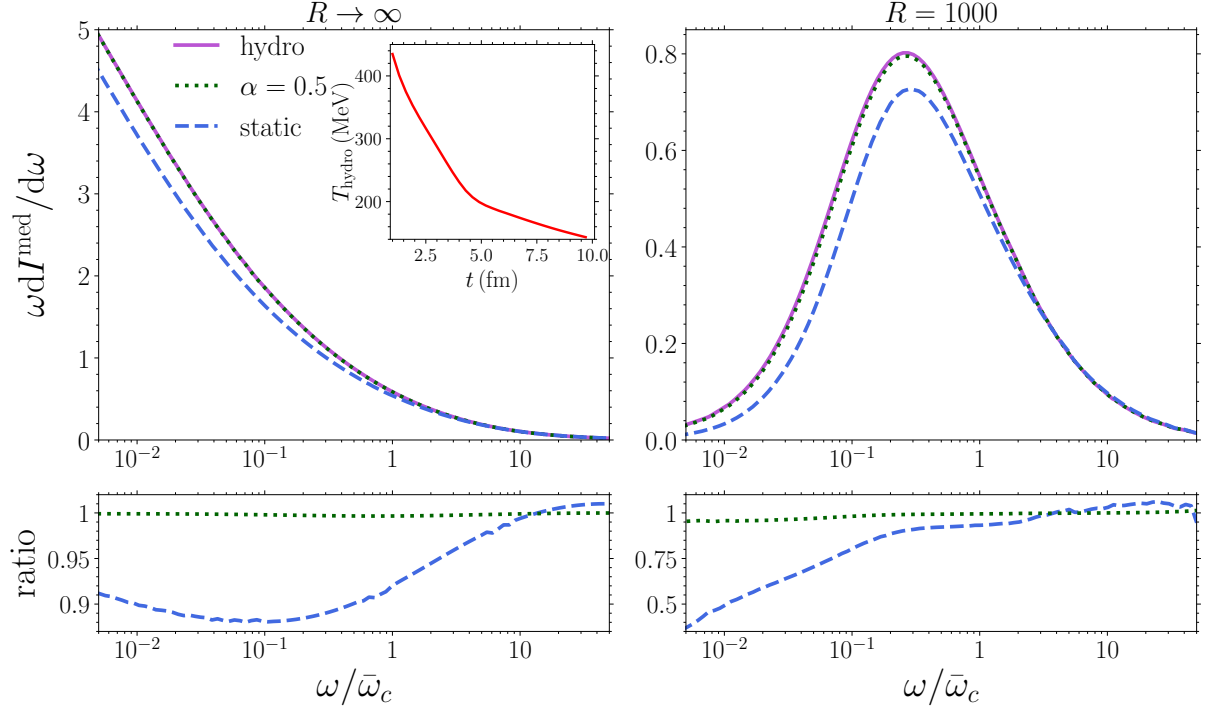


Figure 4.5: Top panels: all order in-medium energy spectrum for the Yukawa collision rate for  $\chi = 5$ , and  $\bar{R} \rightarrow \infty$  (left panel) or  $\bar{R} = 1000$  (right panel) as a function of the rescaled gluon energy  $\omega/\bar{\omega}_c$ . Purple solid lines correspond to the spectra computed along the trajectory whose temperature profile is shown in the inset figure, which was sampled with a central production point over the 0 – 10% centrality class in  $\sqrt{s_{\text{NN}}} = 5.02$  TeV Pb-Pb collisions at the LHC. Blue dashed and green dotted lines correspond, respectively, to the spectrum evaluated using the static (4.19)-(4.21) and power-law (4.23)-(4.25) scaling laws. Bottom panels: green dotted (blue dashed) lines represent the ratio of the spectrum computed with the power-law (static) scaling laws w.r.t. the spectrum evaluated along the path with temperature profile showcased in the inset figure. Figure extracted from [3] under a Creative Commons license, CC-BY 4.0.

the power-law scaling is excellent for all gluon energies, implying a great improvement in comparison with the static scenario for small  $\omega$ . This behavior is observed independently of the kinetic condition, either being removed (left) or imposed (right), which is the case where the static scaling provide the worst results.

We continue our analysis in Fig. 4.6, showing outcomes of the scaling for a different trajectory. The chosen path is still a straight line with a typical temperature profile sampled over the 0 – 10% centrality class in  $\sqrt{s_{\text{NN}}} = 5.02$  TeV Pb-Pb collisions at the LHC, but now having an off-central production point, opposed to the case presented in Fig. 4.5. The corresponding temperature profile along the path is shown in the inset figure. We employed the same values of the free parameters as before  $k_1 = 0.5$  and

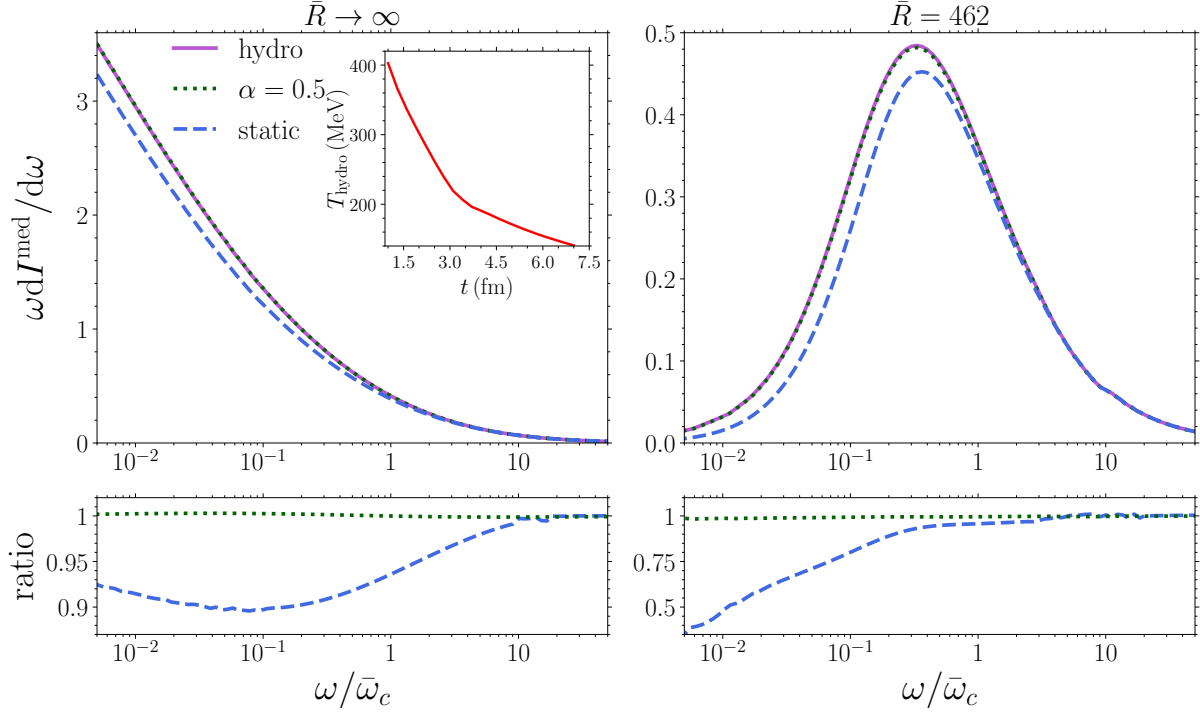


Figure 4.6: Top panels: all order in-medium energy spectrum for the Yukawa collision rate for  $\chi = 3.3$ , and  $\bar{R} \rightarrow \infty$  (left panel) or  $\bar{R} = 462$  (right panel) as a function of the rescaled gluon energy  $\omega/\bar{\omega}_c$ . Purple solid lines correspond to the spectra computed along the trajectory whose temperature profile is shown in the inset figure, which was sampled with a central production point over the 0 – 10% centrality class in  $\sqrt{s_{\text{NN}}} = 5.02$  TeV Pb-Pb collisions at the LHC with an off-central production point. Blue dashed and green dotted lines correspond, respectively, to the spectrum evaluated using the static (4.19)-(4.21) and power-law (4.23)-(4.25) scaling laws. Bottom panels: green dotted (blue dashed) lines represent the ratio of the spectrum computed with the power-law (static) scaling laws w.r.t. the spectrum evaluated along the path with temperature profile showcased in the inset figure. Figure extracted from [3] under a Creative Commons license, CC-BY 4.0.

$k_2 = 45$ , which for this trajectory correspond to  $\chi = 3.3$  and  $\bar{R} \rightarrow \infty$  (left panel), and  $\chi = 3.3$  and  $\bar{R} = 462$  (right panel) in terms of static parameters. The presented results are in perfect agreement with the ones in Fig. 4.5, being the power-law scaling (green dotted lines) the one which better approximates the spectra coming from the evaluation along the actual trajectory (purple solid lines) for any  $\omega$ , greatly improving the performance of the static scenario.

The results we have presented so far correspond to a representative selection of the many straight line trajectories we have studied for the 0 – 10% centrality class in  $\sqrt{s_{\text{NN}}} = 5.02$  TeV Pb-Pb collisions at the LHC. We have repeated this analysis for dif-

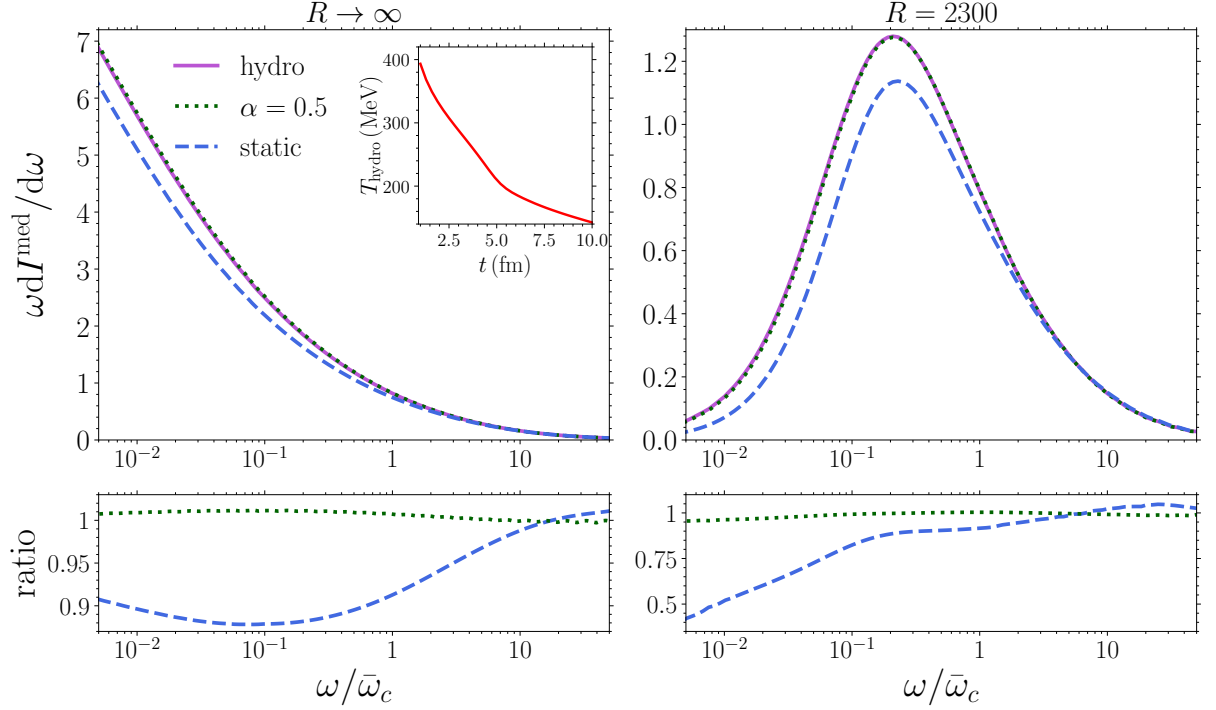


Figure 4.7: Top panels: all order in-medium energy spectrum for the Yukawa collision rate for  $\chi = 7.6$ , and  $\bar{R} \rightarrow \infty$  (left panel) or  $\bar{R} = 2300$  (right panel) as a function of the rescaled gluon energy  $\omega/\bar{\omega}_c$ . Purple solid lines correspond to the spectra computed along the trajectory whose temperature profile is shown in the inset figure, which was sampled with a central production point over the 20–30% centrality class in  $\sqrt{s_{\text{NN}}} = 5.02$  TeV Pb-Pb collision at the LHC. Blue dashed and green dotted lines correspond, respectively, to the spectrum evaluated using the static (4.19)-(4.21) and power-law (4.23)-(4.25) scaling laws. Bottom panels: green dotted (blue dashed) lines represent the ratio of the spectrum computed with the power-law (static) scaling laws w.r.t. the spectrum evaluated along the path with temperature profile showcased in the inset figure. Figure extracted from [3] under a Creative Commons license, CC-BY 4.0.

ferent centrality classes under the same conditions and found similar conclusions: in the case of paths whose temperature profiles decrease monotonically with time the scaling laws employing as reference scenario a power-law expansion provide an excellent description of the full medium-induced emission spectrum in all of the energy range. Actually, for semi-peripheral LHC Pb-Pb collisions at  $\sqrt{s_{\text{NN}}} = 5.02$  TeV discrepancies between the spectra computed along the actual paths coming from the hydrodynamical simulation and the ones resulting from the application of the scaling laws always remain below 2%, thus proving the validity of our approach to be employed in realistic phenomenological studies. We further illustrate this in Fig. 4.7, where we show results for a typical straight-line trajectory sampled over 20 – 30% semi-peripheral  $\sqrt{s_{\text{NN}}} = 5.02$  TeV Pb-Pb collision at

the LHC. As we can see in its temperature profile showcased in the inset figure, it presents a monotonically decreasing behaviour, as in Figs. 4.5 and 4.6. In this case we fixed the free parameters values to  $k_1 = 0.75$  and  $k_2 = 93$ , which in terms of the static variables correspond to  $\chi = 7.6$  and  $\bar{R} \rightarrow \infty$  (left), and  $\chi = 7.6$  and  $\bar{R} = 2300$  (right). Observing the results it is evident that they are in agreement with the previous two figures: for high energies the spectrum computed along the trajectory is well-reproduced regardless of which scenario we use as reference for the scaling laws (as it was expected from the discussion in section 4.4), but for small gluon energies the power-law scenario provides an excellent description, with errors of less than 1%, compared with respect to the static result. The performance of the power-law scaling is the same independently of the chosen kinetic condition, either lifted (left) or imposed (right), whereas that the static scenario provides unacceptable results in the latter case for small gluon energies.

All of the results we presented in Figs. 4.5, 4.6 and 4.7 represent the overall behavior of the majority of trajectories we can sample from Pb-Pb collisions at LHC energies, with the initial hard parton produced near the center of the collision point. However, there are some few extreme cases where the emitter is produced close to the edge of the overlapping region of the participant nuclei and then propagates towards the hotter core, thus resulting in a long-distance propagation through the medium whose temperature profile is non-monotonically decreasing, unlike any trajectory we have seen so far. The thorough investigation of such rare scenarios allowed us to extract two important conclusions: firstly, we measured quantitatively their relative importance with respect to the total amount of possible events, finding out that they constitute about 10% of the events in the 20 – 30% centrality class, and about 7% in the 0 – 10% centrality class; secondly, and more important, even under these extreme conditions our power-law scaling provide a reasonably good description of the full spectrum computed along the actual trajectory, remaining the deviations always below 15%.

We illustrate this last feature in Fig. 4.8, where we show results for a trajectory sampled over the 0 – 10% centrality class in  $\sqrt{s_{NN}} = 5.02$  TeV Pb-Pb collisions at the LHC with a non-central production point. We clearly see in the inset figure that the behavior of the temperature profile is not monotonically decreasing. To derive these results we employed the same values of the free parameters as in Figs. 4.5 and 4.6,  $k_1 = 0.5$  and  $k_2 = 45$ , which for this trajectory corresponds to  $\chi = 6.6$  and  $\bar{R} \rightarrow \infty$  (left), and  $\chi = 6.6$  and  $\bar{R} = 2600$  (right) static parameter values. Fig. 4.8 shows that deviations between the spectrum along the path and the power-law scenario appear for energies smaller than the characteristic gluon energy  $\bar{\omega}_c$  either with the kinematic constraint released (left) or imposed (right). These errors of the power-law scaling remain always below 15%, and it is important to notice that they are of the same order than for the static scenario when no-kinematic condition is imposed (left panel) and much smaller for the finite kinematical cut-off case (right panel), where the static scaling breaks down for small gluon energies. Nevertheless, we would like to remark that in Fig. 4.8 we are showing the most extreme scenario we have found regarding the behaviour of the temperature profile at early times and, therefore, the spectra computed along any other path, including trajectories sampled

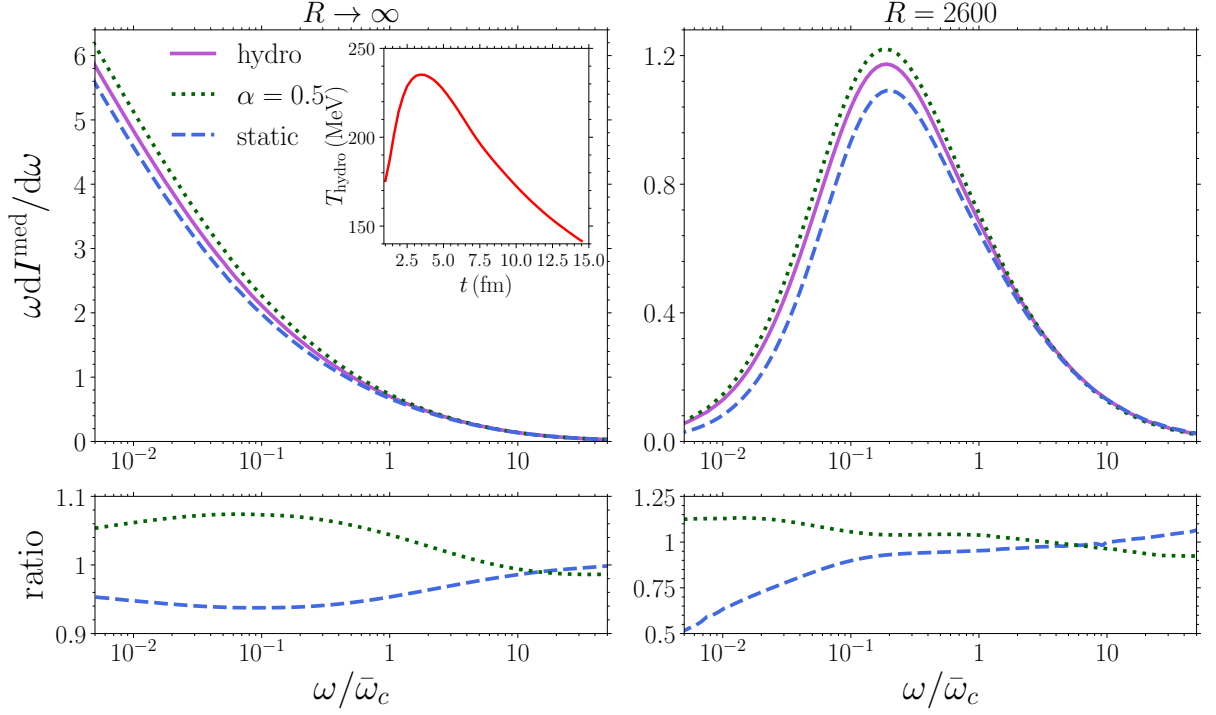


Figure 4.8: Top panels: all order in-medium energy spectrum for the Yukawa collision rate for  $\chi = 6.6$ , and  $\bar{R} \rightarrow \infty$  (left panel) or  $\bar{R} = 2600$  (right panel) as a function of the rescaled gluon energy  $\omega/\bar{\omega}_c$ . Purple solid lines correspond to the spectra computed along the trajectory whose temperature profile is shown in the inset figure, which was sampled with a non-central production point over the 0 – 10% centrality class in  $\sqrt{s_{\text{NN}}} = 5.02$  TeV Pb-Pb collisions at the LHC. Blue dashed and green dotted lines correspond, respectively, to the spectrum evaluated using the static (4.19)-(4.21) and power-law (4.23)-(4.25) scaling laws. Bottom panels: green dotted (blue dashed) lines represent the ratio of the spectrum computed with the power-law (static) scaling laws w.r.t. the spectrum evaluated along the path with temperature profile showcased in the inset figure. Figure extracted from [3] under a Creative Commons license, CC-BY 4.0.

over different centralities, are expected to be even better described by the power-law scaling.

We conclude this section referring the interested reader to appendices 4.A and 4.B. In those appendices we illustrate the robustness of the results of the power-law scaling shown here by analyzing its performance when accounting for EbyE fluctuations (appendix 4.A) or when employing different elastic collision rate models, like the HTL (appendix 4.B). All together, these results highlight the effectiveness of the power-law scaling approach in approximating the spectrum along realistic hydrodynamic paths for a wide range of trajectories and centrality classes in Pb-Pb collisions at the LHC, which will be of crucial importance for future precise phenomenological studies that employ the all-order fully

resummed medium-induced radiation spectrum.

## 4.A Effects of event-by-event fluctuations on the scaling laws

In this appendix we study the effect that event-by-event fluctuations may have on the performance of the power-law scaling laws presented in the previous section (4.23)-(4.25). For this purpose, we make use of the EbyE EKRT hydrodynamics simulation [115], which includes fluctuating initial energy density profiles obtained within the EKRT framework [117]. In this hydrodynamic model the formation time of the initial condition is set to  $\tau_{\text{hydro}} = 0.197$  fm. The simulation uses the s95p parametrization of the lattice QCD results [118] as equation of state, and it parametrizes the sheer viscosity to entropy density ratio  $\eta/s$  as  $\eta/s(T) = \text{param1}$ , as described in Ref. [115]. This hydrodynamical description also implements two different freeze-outs: a chemical freeze-out at  $T_{\text{chem}} = 175$  MeV and a kinetic freeze-out at  $T_{\text{dec}} = 100$  MeV. We highlight here that the results coming from this hydrodynamic simulation present an excellent agreement with experimental measures of soft hadronic observables, such as multiplicity, average transverse momentum, flow coefficients and flow correlations in  $\sqrt{s_{\text{NN}}} = 200$  GeV Au-Au collisions at RHIC, and  $\sqrt{s_{\text{NN}}} = 2.76$  TeV Pb-Pb collisions at the LHC [117].

We focus on two centrality classes to analyze the impact of fluctuations along the trajectory inside the medium on the power-law matching:  $0 - 10\%$  and  $10 - 30\%$ . Our starting point is a set of 400 minimum-bias  $\sqrt{s_{\text{NN}}} = 2.76$  TeV Pb-Pb events from which we select different paths by fixing straight-line trajectories sampled with several production points and directions. In Fig. 4.9 it is shown the average temperature profile obtained from all of the sampled trajectories for the chosen centrality ( $0 - 10\%$  results on the left and  $10 - 30\%$  on the right) with the respective standard deviation (black points with vertical lines). For illustration purposes, we also represent a selection of several temperature profiles (colored dashed curves) sampled from the total list of trajectories shown in the inset figures of Fig. 4.9, which correspond to the ones that present the largest deviations from the average behaviour: on the left, for the  $0 - 10\%$  centrality class, we show trajectories with off-central production points; and on the right, for the  $10 - 30\%$  semi-peripheral events, the chosen paths correspond to central production points. We clearly see that fluctuations are specially relevant for the early stages of the propagation in off-central production points (left panel), resulting in temperature profiles with non-monotonically decreasing behavior. Nonetheless, the relative importance of the fluctuations drastically reduces after a few femtoseconds, being all trajectories well described by the average behavior for mid and late times.

We show in Fig. 4.10 results for the energy distribution computed along one of the paths from the right plot of Fig. 4.9, i.e,  $10 - 30\%$  centrality class with central production point. The results are presented in the same way as in previous figures, with the spectra shown as function of the rescaled gluon energy  $\omega/\bar{\omega}_c$  and with different lines corresponding

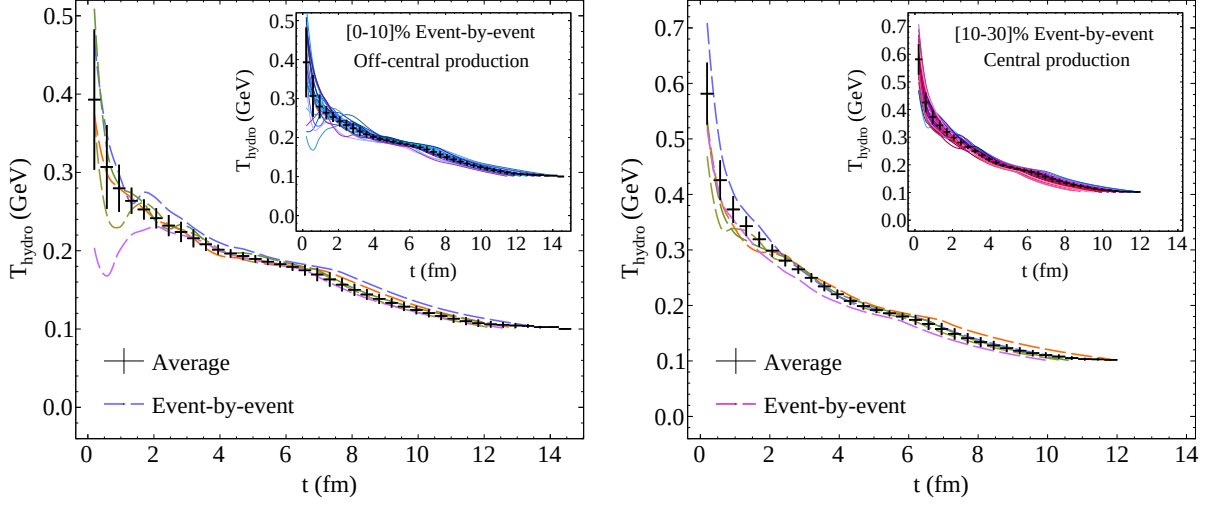


Figure 4.9: Left: average temperature profile with standard deviation (black points with vertical lines) sampled from the 0 – 10% centrality class in  $\sqrt{s_{\text{NN}}} = 2.76$  TeV Pb-Pb collisions at the LHC, together with a sample of temperature profiles corresponding to off-central production points (colored dashed lines). Right: average temperature profile with standard deviation (black points with vertical lines) sampled from the 10 – 30% centrality class in  $\sqrt{s_{\text{NN}}} = 2.76$  TeV Pb-Pb collisions at the LHC together with a sample of temperature profiles corresponding to central production points (colored dashed lines). Inset panels display the full sample of temperature profiles in each centrality class. Figure extracted from [3] under a Creative Commons license, CC-BY 4.0.

to different scenarios: purple solid lines represent the spectrum computed along the actual path whose temperature profile is shown in the inset figure, blue dashed lines correspond to the employment of the scaling laws using the static scenario as reference (4.19)-(4.21), and green dotted lines are computed with the power-law matching given by (4.23)-(4.25). Here we have used as values of the parameters  $k_1 = 0.5$  and  $k_2 = 50$ , which for this trajectory correspond to  $\chi = 5$  and  $\bar{R} \rightarrow \infty$  (left); and  $\chi = 5$  and  $\bar{R} = 1000$  (right), in terms of static parameters. Observing the results we conclude that, regardless of the evident fluctuations that the sampled trajectory shown in the inset figure presents, the scaling laws which use the power-law scenario as reference provide an accurate approximation of the spectrum even for small energies either with the kinetic condition lifted (left) or imposed (right).

Finally, we show in Fig. 4.11 the spectra computed for a trajectory sampled over the 0 – 10% centrality class in  $\sqrt{s_{\text{NN}}} = 2.76$  TeV Pb-Pb collisions at the LHC with a non-central production point. As before, the purple solid line represents the spectrum evaluated along the actual path whose temperature profile is showcased in the inset figure, the blue dashed curve corresponds to the employment of the static scaling laws (4.19)-(4.21), and the green dotted line is obtained using the power-law scenario as reference (4.23)-(4.25). We have used the same parameter values as in the previous figure  $k_1 = 0.5$  and

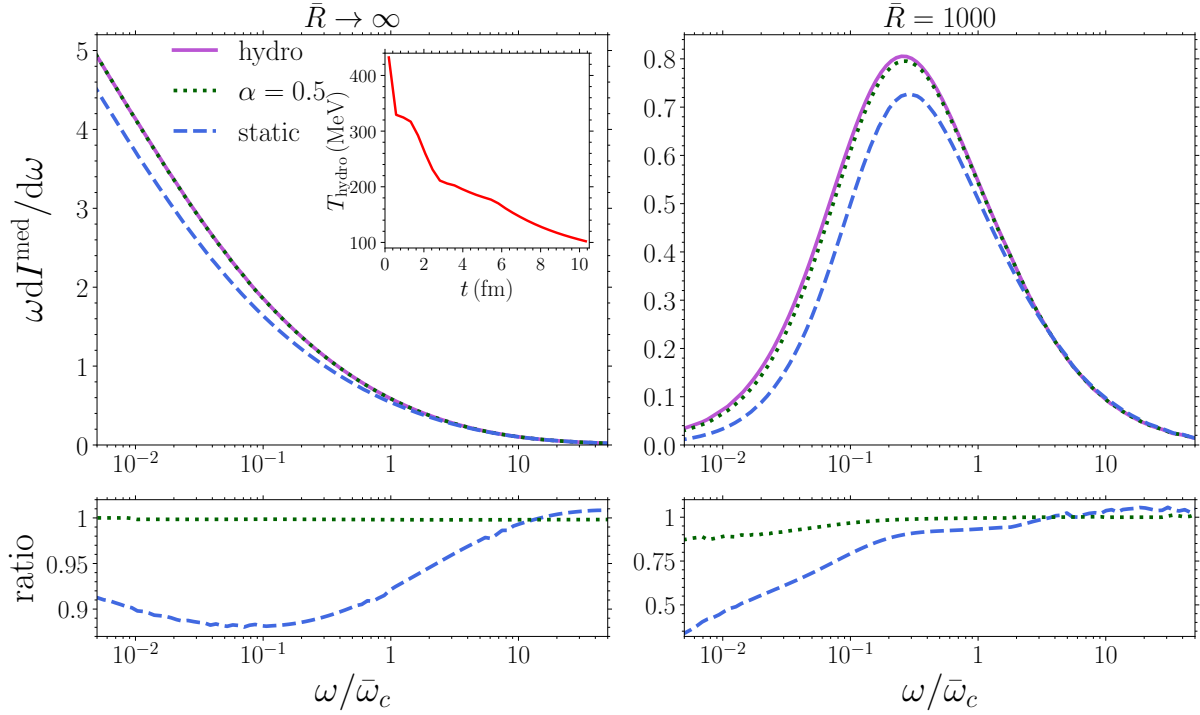


Figure 4.10: Top: all order in-medium energy spectrum for the Yukawa collision rate for  $\chi = 5$ , and  $\bar{R} \rightarrow \infty$  (left) or  $\bar{R} = 1000$  (right) as a function of the rescaled gluon energy  $\omega/\bar{\omega}_c$ . Purple solid lines correspond to the spectra computed along the trajectory whose temperature profile is shown in the inset figure, which was sampled from an event corresponding to the 10 – 30% centrality class in  $\sqrt{s_{\text{NN}}} = 2.76$  TeV Pb-Pb collisions at the LHC. Blue dashed and green dotted lines correspond, respectively, to the spectrum evaluated using the static (4.19)-(4.21) and power-law (4.23)-(4.25) scaling laws. Bottom: green dotted (blue dashed) lines represent the ratio of the spectrum computed with the power-law (static) scaling laws w.r.t. the spectrum evaluated along the path with temperature profile showcased in the inset figure. Figure extracted from [3] under a Creative Commons license, CC-BY 4.0.

$k_2 = 50$ , which in this trajectory correspond to  $\chi = 4.4$  and  $\bar{R} \rightarrow \infty$  (left panel); and  $\chi = 4.4$  and  $\bar{R} = 1100$  (right panel). We clearly see that even in such an extreme scenario the power-law matching provide a really good description of the spectrum computed along the actual path, greatly improving the performance of the static scaling.

The results shown in these two extreme scenarios allow us to conclude that accounting for EbyE fluctuations do not affect the performance of the power-law scaling in a sizeable way. Thus, we can still use the power-law scenario as reference when employing scaling laws and the resulting spectrum should provide a precise description of the spectrum computed along the path.

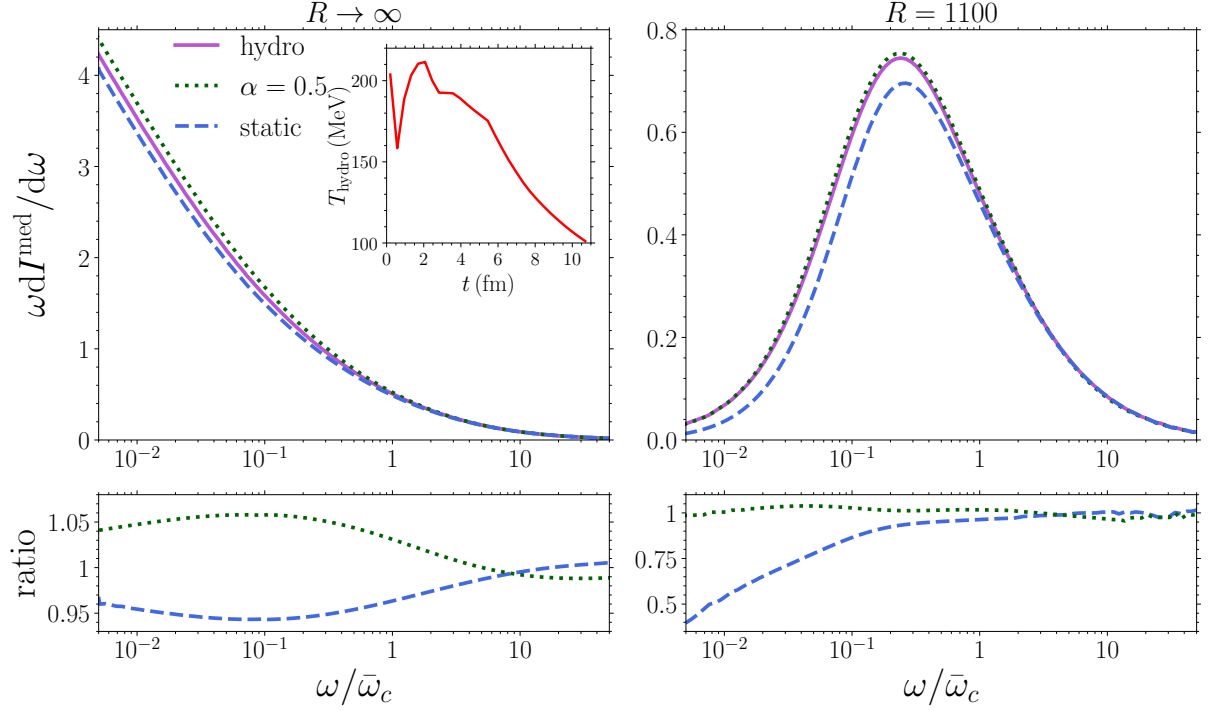


Figure 4.11: Top: all order in-medium energy spectrum for the Yukawa collision rate for  $\chi = 4.4$ , and  $\bar{R} \rightarrow \infty$  (left) or  $\bar{R} = 1100$  (right) as a function of the rescaled gluon energy  $\omega/\bar{\omega}_c$ . Purple solid lines correspond to the spectra computed along the trajectory whose temperature profile is shown in the inset figure, which was sampled over a central event in  $\sqrt{s_{\text{NN}}} = 2.76$  TeV Pb-Pb collisions at the LHC. Blue dashed and green dotted lines correspond, respectively, to the spectrum evaluated using the static (4.19)-(4.21) and power-law (4.23)-(4.25) scaling laws. Bottom: green dotted (blue dashed) lines represent the ratio of the spectrum computed with the power-law (static) scaling laws w.r.t. the spectrum evaluated along the path with temperature profile showcased in the inset figure. Figure extracted from [3] under a Creative Commons license, CC-BY 4.0.

## 4.B Results for the HTL interaction model

In this appendix we provide results of the performance of the power-law matching when employing a different collision rate model, the HTL, in order to illustrate the flexibility of our approach. We start by briefly remembering the expression of the HTL potential

$$\frac{1}{2}n(s)V_{\text{H}}(s;\mathbf{q}) = \frac{g_s^2 N_c m_{\text{D}}^2(s)T(s)}{\mathbf{q}^2(\mathbf{q}^2 + m_{\text{D}}^2(s))}, \quad (4.26)$$

where we have explicitly written the temporal dependence on the Debye mass  $m_{\text{D}}(s)$  and medium temperature  $T(s)$ . We have already explained in chapter 2 how to evaluate the fully resummed medium-induced radiation spectrum using this collision rate. The

interested reader can also see Ref. [58] for further details. We also discussed in chapter 2 that the parameters we need to specify to compute the spectrum with this model are  $T$ ,  $m_D^2$  and  $L$ , but for convenience we define (2.63)

$$\chi_H = TL, \quad \bar{\omega}_c^H = \frac{1}{2}m_D^2 L, \quad \text{and} \quad \bar{R}_H = \bar{\omega}_c^H L. \quad (4.27)$$

When computing the spectrum in realistically evolving media we need to specify a relationship between the Debye mass and the local properties of the medium. This relation at LO in the HTL approach is given by [57]

$$m_{D,\text{hydro}}^2(t) = c_2 T_{\text{hydro}}^2(\xi(t)), \quad (4.28)$$

where  $T_{\text{hydro}}$  is the temperature of the medium along the path followed by the emitter  $\xi(t)$  which will be extracted from the hydrodynamical model [111, 112]. We notice that at LO in the HTL approach the  $c_2$  parameter is given by  $c_2 = g_s^2(1 + N_f/6)$ . However, for convenience and in analogy to the Yukawa potential we decide to leave  $c_2$  as a free parameter.

#### 4.B.1 Scaling laws using the static scenario as reference

In analogy with the work done for the Yukawa-type interaction, we can define the following scaling laws which relate a spectrum computed along a hydrodynamic path with an equivalent static scenario

$$\chi_H = c_1 \int_{\tau_{\text{hydro}}}^{L+\tau_{\text{hydro}}} dt T_{\text{hydro}}(t), \quad (4.29)$$

$$\chi_H \bar{\omega}_c^H = c_1 \int_{\tau_{\text{hydro}}}^{L+\tau_{\text{hydro}}} dt t T_{\text{hydro}}(t) m_{D,\text{hydro}}^2(t), \quad (4.30)$$

$$\chi_H \bar{R}_H = \frac{3c_1}{2} \int_{\tau_{\text{hydro}}}^{L+\tau_{\text{hydro}}} dt t^2 T_{\text{hydro}}(t) m_{D,\text{hydro}}^2(t), \quad (4.31)$$

with  $c_1$  a constant parameter which plays the role of  $k_1$  for the Yukawa case (4.8).

#### 4.B.2 Scaling laws using the power-law scenario as reference

We can also follow the work done in section 4.6 for the Yukawa model and define for the HTL potential a set of relationships between the spectrum computed in a general path and an equivalent power-law expanding scenario. In this case we parametrize the Debye mass  $m_D$  and the medium temperature  $T$  with the following relations

$$T(t) = \frac{\bar{T}_0}{(t + \bar{t}_0)^\alpha}, \quad \text{and} \quad m_D^2(t) = \frac{\bar{m}_{D0}^2}{(t + \bar{t}_0)^{2\alpha}}, \quad (4.32)$$

here  $\bar{T}_0$  and  $\bar{m}_{D0}$  are, respectively, proportional to the maximum value of the temperature and Debye mass, which are reached at the initial time  $\bar{t}_0$ . As done for the Yukawa-type interaction, we set the following values for the extra free parameters  $\alpha = 0.5$  and  $t_0 = \bar{t}_0/\bar{L} = 0.1$ .

Then, now we need to establish three scaling laws, as we have the same amount of parameters as in the static scenario. For the power-law matching we choose

$$\int_0^{\bar{L}} dt T(t) = \int_{\tau_{\text{hydro}}}^{L+\tau_{\text{hydro}}} dt T_{\text{hydro}}(t), \quad (4.33)$$

$$\int_0^{\bar{L}} dt t T(t) m_D^2(t) = \int_{\tau_{\text{hydro}}}^{L+\tau_{\text{hydro}}} dt t T_{\text{hydro}}(t) m_{D,\text{hydro}}^2(t), \quad (4.34)$$

$$\int_0^{\bar{L}} dt t^2 T(t) m_D^2(t) = \int_{\tau_{\text{hydro}}}^{L+\tau_{\text{hydro}}} dt t^2 T_{\text{hydro}}(t) m_{D,\text{hydro}}^2(t), \quad (4.35)$$

with (4.34) ensuring the correct matching of the high energy tails of both spectra (see section 4.4).

Fig. 4.12 illustrates the results obtained for the gluon energy spectrum evaluated along the same path employed in Fig. 4.5 whose temperature profile is showcased in the inset figure, i.e., a straight-line trajectory sampled with a central production point for the 0 – 10% centrality class in  $\sqrt{s_{\text{NN}}} = 5.02$  TeV Pb-Pb collisions at the LHC. Here we plot the different spectra as function of the rescaled gluon energy  $\omega/\bar{\omega}_c^H$ , with  $\bar{\omega}_c^H$  defined through (4.30): the purple solid line stands for the spectrum computed along the path, the blue dashed curve correspond to the spectrum coming from the static scaling laws (4.29)-(4.31), and the green dotted line represents the spectrum evaluated for the power-law matching (4.33)-(4.35). The values of the parameters employed here were  $c_1 = 0.2$  and  $c_2 = 121$ , which for this particular trajectory correspond to  $\chi_H = 2$  and  $\bar{R}_H \rightarrow \infty$  (left), and  $\chi_H = 2$  and  $R_H = 2700$  (right) in terms of static variables. The outcomes presented in Fig. 4.12 are essentially the same we already got in Fig. 4.5 for the Yukawa model: the static scenario has an acceptable performance describing the spectrum computed along realistic paths when the kinematic constraint is lifted (left), but it gives large errors for small gluon energies when it is imposed (right). The power-law approach greatly improves these results, providing an excellent approximation of the spectrum along the trajectory for any condition and energy.

We conclude by showing in Fig. 4.13 the performance of the power-law scaling with the HTL collision rate in an extreme scenario which possess a non-monotonically decreasing temperature, illustrated in the inset figure. The chosen trajectory is the same we previously employed for the Yukawa collision rate in Fig. 4.8. Here we have set the same parameter values as in the previous figure  $c_1 = 0.2$  and  $c_2 = 121$ , which for this path correspond to  $\chi_H = 2.7$  and  $\bar{R}_H \rightarrow \infty$  (left), and  $\chi_H = 2.7$  and  $\bar{R}_H = 7070$  (right). Again, the results obtained for the HTL parton-medium interaction model align perfectly with previous findings for the Yukawa-type collision rate in Fig. 4.8: the power-law scaling

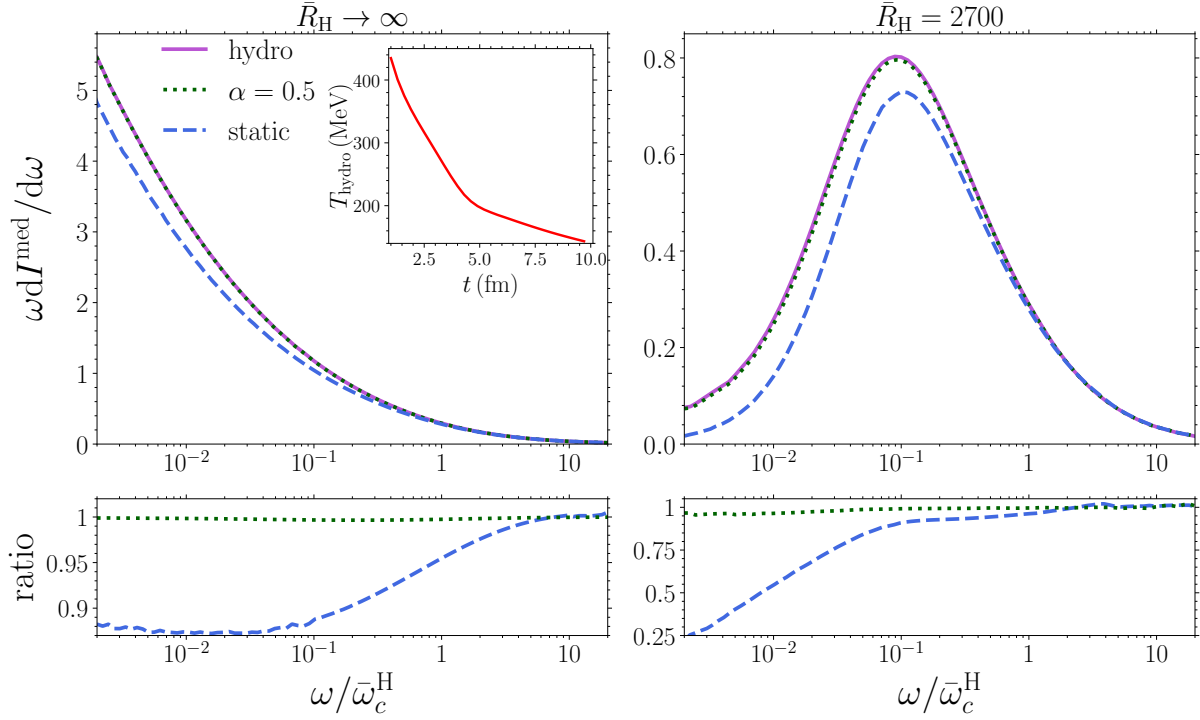


Figure 4.12: Top: all order in-medium energy spectrum for the HTL elastic collision rate with  $\chi_H = 2$ , and  $\bar{R}_H \rightarrow \infty$  (left) or  $\bar{R}_H = 2700$  (right) as a function of the rescaled gluon energy  $\omega/\bar{\omega}_c^H$ . Purple solid lines correspond to the spectra computed along the trajectory whose temperature profile is shown in the inset figure, which was sampled with a central production point over the 0 – 10% centrality class in  $\sqrt{s_{\text{NN}}} = 5.02$  TeV Pb-Pb collisions at the LHC. Blue dashed and green dotted lines correspond, respectively, to the spectrum evaluated using the static (4.29)-(4.31) and power-law (4.33)-(4.35) scaling laws. Bottom: green dotted (blue dashed) lines represent the ratio of the spectrum computed with the power-law (static) scaling laws w.r.t. the spectrum evaluated along the path with temperature profile showcased in the inset figure. Figure extracted from [3] under a Creative Commons license, CC-BY 4.0.

provides a more accurate description of the spectrum computed along the path than the static scenario. For energies smaller than the characteristic gluon energy the deviations observed always remain below 15%, being significantly smaller than the ones observed in the static result.

Taking all of this in account, we can assure that the power-law approach we presented here provides an accurate description of the spectrum across the entire range of gluon energies for realistic trajectories sampled over hydrodynamic simulations, even for the most extreme conditions regarding the temperature behaviour during the initial times. Moreover, the agreement between the results obtained employing the Yukawa and HTL models manifest the robustness of the approach in capturing the behavior of the full medium-

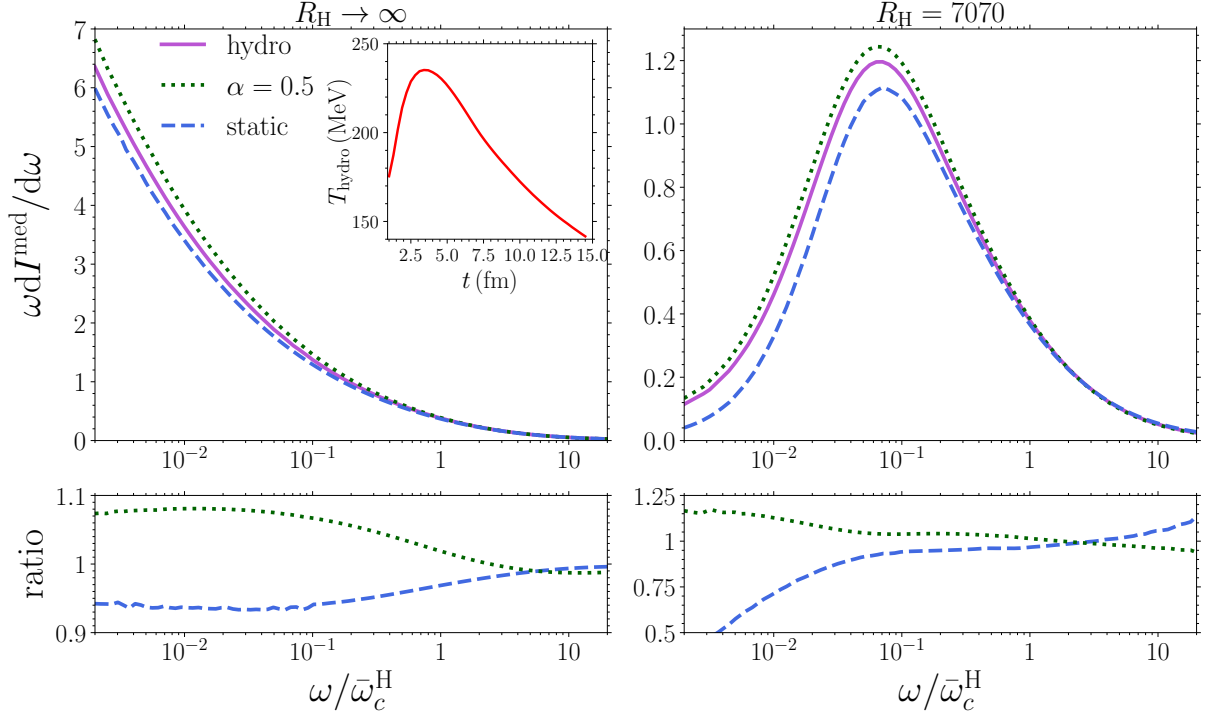


Figure 4.13: Top: all order in-medium energy spectrum for the HTL elastic collision rate with  $\chi_H = 2.7$ , and  $\bar{R}_H \rightarrow \infty$  (left) or  $\bar{R}_H = 7070$  (right) as a function of the rescaled gluon energy  $\omega/\bar{\omega}_c^H$ . Purple solid lines correspond to the spectra computed along the trajectory whose temperature profile is shown in the inset figure, which was sampled with a non-central production point over the 0 – 10% centrality class in  $\sqrt{s_{\text{NN}}} = 5.02$  TeV Pb-Pb collisions at the LHC. Blue dashed and green dotted lines correspond, respectively, to the spectrum evaluated using the static (4.29)-(4.31) and power-law (4.33)-(4.35) scaling laws. Bottom: green dotted (blue dashed) lines represent the ratio of the spectrum computed with the power-law (static) scaling laws w.r.t. the spectrum evaluated along the path with temperature profile showcased in the inset figure. Figure extracted from [3] under a Creative Commons license, CC-BY 4.0.

induced gluon energy distribution, regardless of the specific parton-medium interaction model employed.

MARCOS GONZÁLEZ MARTÍNEZ

# 5

## The role of initial stages on jet quenching

To comply with the regulations for doctoral studies of the University of Santiago de Compostela we include here the full information of the article on which this chapter is based:

Medium-induced radiation with vacuum propagation in the pre-hydrodynamics phase, *J. High Energ. Phys.* **03** (2023) 189

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#### PhD Student Contribution

Active participation in all the calculations presented in the paper, as well as in the discussions, meetings and writing of the paper. All of the authors have the same contribution to the publication.

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The final chapter of the thesis is dedicated to addressing a very important goal of this manuscript: what is the role (if any) that initial stages of heavy ion collisions play in jet quenching? For the first time, the role of vacuum radiation within the short period of time before thermalization is here studied in detail. This contribution has been overlooked in previous works, and its study allow us to obtain information about the temporal structure of the QCD collectivity in the first instants, which is one of the main goals of this thesis, as its title states.

To analyze the impact of the early stages of the collision in the propagation of hard particles travelling through a hot and dense QGP, we will employ the gluon radiation spectrum with full resummation over multiple scatterings explained in chapter 2, which was further extended to more realistic longitudinally expanding media in chapter 4.

The outlook of the chapter is as follows: first, we present the different models for the initial stages that we will analyze. Then, we compute the emission spectrum for every scenario described (both with the full result and analytical approaches). Comparing the spectra evaluated in these different conditions we can obtain information about the effect that early stages have at the level of the spectrum. Finally, we use the spectrum computed for these different assumptions regarding the description of the initial stages of the propagation to analyze the impact that they have when simultaneously describing two jet quenching observables: the single-inclusive particle suppression  $R_{AA}$  and the high- $p_T$  azimuthal asymmetry  $v_2$ . The findings of this chapter are published in [4].

## 5.1 Different scenarios for the initial stages

Throughout the development of this thesis we have been studying jet quenching, one of the few tools that we can employ which witnesses the whole evolution of the system in heavy ion collisions. However, in all of the theoretical calculations we presented here, we have been neglecting the initial part of the evolution, focusing just on the interaction between the hard parton and the QGP in thermal equilibrium. This is a common practice in phenomenological studies, which usually ignore the initial stages and start the propagation when the hydrodynamical description of the medium is applicable. The justification to do so is that the formation time of the QGP is short compared with its length, thus for a jet traveling through it the initial phase of the propagation should be subdominant. Nevertheless, recently it was shown that this assumption breaks when simultaneously describing multiple jet observables [119, 120], resulting in the description of the early stages being crucial. Moreover, there are recent results coming from the Color Glass Condensate community pointing out that the early Glasma phase of the evolution may have a very high impact on jet quenching [121–125].

Here we want to overcome this limitation and allow the original hard parton to propagate before the hydrodynamization of the medium, thus accounting for gluons radiated before the creation of the QGP and its interference with emissions taking place inside the medium. This additional radiation may play an important role in jet quenching studies, as different trajectories through the QGP will have different initial stages propagation and, therefore, different energy losses. We remark that we are not referring to the case where the parent parton is an on-shell particle created at infinity, which was already discussed in [126]. Instead, we assume that our emitter is created at the collision point and before the medium hydrodynamization.

To decouple the production time of the hard parton and the creation of the medium, which are considered to be the same in theoretical calculations, we define two different

time variables. First, if we have a hard parton with energy  $E$  then  $\tau_p \sim 1/E \ll 1$  fm represents its production time. Second, the creation time of the medium is assumed to be the same as the initial time of applicability of the hydrodynamic model and we denote it by  $\tau_m \sim 1$  fm. Since treating in detail the interactions of the emitter in the pre-hydrodynamics phase is beyond the scope of the current manuscript, we simply consider that there are no interactions and it just propagates as in vacuum. By doing so we are able to isolate the contribution from radiation coming from the initial stages and compare it with another scenarios where the production time of the parton is inside the medium but with different starting points. The motivation for this selection is given by novel results [119, 120] pointing out the strong suppression of jet quenching in the pre-hydrodynamic phase required in present energy loss implementations to describe  $R_{AA}$  and high- $p_T$   $v_2$  simultaneously.

The variation of the starting time of the energy loss has already been addressed in previous works [116, 119, 127–129] corroborating the sensitivity of jet quenching to initial stages radiation. Here we compare the medium-induced radiation spectrum derived by changing the starting point of the parton-medium interactions with and without contributions from early stages, thus determining the size of the effect coming from the initial stages emissions in the pre-hydrodynamics phase. The different scenarios we will study to achieve this purpose are represented in Fig. 5.1 and they are characterised by:

**Case A:**  $\tau_p = \tau_m = 0$ . The initial hard parton is produced simultaneously along with the medium. In this scenario the propagation of the emitter is through a medium of length  $L$  which starts at 0. This is the standard setup employed in theoretical computations of the in-medium gluon emission spectrum.

**Case B:**  $\tau_p = \tau_m > 0$ . The emitter is produced simultaneously along with the medium, but the starting point is  $\tau_m$  instead of 0. The only difference between this scenario and the previous one is the length of the medium, which now is  $L - \tau_m$  instead of  $L$ , but both cases are formally equivalent.

**Case C:**  $\tau_m > \tau_p = 0$ . The parent parton is created at  $\tau_p = 0$ , before the hydrodynamization of the medium which occurs at a later time  $\tau_m > \tau_p$ . Thus, the emitter propagates through vacuum during the early stages from 0 to  $\tau_m$ , and once the medium is formed it travels throughout it from  $\tau_m$  to  $L$ . This last case corresponds to a medium of longitudinal extension  $L - \tau_m$ , but taking in account the initial vacuum propagation generates some extra medium-induced emissions in comparison with scenario B. These additional radiations take place before  $\tau_m$  and increase the total emission spectrum. We emphasize that all of the emissions we account for in this manuscript are induced by the medium, vacuum emissions are always neglected unless otherwise specified.

To analyze these three different scenarios we will evaluate their respective medium-induced radiation spectrum and compare them numerically. We are going to compute the

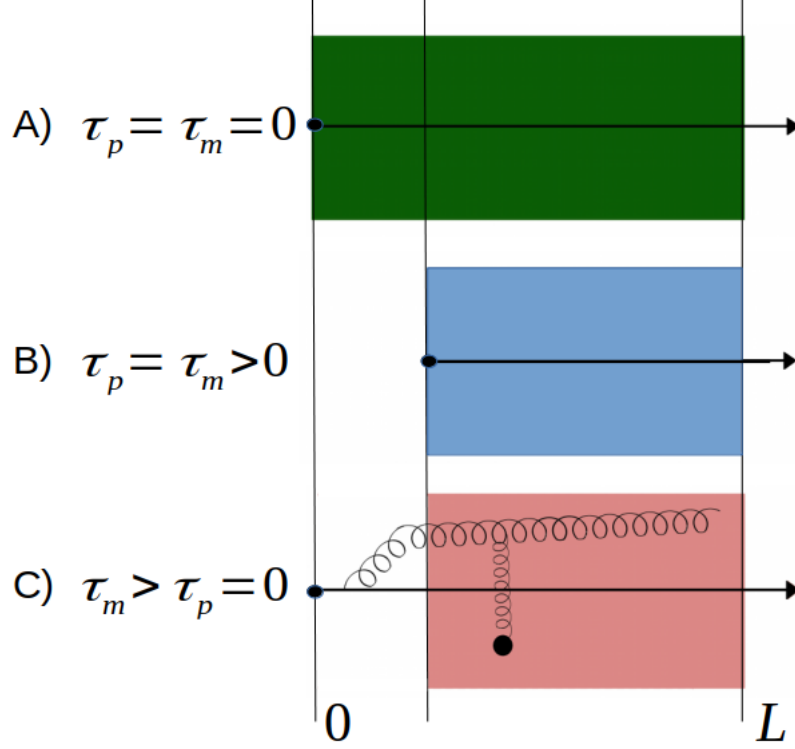


Figure 5.1: Diagrammatic representation of the three different cases studied in this work: scenario A:  $\tau_p = \tau_m = 0$  (top panel); scenario B:  $\tau_p = \tau_m > 0$  (middle panel); and scenario C:  $\tau_m > \tau_p = 0$  (bottom panel). In the last panel, corresponding to case C, the radiated gluon represented illustrates the new kind of contributions we account for only in this scenario in comparison with A and B. They correspond to gluons radiated at times prior to  $\tau_m$  which interfere with the later generated medium. Of course, in this case we also take in account emissions produced inside the medium, which are the only ones considered in the previous two cases A and B. Figure extracted from [4] under a Creative Commons license, CC-BY 4.0.

spectra employing different frameworks: the all-order fully resummed BDMPS-Z spectrum introduced in chapter 2, the  $N = 1$  first opacity result and the HO approximation. Once that we have computed the spectrum, we will employ this energy loss calculation to derive, in a simplified setup, proxies of the charged hadron nuclear modification factor  $R_{AA}$  and the high- $p_T$  azimuthal asymmetry  $v_2$ , describing them simultaneously for the 20 – 30% centrality class in  $\sqrt{s_{NN}} = 2.76$  TeV Pb-Pb collisions at the LHC, hence quantifying the impact of early stages radiation on the simultaneous description of jet quenching observables.

## 5.2 Generalized all-order BDMPS-Z gluon radiation spectrum

We have already exhaustively discussed the BDMPS-Z spectrum in all of the previous chapters of the thesis, but until now we have been implicitly assuming that the initial hard parton and the medium are both created at the same time equal to zero. Here we are going to generalize this computation to the case where the emitter is produced at some time  $\tau_p$  and the medium is formed in a generic time  $\tau_m$ , and both times are allowed to be different. We start the derivation with the in-medium gluon radiation spectrum differential in transverse momentum eq. (2.6)

$$\omega \frac{dI^{\text{Full}}}{d\omega d^2\mathbf{k}} = \frac{\alpha_s C_R}{2\pi^2 \omega} \text{Re} \int_{\tau_p}^L ds n(s) \int_{\tau_p}^s dt \int_{pql} i\mathbf{p} \cdot \left( \frac{\mathbf{l}}{l^2} - \frac{\mathbf{q}}{q^2} \right) \sigma(\mathbf{l} - \mathbf{q}) \tilde{\mathcal{K}}(s, \mathbf{q}; t, \mathbf{p}) \mathcal{P}(L, \mathbf{k}; s, \mathbf{l}), \quad (5.1)$$

where we have set the lower time integral limits to the initial hard parton production point  $\tau_p$ , which is usually assumed to be zero. The linear density of the scattering centers  $n(s)$  is non-zero only inside the medium, which before in (2.6) corresponded to longitudinal coordinates from 0 to  $L$ , but now, as we want to describe the scenario C) shown in the bottom panel of Fig. 5.1, it corresponds to the region between  $\tau_m$  and  $L$ . Then, the time integral over  $s$  has a different lower limit

$$\omega \frac{dI^{\text{Full}}}{d\omega d^2\mathbf{k}} = \frac{\alpha_s C_R}{2\pi^2 \omega} \text{Re} \int_{\tau_m}^L ds n(s) \int_{\tau_p}^s dt \int_{pql} i\mathbf{p} \cdot \left( \frac{\mathbf{l}}{l^2} - \frac{\mathbf{q}}{q^2} \right) \sigma(\mathbf{l} - \mathbf{q}) \tilde{\mathcal{K}}(s, \mathbf{q}; t, \mathbf{p}) \mathcal{P}(L, \mathbf{k}; s, \mathbf{l}). \quad (5.2)$$

Notice that if we set  $\tau_p = \tau_m = 0$  we recover the standard theoretical case where the emitter is produced at time 0 and travels through a medium of length  $L$  (2.6), represented in the top panel of Fig. 5.1. The numerical evaluation of this spectrum was extensively discussed in chapter 2 for a static medium, presenting numerical results in chapter 3, and in chapter 4 we went beyond the brick approximation and computed it for generic longitudinally evolving media. This also applies to the spectrum of the case where the initial stages are completely neglected, corresponding to B) shown in the middle panel of Fig. 5.1, as it is formally analogous to scenario A). Hence, now we focus on the more general case where the emitter is produced before the hydrodynamization of the medium ( $\tau_p < \tau_m$ ). By splitting the integral over  $t$  at  $\tau_m$  in (5.2) we are able to isolate the contribution arising from the initial gluon emissions

$$\begin{aligned} \omega \frac{dI^{\text{Full}}}{d\omega d^2\mathbf{k}} = & \frac{\alpha_s C_R}{2\pi^2 \omega} \text{Re} \int_{\tau_m}^L ds n(s) \int_{\tau_p}^{\tau_m} dt \int_{pql} i\mathbf{p} \cdot \left( \frac{\mathbf{l}}{l^2} - \frac{\mathbf{q}}{q^2} \right) \sigma(\mathbf{l} - \mathbf{q}) \\ & \times \tilde{\mathcal{K}}(s, \mathbf{q}; t, \mathbf{p}) \mathcal{P}(L, \mathbf{k}; s, \mathbf{l}) \\ & + \frac{\alpha_s C_R}{2\pi^2 \omega} \text{Re} \int_{\tau_m}^L ds n(s) \int_{\tau_m}^s dt \int_{pql} i\mathbf{p} \cdot \left( \frac{\mathbf{l}}{l^2} - \frac{\mathbf{q}}{q^2} \right) \sigma(\mathbf{l} - \mathbf{q}) \\ & \times \tilde{\mathcal{K}}(s, \mathbf{q}; t, \mathbf{p}) \mathcal{P}(L, \mathbf{k}; s, \mathbf{l}). \end{aligned} \quad (5.3)$$

Using the following convolution property of the emission kernel we can further simplify the first term in (5.3)

$$\tilde{\mathcal{K}}(s, \mathbf{q}; t, \mathbf{p}) = \int_{\mathbf{k}_1} \tilde{\mathcal{K}}(s, \mathbf{q}; \tau_m, \mathbf{k}_1) \tilde{\mathcal{K}}_0(\tau_m, \mathbf{k}_1; t, \mathbf{p}) = \tilde{\mathcal{K}}(s, \mathbf{q}; \tau_m, \mathbf{p}) e^{-i \frac{\mathbf{p}^2}{2\omega} (\tau_m - t)}, \quad (5.4)$$

here we employed that the kernel, for times prior to  $\tau_m$ , corresponds to the vacuum one  $\tilde{\mathcal{K}}_0$  (we have no medium), which is given by (2.49)

$$\tilde{\mathcal{K}}_0(\tau_m, \mathbf{k}_1; t, \mathbf{p}) = (2\pi)^2 \delta^2(\mathbf{k}_1 - \mathbf{p}) e^{-i \frac{\mathbf{p}^2}{2\omega} (\tau_m - t)}. \quad (5.5)$$

Inserting (5.4) in the first term of eq. (5.3) we can perform the  $t$ -integration, yielding

$$\begin{aligned} \omega \frac{dI^{\text{Full}}}{d\omega d^2\mathbf{k}} &= \frac{\alpha_s C_R}{\pi^2} \text{Re} \int_{\tau_m}^L ds n(s) \int_{pql} \frac{\mathbf{p}}{\mathbf{p}^2} \cdot \left( \frac{\mathbf{l}}{l^2} - \frac{\mathbf{q}}{q^2} \right) \left( 1 - e^{-i \frac{\mathbf{p}^2}{2\omega} (\tau_m - \tau_p)} \right) \sigma(\mathbf{l} - \mathbf{q}) \\ &\quad \times \tilde{\mathcal{K}}(s, \mathbf{q}; \tau_m, \mathbf{p}) \mathcal{P}(L, \mathbf{k}; s, \mathbf{l}) \\ &+ \frac{\alpha_s C_R}{2\pi^2 \omega} \text{Re} \int_{\tau_m}^L ds n(s) \int_{\tau_m}^s dt \int_{pql} i\mathbf{p} \cdot \left( \frac{\mathbf{l}}{l^2} - \frac{\mathbf{q}}{q^2} \right) \sigma(\mathbf{l} - \mathbf{q}) \\ &\quad \times \tilde{\mathcal{K}}(s, \mathbf{q}; t, \mathbf{p}) \mathcal{P}(L, \mathbf{k}; s, \mathbf{l}). \end{aligned} \quad (5.6)$$

In this equation the first term takes in account the additional radiation coming from the propagation during the early stages, whereas that the second contribution is just the spectrum of the scenario where the emitter is produced at the same time as the medium  $\tau_m = \tau_p$ . Clearly, when we set  $\tau_m = \tau_p$  the first term in (5.6) vanishes and (5.6) reduces to the original spectrum (5.1).

In this study we are going to focus on the energy distribution rather than on the transverse momentum differential spectrum, thus we have to integrate the  $\mathbf{k}$ -differential result over transverse momentum

$$\omega \frac{dI^{\text{Full}}}{d\omega} = \int_0^\omega dk k \int_0^{2\pi} d\theta_k \omega \frac{dI^{\text{Full}}}{d\omega d^2\mathbf{k}}. \quad (5.7)$$

As we have done throughout the whole thesis, we will consider both energy distributions, either with the restriction over the transverse momentum phase space of the radiated gluon imposed ( $\bar{R}$  finite) or lifted ( $\bar{R} \rightarrow \infty$ ). We recall again that the later case does not correspond to any physical situation, leading to divergences in the spectrum for small radiated gluon energies, but if we keep the kinematical restriction finite then the divergence vanishes. Integrating (5.6) over all of the transverse momentum phase space the broadening factor disappears, and realizing that the second term inside the brackets is zero for both contributions we get

$$\begin{aligned} \omega \frac{dI^{\text{Full}}}{d\omega} &= 4\alpha_s C_R \text{Re} \int_{\tau_m}^L ds n(s) \int_{pql} \frac{\mathbf{p} \cdot \mathbf{l}}{\mathbf{p}^2 l^2} \left( 1 - e^{-i \frac{\mathbf{p}^2}{2\omega} (\tau_m - \tau_p)} \right) \sigma(\mathbf{l} - \mathbf{q}) \tilde{\mathcal{K}}(s, \mathbf{q}; t, \mathbf{p}) \\ &+ \frac{2\alpha_s C_R}{\omega} \text{Re} \int_{\tau_m}^L ds n(s) \int_{\tau_m}^s dt \int_{pql} i \frac{\mathbf{p} \cdot \mathbf{l}}{l^2} \sigma(\mathbf{l} - \mathbf{q}) \tilde{\mathcal{K}}(s, \mathbf{q}; t, \mathbf{p}). \end{aligned} \quad (5.8)$$

We will employ the Yukawa collision rate (1.26) throughout the development of this chapter. This choice is made because we do not intend to describe the small energy region of the spectrum and, as we discussed in chapter 3, in the intermediate and large energy regimes of the emission spectrum the particular parton-medium interaction model employed to describe the collision rate is irrelevant, as long as it has the correct large- $\mathbf{p}$  behaviour  $V(\mathbf{p}) \sim 1/\mathbf{p}^4$  (see Fig. 3.3). Therefore, we can use either the Yukawa-type (1.26) or the HTL (1.27) model and, for convenience, we choose the former.

In this section we will restrict ourselves to the most simple scenario, employing the brick approximation to describe the medium profile,  $n(s) = n_0$  for  $s \in [\tau_m, L]$ , and the potential screening mass,  $\mu^2(s) = \mu^2$  for  $s \in [\tau_m, L]$ . Given that we are going to analyze only scenarios with either  $\tau_p = 0$  or  $\tau_p = \tau_m$ , then the number of parameters we need to specify to compute the spectrum is four, the three well-know  $n_0$ ,  $L$  and  $\mu^2$ , and the recently introduced  $\tau_m$ . As always, we will choose to work with dimensionless variables when numerically evaluating the spectrum and, therefore, the following parameters are more suitable

$$n_0 L, \quad \frac{\tau_m}{L}, \quad \bar{\omega}_c \equiv \frac{1}{2} \mu^2 L, \quad \bar{R} \equiv \bar{\omega}_c L, \quad (5.9)$$

with these selection for the parameters the energy distribution depends on  $\omega/\bar{\omega}_c$  instead of  $\omega$  and  $\bar{\omega}_c$  independently, as we have already checked in previous chapters.

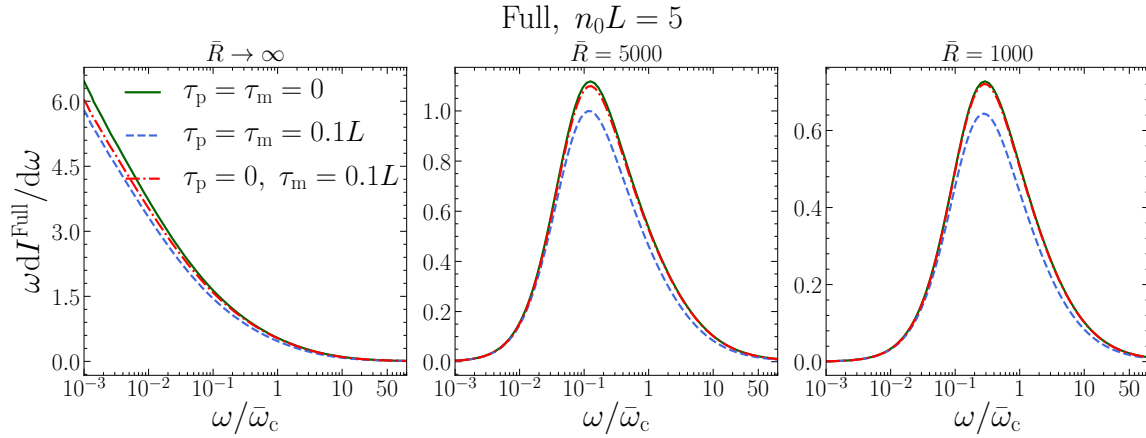


Figure 5.2: All-order in-medium energy distribution with full resummation over multiple scatterings for the three different scenarios analyzed:  $\tau_p = \tau_m = 0$  (green solid lines);  $\tau_p = \tau_m = 0.1 L$  (blue dashed lines); and  $\tau_p = 0$  and  $\tau_m = 0.1 L$  (red dashed dotted lines) as a function of  $\omega/\bar{\omega}_c$  for  $n_0 L = 5$ . Different panels correspond to different values of the kinematical constraint  $\bar{R}$ :  $\bar{R} \rightarrow \infty$  (left),  $\bar{R} = 5000$  (central), and  $\bar{R} = 1000$  (right). Figure extracted from [4] under a Creative Commons license, CC-BY 4.0.

We show in Figs. 5.2 and 5.3 our results for the numerical computation of the energy distribution with full resummation over multiple scatterings in a brick for the three different scenarios illustrated in Fig. 5.1, which correspond to different considerations regarding the propagation during the initial stages. As in the rest of the thesis, we work

with  $C_R = C_F = 4/3$  and  $\alpha_s = 0.3$ , for illustration purposes. We also choose  $n_0 L = 5$  and plot the different spectra as function of the rescaled gluon energy  $\omega/\omega_c$ . The colors of the different curves were chosen to match the scenario they correspond to in Fig. 5.1: green solid curves for case A)  $\tau_p = \tau_m = 0$ , blue dashed lines correspond to scenario B)  $\tau_p = \tau_m > 0$ , and red dashed dotted lines for case C)  $\tau_m > \tau_p = 0$ . In Fig. 5.2 we set  $\tau_m/L = 0.1$ , whereas that in Fig. 5.3 we employed  $\tau_m/L = 0.3$ , except for the green curves which always have  $\tau_m = 0$ . In both figures from left to right we have decreasing values of the kinematical cut-off: on the left plot it is shown the energy distribution with the kinetic constraint removed ( $\bar{R} \rightarrow \infty$ ), on the central panel we set  $\bar{R} = 5000$ , and on the right panel  $\bar{R} = 1000$ .

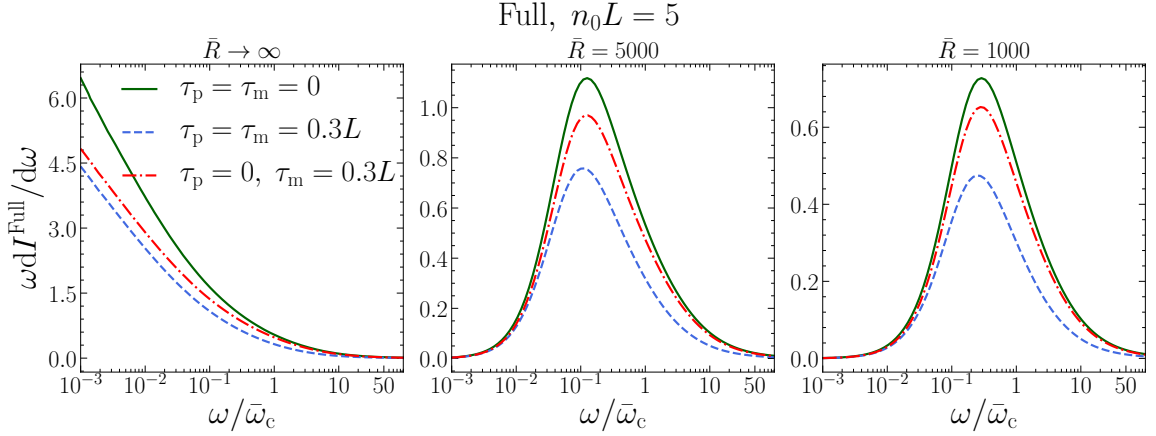


Figure 5.3: All-order in-medium energy distribution with full resummation over multiple scatterings for the three different scenarios analyzed:  $\tau_p = \tau_m = 0$  (green solid lines);  $\tau_p = \tau_m = 0.3L$  (blue dashed lines); and  $\tau_p = 0$  and  $\tau_m = 0.3L$  (red dashed dotted lines) as a function of  $\omega/\bar{\omega}_c$  for  $n_0 L = 5$ . Different panels correspond to different values of the kinematical constraint  $\bar{R}$ :  $\bar{R} \rightarrow \infty$  (left),  $\bar{R} = 5000$  (central), and  $\bar{R} = 1000$  (right). Figure extracted from [4] under a Creative Commons license, CC-BY 4.0.

The first important feature we want to comment is the fact that, under the same conditions, the spectrum corresponding to completely neglect the initial stages  $\tau_p = \tau_m > 0$  (blue dashed curves) is always smaller than the one which considers  $\tau_p = \tau_m = 0$  (green solid lines), with the difference between curves increasing for bigger values of  $\tau_m$ . This is clearly seen by comparing Figs. 5.2 and 5.3, and is something that we anticipated. As we previously discussed, both cases are formally equivalent, being the only difference between them the length of the medium, which is  $L$  for  $\tau_p = \tau_m = 0$ , but in the  $\tau_p = \tau_m > 0$  case it corresponds to  $L - \tau_m$ , being a shorter length than for the former scenario when  $\tau_m \neq 0$ . Hence, if we employ the same medium parameters in both cases to compute the spectrum, it is obvious that the one corresponding to  $\tau_p = \tau_m > 0$  is always going to be smaller.

Analyzing now the scenario which includes the initial stages radiation in vacuum  $\tau_m > \tau_p = 0$  (red dashed dotted curves) we see in Figs. 5.2 and 5.3 that this case is

always somewhere in between the other two. When considering a small  $\tau_m/L$ , as in Fig. 5.2, the differences between this scenario (red dashed dotted lines) and the one in which the medium is formed at  $\tau_m = 0$  (green solid curves) are almost negligible, only been noticeable on the left panel of Fig. 5.2 for very small gluon energies. However, when increasing the value of  $\tau_m/L$  these differences are more notorious, as it can be seen in Fig. 5.3, where both spectra only have a good agreement in the high energy region for all panels.

The fact that highly energetic gluons seem to be insensitive to the subtraction of part of the medium during the early times of the propagation can be understood in terms of the formation time (1.39). As we discussed in section 3.1, harder gluons need larger transverse momentum transfers to decorrelate from the emitter, thus having bigger formation times. Gluons which have a formation time larger than  $\tau_m$  can be radiated before the creation of the medium and still undergo its effects before being completely decorrelated from the emitter. However, gluons with a small formation time only contribute to the medium-induced radiation spectrum if they are emitted after the medium is created. In following sections, 5.3 and 5.4, we will prove how the relevant energy scales that separate both regimes appear when we use analytical approaches, like the single hard scattering (GLV, section 5.3) and multiple soft scatterings (HO, section 5.4) approximations.

### 5.3 GLV limit

As we already discussed in chapter 2, the single scattering contribution to the full spectrum can be derived by replacing the emission kernel  $\tilde{\mathcal{K}}$  and the momentum broadening  $\mathcal{P}$  by their vacuum versions (2.49) and (2.48), respectively. Introducing eqs. (2.48) and (2.49) in the general spectrum (5.2) we can perform the integral over time  $t$ , as all of the dependence on this variable is in the exponential of the kernel (2.49). By doing so we get that the GLV transverse momentum dependent spectrum is given by

$$\omega \frac{dI^{\text{GLV}}}{d\omega d^2\mathbf{k}} = \frac{\alpha_s C_R}{\pi^2} \text{Re} \int_{\tau_m}^L ds n(s) \int_{\mathbf{p}} \frac{1}{\mathbf{p}^2} \left( \frac{\mathbf{p} \cdot \mathbf{k}}{\mathbf{k}^2} - 1 \right) \left( 1 - e^{-i \frac{\mathbf{p}^2}{2\omega} (s - \tau_p)} \right) \sigma(\mathbf{k} - \mathbf{p}). \quad (5.10)$$

With this equation we can see that the generalized GLV medium-induced radiation spectrum where we allow the emitter to be produced before the hydrodynamization of the medium ( $\tau_p < \tau_m$ , illustrated in the bottom picture of Fig. 5.1), can be written as two different contributions off hard partons produced inside the medium (top picture of Fig. 5.1) with different longitudinal extensions

$$\begin{aligned} \omega \frac{dI^{\text{GLV}}}{d\omega d^2\mathbf{k}} \Big|_{\tau_m > \tau_p} &= \omega \frac{dI^{\text{GLV}}}{d\omega d^2\mathbf{k}} \Big|_{\tau_m = \tau_p} - \frac{\alpha_s C_R}{\pi^2} \text{Re} \int_{\tau_p}^{\tau_m} ds n(s) \int_{\mathbf{p}} \frac{1}{\mathbf{p}^2} \left( \frac{\mathbf{p} \cdot \mathbf{k}}{\mathbf{k}^2} - 1 \right) \\ &\quad \times \left( 1 - e^{-i \frac{\mathbf{p}^2}{2\omega} (s - \tau_p)} \right) \sigma(\mathbf{k} - \mathbf{p}), \end{aligned} \quad (5.11)$$

where the last term is just the emission spectrum of an emitter travelling inside a medium from  $\tau_p$  to  $\tau_m$ . The difference between the initial propagation being either in vacuum or in medium is precisely the amount of radiation one gets from an in-medium propagation during that initial slab ( $\tau_m - \tau_p$ ). This is a particular result that holds only for the GLV spectrum and not for the all-order result presented in the previous section, because in the full spectrum case multiple scatterings can occur anywhere and, hence, we can not separate between emissions interacting with the medium before or after  $\tau_m$ . We recall that it was proven in section 3.2 (see also [2, 58]) that the GLV limit provides the correct description of the fully resummed spectrum over multiple scatterings in the large energy regime. Therefore, this relation which is specific for the first opacity result is expected to hold for the complete BDMPS-Z spectrum at high energies.

Moving on to the energy spectrum, we can perform the integration over transverse momentum  $\mathbf{k}$  in (5.10) employing the relation between the dipole cross section  $\sigma$  and the elastic collision rate  $V$  (1.32) and the Yukawa potential (1.26). Integrating also the angular part of the transverse momentum  $\mathbf{p}$  we reach (see chapter 2 for details about simplifying the GLV spectrum for the Yukawa collision rate)

$$\omega \frac{dI^{\text{GLV}}}{d\omega} = \frac{4\alpha_s C_R}{\pi} \text{Re} \int_{\tau_m}^L ds n(s) \int_0^\infty \frac{dp}{p} \left[ \frac{\mu^2}{\mu^2 + p^2} - \frac{\mu^2}{\sqrt{(\mu^2 + p^2 + \omega^2)^2 - 4\omega^2 p^2}} \right] \times \left( 1 - e^{-i\frac{p^2}{2\omega}(s-\tau_p)} \right), \quad (5.12)$$

where we integrated  $k$  in a finite domain  $0 \leq k < \omega$  in order to keep the kinematic cut-off. We can remove the kinematical condition by allowing  $k$  to go up to infinity, which results in the fading of the second term in square brackets in (5.12), thus yielding

$$\omega \frac{dI^{\text{GLV}}}{d\omega} = \frac{4\alpha_s C_R}{\pi} \text{Re} \int_{\tau_m}^L ds n(s) \int_0^\infty \frac{dp}{p} \frac{\mu^2}{\mu^2 + p^2} \left( 1 - e^{-i\frac{p^2}{2\omega}(s-\tau_p)} \right). \quad (5.13)$$

The relation between the spectrum with initial vacuum propagation and the scenario where the emitter is produced inside the medium obtained in (5.11) holds as well in both energy distributions (5.12) and (5.13).

### 5.3.1 GLV limit for the brick

As we did in the previous section, to give numerical results of the evaluation of the spectrum we restrict ourselves, for simplicity, to the most basic scenario where the medium is described by a brick of static linear density of scattering centers  $n(s) = n_0$  for  $s \in [\tau_m, L]$ , and with a constant screening mass  $\mu(s) = \mu$  for  $s \in [\tau_m, L]$ . With this simple consideration the integration over  $s$  in (5.10) is trivial to perform, yielding the following

result for the GLV transverse momentum dependent spectrum in a brick

$$\omega \frac{dI^{\text{GLV}}}{d\omega d^2\mathbf{k}} = \frac{\alpha_s C_R n_0}{\pi^2} \int_p \frac{1}{p^2} \left( \frac{\mathbf{p} \cdot \mathbf{k}}{k^2} - 1 \right) \sigma(\mathbf{k} - \mathbf{p}) \left\{ L - \tau_m - \frac{2\omega}{p^2} \left[ \sin \left( \frac{p^2(L - \tau_p)}{2\omega} \right) - \sin \left( \frac{p^2(\tau_m - \tau_p)}{2\omega} \right) \right] \right\}. \quad (5.14)$$

We can also simplify the energy distribution employing the brick approximation. First we focus on the energy spectrum with kinematical constraint (5.12). Integrating over  $s$  we find that the energy distribution is given by

$$\omega \frac{dI^{\text{GLV}}}{d\omega} = \frac{4\alpha_s C_R n_0}{\pi} \int_0^\infty \frac{dp}{p} \left[ \frac{\mu^2}{\mu^2 + p^2} - \frac{\mu^2}{\sqrt{(\mu^2 + p^2 + \omega^2)^2 - 4\omega^2 p^2}} \right] \times \left( L - \tau_m - \frac{2\omega}{p^2} \left[ \sin \left( \frac{p^2(L - \tau_p)}{2\omega} \right) - \sin \left( \frac{p^2(\tau_m - \tau_p)}{2\omega} \right) \right] \right). \quad (5.15)$$

As we have done throughout the whole thesis, we work with dimensionless quantities to reduce the number of free parameters. Following that spirit, we rescale the dummy momentum variable  $p \rightarrow p\sqrt{2\omega/\bar{L}}$  in (5.15), getting

$$\omega \frac{dI^{\text{GLV}}}{d\omega} = \frac{4\alpha_s C_R n_0 L}{\pi} \int_0^\infty \frac{dp}{p} \left[ \frac{1}{1 + xp^2} - \frac{1}{\sqrt{(1 + xp^2 + x^2 \bar{R}/2)^2 - 2x^3 \bar{R} p^2}} \right] \times \left( 1 - \frac{\tau_m}{L} - \frac{1}{p^2} \left[ \sin \left( p^2 \left( 1 - \frac{\tau_p}{L} \right) \right) - \sin \left( \frac{p^2}{L} (\tau_m - \tau_p) \right) \right] \right), \quad (5.16)$$

with  $x = \omega/\bar{\omega}_c$ , and  $\bar{\omega}_c$  and  $\bar{R}$  defined in (5.9). In this case we will also work always with  $\tau_p$  being either  $\tau_p = 0$  or  $\tau_p = \tau_m$ . Therefore, the general GLV energy distribution (5.16) is function of the same variables as the full result (5.9):  $n_0 L$ ,  $x$ ,  $\bar{R}$  and  $\tau_m/L$ .

Let us now move on to the case where the kinematical constraint is removed. With this consideration, the second term inside the square brackets in (5.15) vanishes, hence we get

$$\omega \frac{dI^{\text{GLV}}}{d\omega} = \frac{4\alpha_s C_R n_0}{\pi} \int_0^\infty \frac{dp}{p} \frac{\mu^2}{\mu^2 + p^2} \times \left( L - \tau_m - \frac{2\omega}{p^2} \left[ \sin \left( \frac{p^2(L - \tau_p)}{2\omega} \right) - \sin \left( \frac{p^2(\tau_m - \tau_p)}{2\omega} \right) \right] \right). \quad (5.17)$$

Setting  $\tau_p = 0$  in (5.17) we clearly see that this expression corresponds to the difference between the GLV spectra for an emitter created in the medium travelling through bricks of longitudinal extensions  $L$  and  $\tau_m$ , respectively, as it occurred in (5.11). We can

rewrite (5.17) by separating the two different terms and rescaling the momentum variables accordingly, obtaining

$$\omega \frac{dI^{\text{GLV}}}{d\omega} = \frac{4\alpha_s C_R}{\pi} \left( n_0 L \int_0^\infty dp \frac{p^2 - \sin p^2}{p^3} \frac{1}{1 + \frac{\omega}{\bar{\omega}_c} p^2} - n_0 \tau_m \int_0^\infty dp \frac{p^2 - \sin p^2}{p^3} \frac{1}{1 + \frac{\omega}{\bar{\omega}_m} p^2} \right), \quad (5.18)$$

where we have defined, in analogy with  $\bar{\omega}_c$ , the following energy scale  $\bar{\omega}_m \equiv \mu^2 \tau_m / 2 = \bar{\omega}_c \tau_m / L$ . When  $\omega \gg \bar{\omega}_m$  the second term in (5.18) becomes small compared with the first one. Therefore, we explicitly see in an analytical way that the initial part of the propagation is not relevant for radiated gluons with large energies, obtaining identical spectra for hard partons produced inside or outside the medium.

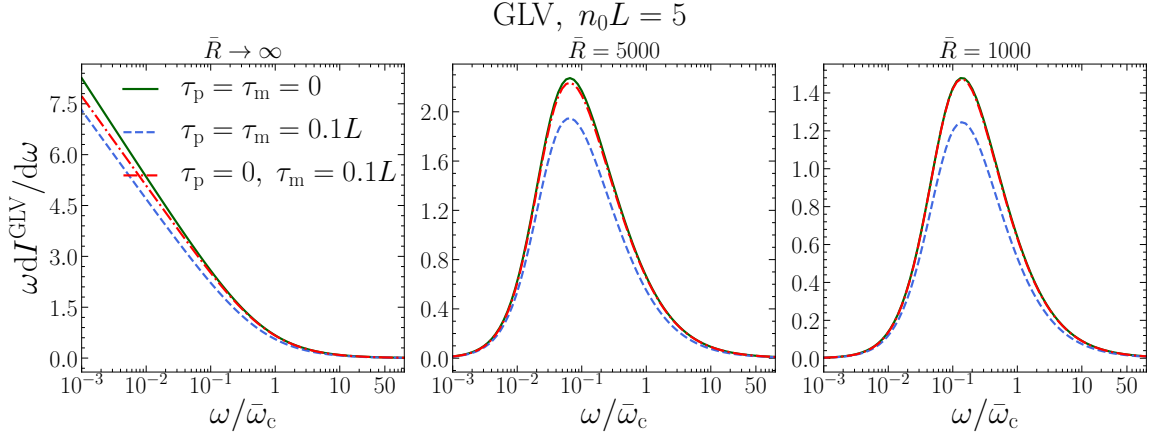


Figure 5.4: In-medium energy distribution for the GLV limit for  $\tau_p = \tau_m = 0$  (green solid lines);  $\tau_p = \tau_m = 0.1 L$  (blue dashed lines); and  $\tau_p = 0$  and  $\tau_m = 0.1 L$  (red dashed dotted lines) as a function of the rescaled gluon energy  $x = \omega / \bar{\omega}_c$  for  $n_0 L = 5$ . Different panels correspond to different values of the kinematical cut-off  $\bar{R}$ :  $\bar{R} \rightarrow \infty$  (left),  $\bar{R} = 5000$  (central), and  $\bar{R} = 1000$  (right). Figure extracted from [4] under a Creative Commons license, CC-BY 4.0.

We show in Figs. 5.4 and 5.5 results of the numerical evaluation of the GLV medium-induced energy distribution off a hard quark as a function of the rescaled gluon energy  $\omega / \bar{\omega}_c$  for the three different scenarios illustrated in Fig. 5.1, which correspond to different considerations regarding the description of the initial stages. We employ the same line styles as in section 5.2: green solid curves for  $\tau_p = \tau_m = 0$ ; blue dashed lines for  $\tau_p = \tau_m > 0$ ; and red dashed dotted curves for  $\tau_m > \tau_p = 0$ . Different panels in each figure correspond to different values of the kinematical cut-off, chosen to match the same ones employed with the full spectrum:  $\bar{R} \rightarrow \infty$  (no kinematical condition imposed) on the left,  $\bar{R} = 5000$  on the central panel and  $\bar{R} = 1000$  on the right. The difference between

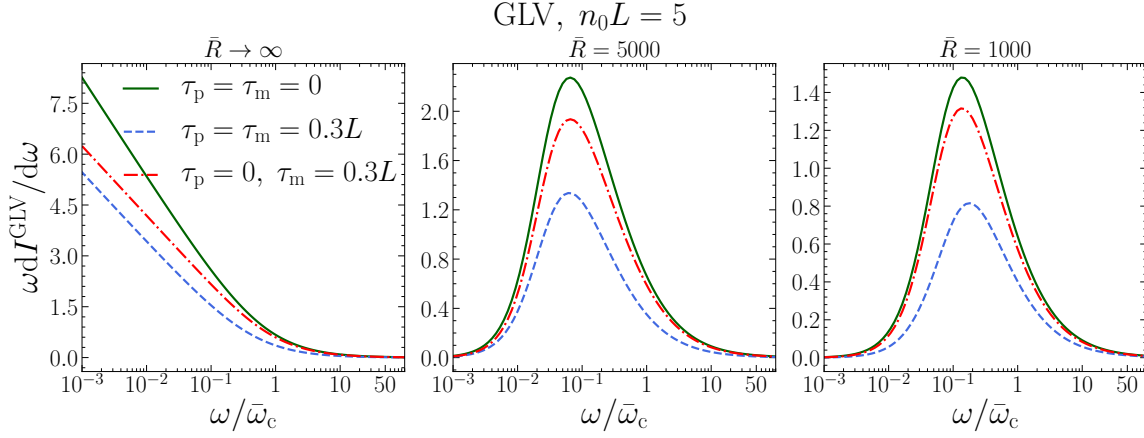


Figure 5.5: In-medium energy distribution for the GLV limit for  $\tau_p = \tau_m = 0$  (green solid lines);  $\tau_p = \tau_m = 0.3L$  (blue dashed lines); and  $\tau_p = 0$  and  $\tau_m = 0.3L$  (red dashed dotted lines) as a function of the rescaled gluon energy  $x = \omega/\bar{\omega}_c$  for  $n_0 L = 5$ . Different panels correspond to different values of the kinematical cut-off  $\bar{R}$ :  $\bar{R} \rightarrow \infty$  (left),  $\bar{R} = 5000$  (central), and  $\bar{R} = 1000$  (right). Figure extracted from [4] under a Creative Commons license, CC-BY 4.0.

Figs. 5.4 and 5.5 is the value of  $\tau_m/L$  employed to compute the curves where it is not zero (blue dashed and red dashed dotted). In Fig. 5.4 we use a small value of that parameter  $\tau_m/L = 0.1$ , whereas that in Fig. 5.5 we increase it and set its value to  $\tau_m/L = 0.3$ .

The results presented in Fig. 5.4 and 5.5 are in agreement with the findings obtained in the previous section for the fully resummed spectrum: the scenario where initial stages are completely neglected ( $\tau_p = \tau_m > 0$ , blue dashed lines) is always below the case where the initial propagation is done through medium ( $\tau_p = \tau_m = 0$ , green solid curves), and the scenario with vacuum propagation before entering the medium ( $\tau_m > \tau_p = 0$ , red dashed dotted lines) is somewhere in between those two. We have an agreement between  $\tau_m > \tau_p = 0$  and  $\tau_p = \tau_m = 0$  cases for high energies, showing again the insensitivity of large energy gluons to the details of the propagation during the initial stages. However, we have differences between both spectra at lower energies. Comparing Figs. 5.4 and 5.5 we can see that the transition region is located at  $\omega/\bar{\omega}_c \sim \tau_m/L$  (or, equivalently,  $\omega \sim \bar{\omega}_m$ ), as we anticipated in (5.18) when we defined the new energy scale  $\bar{\omega}_m = \mu^2 \tau_m/2$ .

## 5.4 HO approach

As we previously mentioned in chapter 3, the HO approximation, which accounts for multiple soft scatterings during the emission of the gluon, relies on taking just the leading logarithmic behavior of the dipole cross section in coordinate space  $\sigma(\mathbf{r})$  (3.19)

$$n(s)\sigma(\mathbf{r}) \equiv n(s) \int_{\mathbf{q}} e^{i\mathbf{q}\cdot\mathbf{r}} \sigma(\mathbf{q}) \approx \frac{1}{2} \hat{q}(s) \mathbf{r}^2 + \mathcal{O}(\mathbf{r}^2 \ln \mathbf{r}^2). \quad (5.19)$$

The natural way of expressing the HO medium-induced radiation spectrum is in coordinate space, thus we take the Fourier transform of the full spectrum written in momentum space (5.2)

$$\omega \frac{dI}{d\omega d^2\mathbf{k}} = \frac{2\alpha_s C_R}{(2\pi)^3 \omega} \text{Re} \int_{\tau_m}^L ds n(s) \int_{\tau_p}^s dt \int d^2\mathbf{x} d^2\mathbf{y} i [\sigma(\mathbf{x}) - \sigma(\mathbf{y})] \times \frac{\mathbf{x} - \mathbf{y}}{(\mathbf{x} - \mathbf{y})^2} \cdot \partial_r \mathcal{K}(s, \mathbf{x}; t, \mathbf{r})|_{r=0} e^{-\frac{1}{2} \int_s^L d\xi n(\xi) \sigma(\mathbf{y})} e^{-i\mathbf{k} \cdot \mathbf{y}}, \quad (5.20)$$

where we employed the following relations

$$\mathcal{P}(L, \mathbf{k}; s, \mathbf{l}) = \int d^2\mathbf{y} e^{-i(\mathbf{k}-\mathbf{l}) \cdot \mathbf{y}} e^{-\frac{1}{2} \int_s^L d\xi n(\xi) \sigma(\mathbf{y})}, \quad (5.21)$$

$$\tilde{\mathcal{K}}(s, \mathbf{q}; t, \mathbf{p}) = \int d^2\mathbf{x} d^2\mathbf{r} e^{-i(\mathbf{q} \cdot \mathbf{x} - \mathbf{p} \cdot \mathbf{r})} \mathcal{K}(s, \mathbf{x}; t, \mathbf{r}). \quad (5.22)$$

We proceed as we did for the full spectrum and isolate the contribution coming from the radiation of the early stages in the scenario where the initial hard parton is created before the hydrodynamization of the medium  $\tau_m \geq \tau_p$  by diving the time integral over  $t$  in two terms at  $\tau_m$

$$\begin{aligned} \omega \frac{dI}{d\omega d^2\mathbf{k}} &= \frac{2\alpha_s C_R}{(2\pi)^3 \omega} \text{Re} \int_{\tau_m}^L ds n(s) \int_{\tau_p}^{\tau_m} dt \int d^2\mathbf{x} d^2\mathbf{y} i [\sigma(\mathbf{x}) - \sigma(\mathbf{y})] \\ &\quad \times \frac{\mathbf{x} - \mathbf{y}}{(\mathbf{x} - \mathbf{y})^2} \cdot \partial_r \mathcal{K}(s, \mathbf{x}; t, \mathbf{r})|_{r=0} e^{-\frac{1}{2} \int_s^L d\xi n(\xi) \sigma(\mathbf{y})} e^{-i\mathbf{k} \cdot \mathbf{y}} \\ &+ \frac{2\alpha_s C_R}{(2\pi)^3 \omega} \text{Re} \int_{\tau_m}^L ds n(s) \int_{\tau_m}^s dt \int d^2\mathbf{x} d^2\mathbf{y} i [\sigma(\mathbf{x}) - \sigma(\mathbf{y})] \\ &\quad \times \frac{\mathbf{x} - \mathbf{y}}{(\mathbf{x} - \mathbf{y})^2} \cdot \partial_r \mathcal{K}(s, \mathbf{x}; t, \mathbf{r})|_{r=0} e^{-\frac{1}{2} \int_s^L d\xi n(\xi) \sigma(\mathbf{y})} e^{-i\mathbf{k} \cdot \mathbf{y}}. \end{aligned} \quad (5.23)$$

In this expression we can simplify the first term employing the convolution property of the kernel in coordinate space

$$\mathcal{K}(s, \mathbf{x}; t, \mathbf{r}) = \int d^2\mathbf{z} \mathcal{K}(s, \mathbf{x}; \tau_m, \mathbf{z}) \mathcal{K}_0(\tau_m, \mathbf{z}; t, \mathbf{r}), \quad (5.24)$$

where  $\mathcal{K}_0$  is the vacuum kernel in coordinate space, given by the Fourier transform of eq. (2.49)

$$\mathcal{K}_0(\tau_m, \mathbf{z}; t, \mathbf{r}) = \frac{-i\omega}{2\pi(\tau_m - t)} \exp \left\{ i \frac{\omega}{2} \frac{(\mathbf{z} - \mathbf{r})^2}{(\tau_m - t)} \right\}. \quad (5.25)$$

Plugging (5.24) and (5.25) into the first term of (5.23) we get

$$\begin{aligned}
 \omega \frac{dI}{d\omega d^2\mathbf{k}} &= \frac{2\alpha_s C_R \omega}{(2\pi)^4} \text{Re} \int_{\tau_m}^L ds n(s) \int_{\tau_p}^{\tau_m} \frac{dt}{(\tau_m - t)^2} \int d^2\mathbf{x} d^2\mathbf{y} d^2\mathbf{z} i [\sigma(\mathbf{x}) - \sigma(\mathbf{y})] \\
 &\quad \times \frac{\mathbf{z} \cdot (\mathbf{y} - \mathbf{x})}{(\mathbf{x} - \mathbf{y})^2} \exp \left\{ i \frac{\omega}{2} \frac{\mathbf{z}^2}{\tau_m - t} \right\} \mathcal{K}(s, \mathbf{x}; \tau_m, \mathbf{z}) e^{-\frac{1}{2} \int_s^L d\xi n(\xi) \sigma(\mathbf{y})} e^{-i\mathbf{k} \cdot \mathbf{y}} \\
 &+ \frac{2\alpha_s C_R}{(2\pi)^3 \omega} \text{Re} \int_{\tau_m}^L ds n(s) \int_{\tau_m}^s dt \int d^2\mathbf{x} d^2\mathbf{y} i [\sigma(\mathbf{x}) - \sigma(\mathbf{y})] \\
 &\quad \times \frac{\mathbf{x} - \mathbf{y}}{(\mathbf{x} - \mathbf{y})^2} \cdot \partial_r \mathcal{K}(s, \mathbf{x}; t, \mathbf{r})|_{r=0} e^{-\frac{1}{2} \int_s^L d\xi n(\xi) \sigma(\mathbf{y})} e^{-i\mathbf{k} \cdot \mathbf{y}}.
 \end{aligned} \tag{5.26}$$

As it happened with the fully resummed spectrum (5.6), the above expression has two contributions, the second term is just the emission spectrum in the situation where the emitter is created at the same time as the medium  $\tau_p = \tau_m$ , whereas that the first contribution accounts for the additional radiation coming from the propagation through vacuum during the initial stages.

### 5.4.1 HO for the brick

Now we restrict ourselves to the brick scenario where the medium is considered to be static with a constant jet quenching parameter  $\hat{q}(s) = \hat{q}_0$  for  $s \in [\tau_m, L]$ . The static case is one of the few in which the HO emission kernel can be computed analytically, and it is given by [70, 106]

$$\begin{aligned}
 \mathcal{K}^{\text{HO}}(s, \mathbf{x}; t, \mathbf{r}) &= \\
 &\frac{-i\omega\Omega}{2\pi \sin[\Omega(s-t)]} \exp \left\{ \frac{i\omega\Omega}{2 \sin[\Omega(s-t)]} (\cos[\Omega(s-t)] \mathbf{x}^2 + \cos[\Omega(s-t)] \mathbf{r}^2 - 2\mathbf{x} \cdot \mathbf{r}) \right\},
 \end{aligned} \tag{5.27}$$

with  $\Omega$  defined as

$$\Omega \equiv \frac{(1-i)}{2} \sqrt{\frac{\hat{q}_0}{\omega}}. \tag{5.28}$$

We will show in the next section the expressions of the kernel and spectrum obtained employing the HO approach in the case where the jet quenching parameter has a power-law dependence on  $s$ . These expressions will be used later in section 5.5 when discussing the simultaneous description of  $R_{\text{AA}}$  and high- $p_T$   $v_2$ . For other profiles we refer the reader, for instance, to Refs. [106, 107].

Introducing (5.19) and (5.27) in (5.26) we can perform the integrals over position variables  $\mathbf{x}$  and  $\mathbf{y}$ . By doing so we get the following expression for the HO transverse

momentum dependent spectrum

$$\begin{aligned}
\omega \frac{dI^{\text{HO}}}{d\omega d^2\mathbf{k}} &= \frac{\alpha_s C_R}{\pi^2 \omega} \text{Re} \int_{\tau_m}^L ds \int_{\tau_p}^{\tau_m} dt \frac{i\hat{q}_0}{\left[\tilde{A}(s,t)\right]^2} \frac{1}{\hat{q}_0(L-s) - 2i\omega\Omega \frac{\tilde{A}(s,t)}{\tilde{B}(s,t)}} \\
&\times \exp \left\{ \frac{-k^2}{\hat{q}_0(L-s) - 2i\omega\Omega \frac{\tilde{A}(s,t)}{\tilde{B}(s,t)}} \right\} \\
&+ \frac{\alpha_s C_R}{\pi^2 \omega} \text{Re} \int_{\tau_m}^L ds \int_{\tau_m}^s dt \frac{i\hat{q}_0 \sec^2[\Omega(s-t)]}{\hat{q}_0(L-s) - 2i\omega\Omega \cot[\Omega(s-t)]} \\
&\times \exp \left\{ \frac{-k^2}{\hat{q}_0(L-s) - 2i\omega\Omega \cot[\Omega(s-t)]} \right\},
\end{aligned} \tag{5.29}$$

where we defined the following functions

$$\tilde{A}(s,t) \equiv \cos[\Omega(s - \tau_m)] - \Omega(\tau_m - t) \sin[\Omega(s - \tau_m)], \tag{5.30}$$

$$\tilde{B}(s,t) \equiv \sin[\Omega(s - \tau_m)] + \Omega(\tau_m - t) \cos[\Omega(s - \tau_m)]. \tag{5.31}$$

Now we have to integrate the  $\mathbf{k}$ -dependent HO spectrum (5.29) over transverse momentum to get the energy distribution. Starting with the case where the kinematical condition is kept  $0 \leq k < \omega$  we get (after rescaling time variables  $t \rightarrow tL$  and  $s \rightarrow sL$ )

$$\begin{aligned}
\omega \frac{dI^{\text{HO}}}{d\omega} &= \\
&- \frac{2\alpha_s C_R}{\pi} \text{Re} \int_{\tau_m/L}^1 ds \int_{\tau_p/L}^{\tau_m/L} dt \frac{i\omega_c}{\omega A^2(s,t)} \left[ \exp \left\{ \frac{-R\omega^2}{2\omega_c^2 \left(1 - s - \frac{i\omega\Omega L A(s,t)}{\omega_c B(s,t)}\right)} \right\} - 1 \right] \\
&- \frac{2\alpha_s C_R}{\pi} \text{Re} \int_{\frac{\tau_m}{L}}^1 ds \int_{\frac{\tau_m}{L}}^s dt \frac{i\omega_c}{\omega \cos^2[\Omega L(s-t)]} \left[ \exp \left\{ \frac{-R\omega^2}{2\omega_c^2 \left[1 - s - \frac{\cot[\Omega L(s-t)]}{\Omega L}\right]} \right\} - 1 \right],
\end{aligned} \tag{5.32}$$

with  $A$  and  $B$ , respectively, the rescaled versions of the previously defined  $\tilde{A}$  and  $\tilde{B}$  functions

$$A(s,t) \equiv \cos[\Omega L(s - \tau_m/L)] - \Omega L(\tau_m/L - t) \sin[\Omega L(s - \tau_m/L)], \tag{5.33}$$

$$B(s,t) \equiv \sin[\Omega L(s - \tau_m/L)] + \Omega L(\tau_m/L - t) \cos[\Omega L(s - \tau_m/L)], \tag{5.34}$$

being  $\omega_c$  and  $R$

$$\omega_c \equiv \frac{1}{2}\hat{q}_0 L^2, \quad \text{and} \quad R \equiv \omega_c L. \tag{5.35}$$

Here  $R$  plays the role of the kinematical constraint, as  $\bar{R}$  did before. Thus, performing the integration over all transverse momentum space is equivalent to taking the  $R \rightarrow \infty$  limit with  $\omega_c$  fixed. Imposing  $R \rightarrow \infty$  in (5.32) we obtain

$$\omega \frac{dI^{\text{HO}}}{d\omega} = \frac{2\alpha_s C_R}{\pi} \ln \left| \cos \left[ \Omega L \left( 1 - \frac{\tau_m}{L} \right) \right] - \Omega L \frac{\tau_m - \tau_p}{L} \sin \left[ \Omega L \left( 1 - \frac{\tau_m}{L} \right) \right] \right|. \quad (5.36)$$

One should notice that the quantity  $\Omega L$  can be written as a function of  $\omega/\omega_c$  only

$$\Omega L = (1 - i) \sqrt{\frac{\omega_c}{2\omega}}, \quad (5.37)$$

thus, if  $\tau_p$  is set to  $\tau_p = \tau_m$  or  $\tau_p = 0$ , as we always do in this manuscript, then the energy distribution without kinematic cut-off depends just on  $\omega/\omega_c$  and  $\tau_m/L$ . If we keep the kinematical condition finite then  $R$  is also a free parameter.

As we did with the GLV result, we can employ the general HO spectrum without kinematical constraint where the emitter is created before the medium (bottom panel in Fig. 5.1) (5.36) to derive a new energy scale which will determine the border between the regions where the details about the propagation during the initial stages are relevant or not. We just need to realize that (5.36) can be written in terms of the spectrum where the initial hard parton is produced inside the medium (scenario A) in the top panel of Fig. 5.1) in the following way

$$\begin{aligned} \omega \frac{dI^{\text{HO}}}{d\omega} \Big|_{\tau_m > \tau_p = 0} &= \omega \frac{dI^{\text{HO}}}{d\omega} \Big|_{\tau_m = \tau_p = 0} \\ &- \frac{2\alpha_s C_R}{\pi} \ln \left| \frac{\cos(\Omega L)}{\cos[\Omega L(1 - \tau_m/L)] - \Omega \tau_m \sin[\Omega L(1 - \tau_m/L)]} \right|, \end{aligned} \quad (5.38)$$

where, unlike what happened for the GLV result (5.11), now the last term is not the medium-induced radiation spectrum for the case where the emitter is created at the same time as the medium, through which it propagates from  $\tau_p = 0$  to  $\tau_m$ . Once that we have (5.38), we rewrite the numerator inside the logarithm of the last term

$$\cos(\Omega L) = \cos[\Omega L(1 - \tau_m/L)] \cos(\Omega L \tau_m/L) - \sin[\Omega L(1 - \tau_m/L)] \sin(\Omega L \tau_m/L), \quad (5.39)$$

in the  $|\Omega \tau_m| \ll 1$  limit this equation becomes

$$\cos(\Omega L) \xrightarrow{|\Omega \tau_m| \ll 1} \cos[\Omega L(1 - \tau_m/L)] - \Omega \tau_m \sin[\Omega L(1 - \tau_m/L)], \quad (5.40)$$

which is exactly the expression of the denominator of the second term on the r.h.s. of eq. (5.38). Therefore, this term goes to 0 in the  $|\Omega \tau_m| \ll 1$  limit, setting a new energy scale  $\omega_m$  defined by

$$\omega_m \equiv \frac{1}{2} \hat{q}_0 \tau_m^2 = \omega_c \frac{\tau_m^2}{L^2}, \quad (5.41)$$

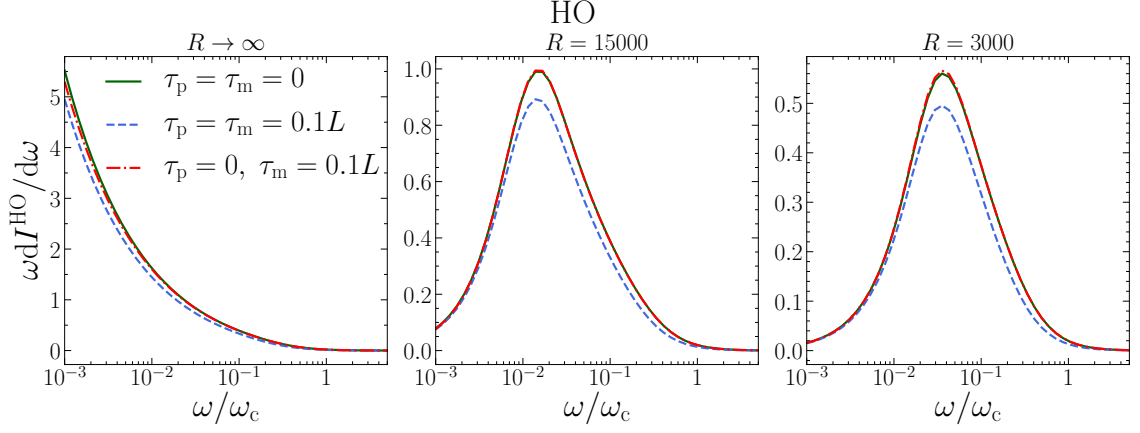


Figure 5.6: In-medium energy distribution for the HO approach as a function of the rescaled gluon energy  $\omega/\omega_c$  for the three different scenarios analyzed:  $\tau_p = \tau_m = 0$  (green solid lines);  $\tau_p = \tau_m = 0.1L$  (blue dashed lines); and  $\tau_p = 0$  and  $\tau_m = 0.1L$  (red dashed dotted lines). Different panels correspond to different values of the kinematical constraint  $R$ :  $R \rightarrow \infty$  (left),  $R = 15000$  (central), and  $R = 3000$  (right). Figure extracted from [4] under a Creative Commons license, CC-BY 4.0.

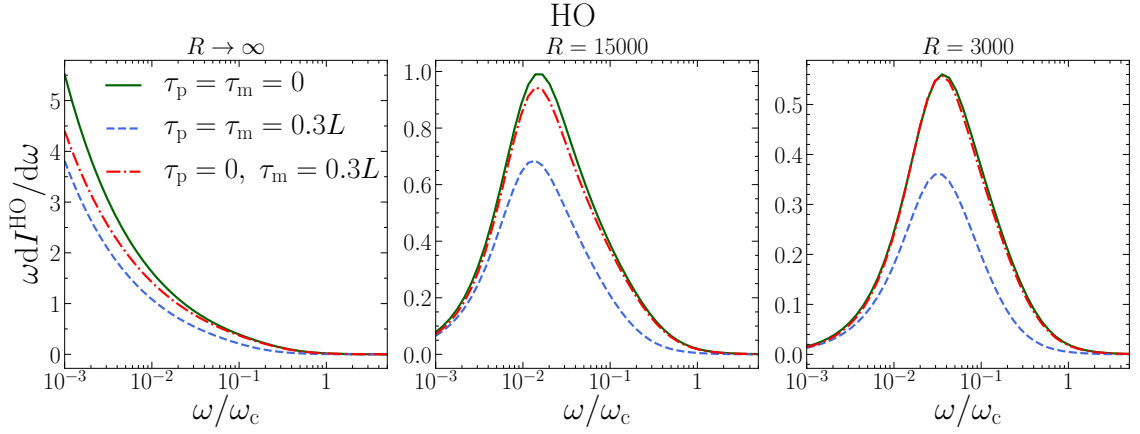


Figure 5.7: In-medium energy distribution for the HO approach as a function of the rescaled gluon energy  $\omega/\omega_c$  for the three different scenarios analyzed:  $\tau_p = \tau_m = 0$  (green solid lines);  $\tau_p = \tau_m = 0.3L$  (blue dashed lines); and  $\tau_p = 0$  and  $\tau_m = 0.3L$  (red dashed dotted lines). Different panels correspond to different values of the kinematical constraint  $R$ :  $R \rightarrow \infty$  (left),  $R = 15000$  (central), and  $R = 3000$  (right). Figure extracted from [4] under a Creative Commons license, CC-BY 4.0.

which, as in the GLV limit, explicitly shows the fact that highly energetic gluons  $\omega \gg \omega_m$  do not feel the difference about the details of the propagation during the early stages, being unable to differentiate propagation through medium or vacuum.

We show in Figs. 5.6 and 5.7 our results for the in-medium radiation spectrum off a hard quark obtained within the HO approach as a function of the rescaled gluon energy  $\omega/\omega_c$  for the three different scenarios studied in this work which encode different considerations about the initial stages propagation. The lines style is chosen to match the one used in the previous cases: green solid lines for  $\tau_p = \tau_m = 0$ ; blue dashed curves for  $\tau_p = \tau_m > 0$ ; and red dashed dotted lines for  $\tau_m > \tau_p = 0$ . In both figures we consider the medium to be static, being the difference between them the value of  $\tau_m/L$  when different from 0. In Fig. 5.6 we set it to a small value  $\tau_m/L = 0.1$ , whereas that in Fig. 5.7 we increase its value and set it to  $\tau_m/L = 0.3$ . Different panels in each figure correspond to different values of the kinematical parameter  $R$ : energy distribution without kinematic constraint  $R \rightarrow \infty$  (left),  $R = 15000$  (central panel) and  $R = 3000$  (right).

Figs. 5.6 and 5.7 show the same features as those we found with previous approaches. In the large energy region there is no difference between the spectrum for the case where the emitter is created before the medium ( $\tau_m > \tau_p = 0$ , red dashed dotted lines) and the one obtained in the scenario where hard parton and medium are produced at the same time ( $\tau_p = \tau_m = 0$ , green solid lines), whereas that for small energies the former is always below the latter and above the spectrum of the scenario which neglects initial stages propagation ( $\tau_p = \tau_m > 0$ , blue dashed curves). We see, however, that differences between green solid and red dashed dotted lines are smaller than in previous cases, which is a direct consequence of the fact that the relevant energy scale in the HO approach is  $\omega_m \sim (\tau_m/L)^2$  instead of  $\bar{\omega}_m \sim \tau_m/L$ . Hence, the point where green and red dashed dotted lines start to differ is much lower in the HO approximation than in the GLV and full frameworks, and for the  $R$  finite spectrum this region is suppressed.

### 5.4.2 HO for a power-law longitudinally evolving medium

In this short section we will provide the solution of the HO energy distribution for an evolving medium whose jet quenching parameter follows a power-law behaviour. We focus now on dynamic media because in the next section we will perform a phenomenological study which requires more precise results for the spectrum compared to the brick case. To not extend the length of the manuscript unnecessarily, we will not provide numerical results for the evaluation of the spectrum, as we would get the same conclusions we already achieved for static spectra.

The parametric form of  $\hat{q}$  we consider here is a power law

$$\hat{q}(s) = \hat{q}_0 \left( \frac{t_0}{s + t_0} \right)^\alpha, \quad (5.42)$$

with  $t_0$  and  $\alpha$  constant parameters and for  $s \in [\tau_m, L]^1$ . We see in (5.42) that for  $\alpha = 0$  we recover the static case we analyzed in detail in the previous section. The emission

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<sup>1</sup>In practice we use  $t_0 = \tau_m$ .

kernel for the HO approximation with a  $\hat{q}$  given by (5.42) can be written as [70, 106]

$$\mathcal{K}^{\text{HO}}(s, \mathbf{x}; t, \mathbf{r}) = \frac{-i\omega}{2\pi D(s, t)} \exp \left\{ \frac{i\omega}{2D(s, t)} (C_1(s, t) \mathbf{x}^2 + C_2(s, t) \mathbf{r}^2 - 2\mathbf{x} \cdot \mathbf{r}) \right\}, \quad (5.43)$$

where  $\alpha \neq 2^2$  and  $C_1$ ,  $C_2$  and  $D$  are defined as

$$D(s, t) \equiv \frac{2\nu}{[2\nu\Omega_\alpha(t_0)]^{2\nu}} (z_s z_t)^\nu [I_\nu(z_s)K_\nu(z_t) - K_\nu(z_s)I_\nu(z_t)], \quad (5.44)$$

$$C_1(s, t) \equiv z_s \left( \frac{z_t}{z_s} \right)^\nu [I_{\nu-1}(z_s)K_\nu(z_t) + K_{\nu-1}(z_s)I_\nu(z_t)], \quad (5.45)$$

$$C_2(s, t) \equiv z_t \left( \frac{z_s}{z_t} \right)^\nu [K_\nu(z_s)I_{\nu-1}(z_t) + I_\nu(z_s)K_{\nu-1}(z_t)], \quad (5.46)$$

being  $I_\nu(z)$  and  $K_\nu(z)$  the modified Bessel functions of first and second kind of order  $\nu \equiv 1/(2 - \alpha)$ , respectively, with the argument  $z$  given by

$$z(\xi) = -2i|\nu|\Omega_\alpha(t_0)(\xi + t_0)^{\frac{1}{2\nu}}, \quad (5.47)$$

where  $\Omega_\alpha$  is

$$\Omega_\alpha(\xi) = \frac{(1-i)}{2} \sqrt{\frac{\hat{q}_0}{\omega}} \xi^\alpha, \quad (5.48)$$

and we also defined  $z_s \equiv z(s)$  and  $z_t \equiv z(t)$ . By setting  $\alpha = 0$  in this set of equations it is easy to check that the kernel (5.43) becomes the static one (5.27).

Introducing the dynamic HO kernel (5.43) in the medium-induced radiation spectrum differential in  $\mathbf{k}$  written in coordinate space (5.20) we can perform all of the integrations except the ones over  $s$  and  $t$ , reaching the following result

$$\begin{aligned} \omega \frac{dI}{d\omega d^2\mathbf{k}} &= \frac{\alpha_s C_R}{\pi^2 \omega} \text{Re} \int_{\tau_m}^L ds \hat{q}(s) \int_{\tau_p}^{\tau_m} dt \frac{i}{A_\alpha^2(s, t)} \\ &\quad \times \frac{1}{\int_s^L d\xi \hat{q}(\xi) - 2i\omega\Omega \frac{A_\alpha(s, t)}{B_\alpha(s, t)}} \exp \left\{ \frac{-\mathbf{k}^2}{\int_s^L d\xi \hat{q}(\xi) - 2i\omega\Omega \frac{A_\alpha(s, t)}{B_\alpha(s, t)}} \right\} \\ &\quad + \frac{\alpha_s C_R}{\pi^2 \omega} \text{Re} \int_{\tau_m}^L ds \hat{q}(s) \int_{\tau_m}^s dt \frac{i}{C_1^2(s, t) F_\alpha(s, t)} \exp \left\{ \frac{-\mathbf{k}^2}{F_\alpha(s, t)} \right\}, \end{aligned} \quad (5.49)$$

where we have defined the generalized versions of the functions  $\tilde{A}$  and  $\tilde{B}$  (5.30) and (5.31) (notice that in (5.49) we did not rescale)

$$A_\alpha(s, t) \equiv C_1(s, \tau_m) - \frac{\tau_m - t}{D(s, \tau_m)} [1 - C_1(s, \tau_m)C_2(s, \tau_m)], \quad (5.50)$$

<sup>2</sup>See appendix C of Ref. [70] for the solution of the kernel in the  $\alpha = 2$  case.

$$B_\alpha(s, t) \equiv \Omega [D(s, \tau_m) + (\tau_m - t)C_2(s, \tau_m)] , \quad (5.51)$$

and we also defined

$$F_\alpha(s, t) \equiv \int_s^L d\xi \hat{q}(\xi) - \frac{2i\omega C_1(s, t)}{D(s, t)} . \quad (5.52)$$

The second term in (5.49) is the in-medium radiation spectrum dependent on transverse momentum in scenarios where the emitter is created at the same time as the medium ( $\tau_p = \tau_m$ ), while the first term accounts for the initial propagation through vacuum, generating extra gluon emissions. If we set  $\tau_p = \tau_m$  then this first term is clearly 0. Moreover, by setting  $\alpha = 0$  in (5.49) we recover the static result (5.29). These two consistency checks indicate that we have obtained reliable results.

We want to compute the energy distribution, thus we have to perform the  $\mathbf{k}$ -integral in (5.49). Imposing the kinematical constraint  $0 \leq k \leq \omega$  we get

$$\begin{aligned} \omega \frac{dI^{\text{HO}}}{d\omega} = & \frac{\alpha_s C_R}{\pi\omega} \text{Re} \int_{\tau_m}^L ds \hat{q}(s) \int_{\tau_p}^{\tau_m} dt \frac{i}{A_\alpha^2(s, t)} \left[ 1 - \exp \left\{ \frac{-\omega^2}{\int_s^L d\xi \hat{q}(\xi) - 2i\omega\Omega \frac{A_\alpha(s, t)}{B_\alpha(s, t)}} \right\} \right] \\ & + \frac{\alpha_s C_R}{\pi\omega} \text{Re} \int_{\tau_m}^L ds \hat{q}(s) \int_{\tau_m}^s dt \frac{i}{C_1^2(s, t)} \left[ 1 - \exp \left\{ \frac{-\omega^2}{\int_s^L d\xi \hat{q}(\xi) - \frac{2i\omega C_1(s, t)}{D(s, t)}} \right\} \right] . \end{aligned} \quad (5.53)$$

If in this equation we set  $\alpha = 0$  and rescale the time variables as  $t \rightarrow tL$  we get the static result (5.32), as expected.

The final case we compute is the no-kinematic condition energy distribution, which is obtained by integrating (5.49) over  $\mathbf{k}$  with  $0 \leq k < \infty$

$$\omega \frac{dI^{\text{HO}}}{d\omega} = \frac{\alpha_s C_R}{\pi\omega} \text{Re} \int_{\tau_m}^L ds \hat{q}(s) \left[ \int_{\tau_p}^{\tau_m} dt \frac{i}{A_\alpha^2(s, t)} + \int_{\tau_m}^s dt \frac{i}{C_1^2(s, t)} \right] , \quad (5.54)$$

where, again, the first term accounts for the initial propagation in vacuum which generates additional gluon radiation. This term vanishes when the production of the emitter coincides with the hydrodynamization of the medium ( $\tau_p = \tau_m$ ). By setting  $\tau_p = \tau_m$  and  $\alpha = 0$  this equation becomes the usual  $R \rightarrow \infty$  HO energy spectrum (3.25), as one should expect.

## 5.5 Simultaneous description of $R_{AA}$ and high- $p_T$ $v_2$

Having accounted for the additional propagation of the hard parton during the initial stages prior to the formation of the medium, we now analyze the potential impact of this extra radiation on phenomenological studies. Specifically, we focus on the simultaneous determination of two observables: the single-inclusive particle suppression  $R_{AA}$  and the high- $p_T$  azimuthal asymmetry  $v_2$ , as it has been demonstrated in recent studies that these two observables are sensitive to the particular description of the initial stages.

Even though the simultaneous description of these two observables has been the subject of extensive exploration in the past decade in several independent publications [127, 130–133], it was only pointed out in recent works [119] that it is highly sensitive to the starting point of the energy loss process. Actually, all of the works which are able to satisfactorily describe both observables have modeled the initial stages according to the scenario B) of Fig. 5.1 that we analyzed in previous sections, i.e., they completely ignore the propagation during the early stages and set the production point of the emitter to be equal to the hydrodynamization time of the medium  $\tau_m = \tau_p$  [120, 130, 134–136], which is usually taken as<sup>3</sup>  $\tau_{\text{hydro}} = 0.6 - 1\text{fm}$ .<sup>4</sup>

We remark that we do not wish for accomplishing a precise description of experimental data in this thesis. Our solely purpose is to illustrate the impact that the initial stages of the propagation may have in phenomenological studies by performing simple calculations, thus our results should not be taken for quantitative comparisons, rather qualitative results are the only ones we will analyze, leaving the quantitative description of data for future publications. Thus, the procedure we follow for describing  $R_{AA}$  and high- $p_T$   $v_2$  will be the same for the three scenarios analyzed in this chapter (see Fig. 5.1), and we describe the behavior of the medium either with a hydrodynamical simulation or with a power-law expansion like the one employed in the previous section for the HO. We then compute the energy loss of the emitter in particular trajectories and use the results for estimating both observables with a simple model.

### 5.5.1 The formalism

Starting from the beginning, the suppression experienced by a parton species  $i$  in a particular trajectory will be estimated using the quenching factor  $Q_i(p_T)$ , which is defined as the ratio of the partonic spectrum between medium and vacuum scenarios

$$Q_i(p_T) = \frac{d\sigma^{\text{AA}}(p_T)/dp_T}{d\sigma^{\text{PP}}(p_T)/dp_T} = \int d\epsilon P_i(\epsilon) \frac{d\sigma^{\text{PP}}(p_T + \epsilon)/dp_T}{d\sigma^{\text{PP}}(p_T)/dp_T}, \quad (5.55)$$

where  $\sigma^{\text{AA}}(p_T)$  is the nucleus-nucleus transverse momentum dependent spectrum,  $\sigma^{\text{PP}}(p_T)$  is the proton-proton one, the subscript  $i$  labels the species of the travelling parton we are considering and  $P(\epsilon)$  are the quenching weights (QWs) which encode the probability that a hard parton loses an additional energy fraction  $\epsilon$  due to scatterings with the medium. The main assumption in which these QWs rely is that consecutive gluon emissions coming from the same parent parton are independent one from each other. This consideration trivially leads to the fact that the probability distribution  $P(\epsilon)$  which describes the energy

<sup>3</sup>We notice that the range we present here is the one employed in traditional jet quenching studies. However, it was shown by recent studies that a hydrodynamical description of the medium at earlier times  $\tau_{\text{hydro}} \sim 0.2\text{fm}$  is also possible [137].

<sup>4</sup>In [138, 139] it was introduced a magnetic monopole producing a huge increase of the jet quenching parameter  $\hat{q}$  for temperatures close to the deconfinement temperature which has effects that resemble the ones originated by delaying the interaction between the medium and the initial hard parton.

loss is a Poisson distribution. Consequently, the definition of the QWs is [70, 110]

$$P(\epsilon) = \sum_{n=0}^{\infty} \frac{1}{N!} \prod_{i=1}^N \left[ \int d\omega_i \frac{dI(\omega_i)}{d\omega} \right] \delta \left( \epsilon - \sum_{i=1}^N \omega_i \right) \exp \left\{ - \int_0^{\infty} d\omega \frac{dI}{d\omega} \right\}, \quad (5.56)$$

where  $N$  is the number of gluons emitted during the propagation,  $\epsilon$  is the total amount of energy taken by those gluons and  $\omega_i$  are their respective individual energies. Also, here we use as  $dI/d\omega$  the medium-induced energy spectrum for a fixed trajectory with the kinematic cut-off imposed.

Under these assumptions and approximating the initial parton spectrum by a power law  $d\sigma^{\text{pp}}/dp_T \propto p_T^{-n}$ , being  $n$  a constant, we can estimate the quenching factor (5.55) by computing just the QWs

$$Q_i(p_T) = \int d\epsilon P_i(\epsilon) \left( \frac{p_T + \epsilon}{p_T} \right)^n. \quad (5.57)$$

Here we will use this  $Q_i(p_T)$  to estimate the inclusive suppression of charged particles by computing it for a quark  $q$  along two perpendicular directions, the in-plane (shortest path length) and the out-of-plane direction (longest path length), and using as proxy for  $R_{\text{AA}}$  the average value between both quenching factors

$$Q_{\text{AA}}(p_T) = \frac{Q_q^{\text{in}}(p_T) + Q_q^{\text{out}}(p_T)}{2}, \quad (5.58)$$

where  $Q_q^{\text{in}}(p_T)$  and  $Q_q^{\text{out}}(p_T)$  denote, respectively, the (partonic) quenching factors of a quark  $q$  along the direction of the event plane and along the direction perpendicular to the event plane.

Also, the high- $p_T$  azimuthal asymmetry can be estimated with these quenching factors employing a simple model [108, 140]

$$w_2 = \frac{1}{2} \frac{Q_q^{\text{in}}(p_T) - Q_q^{\text{out}}(p_T)}{Q_q^{\text{in}}(p_T) + Q_q^{\text{out}}(p_T)}, \quad (5.59)$$

which is going to be our proxy for  $v_2$  in the hard sector.

In the following sections we will employ this formalism to compute  $Q_{\text{AA}}$  for the three different initial stages scenarios illustrated in Fig. 5.1 considering that the initial parton is a hard quark. The energy distributions will be computed with the HO framework, derived in section 5.4.2, and with the spectrum fully resummed over multiple scatterings, computed in section 5.2. The parameterization is done using the local temperature of the medium, and the last free parameter is fitted to ALICE data on the charged hadron  $R_{\text{AA}}$  for the 20 – 30% centrality class in  $\sqrt{s_{\text{NN}}} = 2.76$  TeV Pb-Pb collisions at the LHC [141]. We will explain this method in detail in section 5.5.2 for the HO approximation, and in section 5.5.3 for the full result. Once that we have fitted the only free parameter to experimental data, we use this value to derive  $w_2$  (5.59) for the same conditions as the fit is done.

### 5.5.2 HO approach

We decided to employ the HO framework in our analysis because the QWs formalism and its phenomenological applications have been mostly developed in this approximation. As we have been considering during this section, the initial hard quark is created at time  $\tau_p$ , and then propagates following a straight line. This trajectory can be either along the in-plane or the out-of-plane directions of the QGP created in the heavy ion collision. We can see in (5.53) that the only parameters we need to specify to compute the HO energy distribution for an expanding medium following a power-law behaviour are  $\omega$  and  $\hat{q}$ . The gluon energy  $\omega$  is also used as a variable for evaluating the QWs (5.56), so we compute the spectrum for a large range of  $\omega$  values. However, regarding the jet quenching parameter  $\hat{q}(\xi)$ , we need to specify the relation between this local transport coefficient and the temperature  $T(\xi)$  for a given proper time of the trajectory  $\xi$ . Motivated by [142] we set this relation between the two quantities in the QGP phase to be<sup>5</sup>

$$\hat{q}(\xi) = K_1 T^3(\xi), \quad (5.60)$$

and, as  $\hat{q}(\xi)$  needs to follow a power-law dependence with respect to  $\xi$  to be able to have an analytical solution for the energy spectrum, we establish the following relation for the temperature

$$T(\xi) = T_0 \left( \frac{\xi_0}{\xi + \xi_0} \right)^c, \quad (5.61)$$

with  $\xi_0$ ,  $T_0$  and  $c$  some constant parameters that we determine by comparing (5.61) with temperature profiles extracted from taking straight lines within the 2+1 viscous hydrodynamic model for  $\sqrt{s_{NN}} = 2.76$  TeV Pb-Pb collisions described in Refs. [111, 112]. We considered several trajectories and the power-law decay was a good approximation for the behaviour of the temperature in all of them. As mentioned in chapter 4, the starting point of this hydrodynamic model is at proper time  $\tau_{\text{hydro}} = 1$  fm, its initial condition is given by a factorized Kharzeev-Levin-Nardi model [143] and fixes the ratio of shear viscosity to entropy density  $\eta/s$  to  $\eta/s = 0.16$ , being constant for the whole evolution. The simulation considers the medium to be in chemical equilibrium described by an equation of state inspired by Lattice QCD calculations and it sets a freeze-out temperature at  $T_f = 140$  MeV. Further details concerning the description of the hydrodynamic model can be found in [111, 112]. Taking this in account, we decided to set  $\xi_0 = \tau_{\text{hydro}} = 1$  fm and then determine the values of the other two constants  $c$  and  $T_0$  by identifying (5.61) to the value of the temperature at initial and final proper time of the trajectory. Employing this procedure we get  $c^{\text{in}} = 0.72$  and  $T_0^{\text{in}} = 610$  MeV for the in-plane direction, and  $c^{\text{out}} = 0.67$  and  $T_0^{\text{out}} = 589$  MeV for the out-of-plane direction.

With this consideration we can compute the HO energy distribution (5.53) for the three initial stages scenarios that we are analyzing (see Fig. 5.1) in both paths. Once that we have the gluon energy spectrum, we calculate the corresponding QWs  $P(\epsilon)$ , as

<sup>5</sup>In this study we do not take in account any quenching in the hadronic phase.

| Case                                | $K_1$ |
|-------------------------------------|-------|
| A: $\tau_p = \tau_m = 0$ fm         | 1.5   |
| B: $\tau_p = \tau_m = 1$ fm         | 16    |
| C: $\tau_p = 0$ and $\tau_m = 1$ fm | 8     |

Table 5.1:  $K_1$  values obtained by fitting eq. (5.58) to ALICE  $R_{AA}$  data [141] with  $p_T > 6$  GeV for the 20 – 30% centrality class in  $\sqrt{s_{NN}} = 2.76$  TeV Pb-Pb collisions. We present results for the three different initial stages scenarios analyzed here, which are illustrated in Fig. 5.1.

it is done in [70], for the in-plane and out-of-plane directions. The QWs are then used to derive the quenching factors (5.57) in both paths employing  $n = 4$ , which are later employed to compute  $Q_{AA}$  (5.58). We repeat the procedure several times for different  $K_1$  values and perform a  $\chi^2$ -fit to the ALICE single-inclusive suppression  $R_{AA}$  data for the 20 – 30% centrality class in  $\sqrt{s_{NN}} = 2.76$  TeV Pb-Pb collisions [141] in order to obtain the optimal  $K_1$  value. This best value of the fitting parameter  $K_1$  is finally used to compute the azimuthal asymmetry  $w_2$  (5.59), following [119].

We present in table 5.1 the results obtained for the fitting parameter  $K_1$  for the three different initial stages analyzed in this manuscript: emitter and medium created at the same time filling the initial stages with medium  $\tau_p = \tau_m = 0$  fm; emitter and medium produced at the same time neglecting the propagation prior to the hydrodynamization  $\tau_p = \tau_m = 1$  fm; and allowing the emitter to propagate through vacuum before the creation of the medium  $\tau_m = 1$  fm and  $\tau_p = 0$  fm. It is important to notice that for the first scenario (top picture in Fig. 5.1) we need to perform some extrapolation of the temperature to have non-zero results before the hydrodynamical model is applicable, which we do by employing (5.61) outside the range where it is derived  $\xi < \tau_{hydro}$ . For the other two cases we simply take  $\tau_m = \tau_{hydro}$  and then apply (5.61) for  $\tau_m \leq \xi \leq L$ . The results in table 5.1 manifest the great discrepancies for the transport coefficient  $\hat{q}$  between the three different cases, demonstrating that the jet quenching parameter highly depends on the details of the initial stages modeling (this can also be seen in [15]). As it is obvious, the largest medium (case A),  $\tau_p = \tau_m = 0$  is the one which needs the smallest  $\hat{q}$  value to produce the same amount of quenching as the other two scenarios. The difference between having vacuum propagation before the creation of the medium or not (cases C)  $\tau_p = 0$ ,  $\tau_m = 1$  fm and B)  $\tau_p = \tau_m = 1$  fm, respectively) results in just a factor 2 of difference in the value of  $K_1$ . With these results it is evident that the treatment of the early stages of the collision has a crucial importance in jet quenching studies which extract the value of the jet quenching parameter by fitting the data of single-inclusive suppression observables.

We present in Fig. 5.8 the results for our proxies of  $R_{AA}$  (left) and high- $p_T$   $v_2$  (right) for the three different initial stages scenarios considered as a function of the transverse

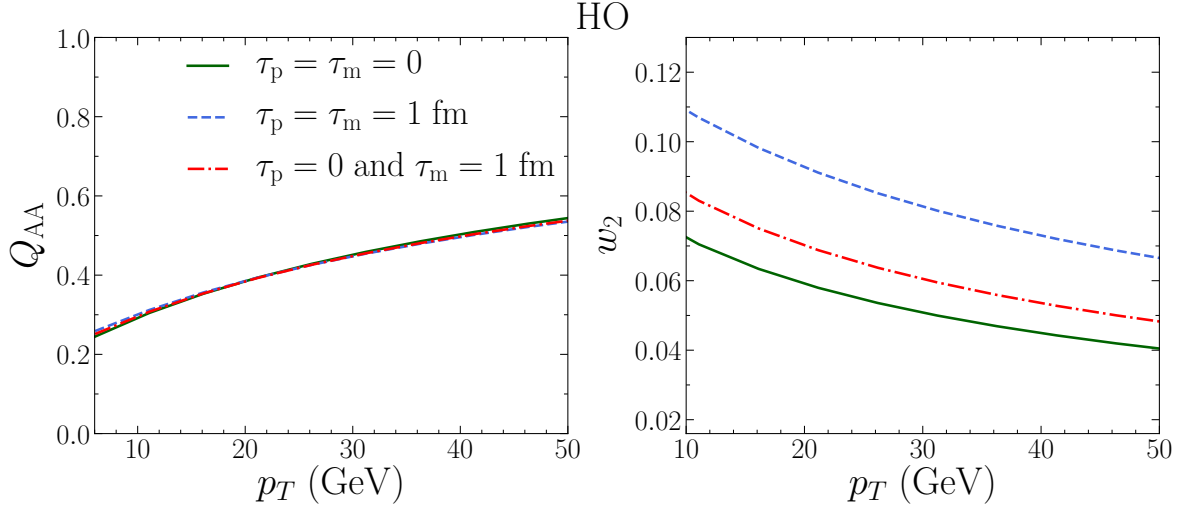


Figure 5.8: Left:  $Q_{AA}$ , our proxy for the single-inclusive particle suppression  $R_{AA}$ , as a function of  $p_T$ . Right:  $w_2$ , our proxy for the high- $p_T$  azimuthal asymmetry  $v_2$ , as a function of  $p_T$ . In each panel we show the results obtained employing the HO approach for the three different scenarios discussed in the thesis:  $\tau_p = \tau_m = 0$  fm (green solid lines),  $\tau_p = \tau_m = 1$  fm (blue dashed lines); and  $\tau_p = 0$  fm and  $\tau_m = 1$  fm (red dashed dotted lines). All of the results are for the 20 – 30% centrality class in  $\sqrt{s_{NN}} = 2.76$  TeV Pb-Pb collisions at the LHC. Figure extracted from [4] under a Creative Commons license, CC-BY 4.0.

momentum  $p_T$ . As we saw before, the values of the fitting parameter  $K_1$  obtained for the different cases, which are shown in table 5.1, presented large discrepancies. However, the left plot of Fig. 5.8 shows that all of the scenarios reproduce (almost) the same quenching  $Q_{AA}$ , which is in agreement with previous findings [116,119,128,129]. Moreover, even though all of the initial stages scenarios have the same amount of energy loss, they yield different results for the azimuthal asymmetry, as it can be seen on the right panel. These results are in agreement with [119], where the authors showed that delaying the starting point of the hydrodynamic model increased the azimuthal asymmetry, and they evidence the importance of properly describing the medium-induced radiation during the initial stages in phenomenological studies. Nonetheless, the correct way of eliminating the parton-medium interactions during the early stages is not to neglect the propagation until the hydrodynamization time like it is usually done (blue dashed curves in Fig. 5.8), but to allow the hard parton to travel through vacuum before entering the medium, as we propose in this work (red dashed dotted curves), and which results in a smaller decrease of the azimuthal asymmetry with respect to the previous case.

### 5.5.3 Fully resummed spectrum

We conclude the study of the effect of initial stages on jet quenching observables by repeating the analysis done in the previous section employing now a better description of the medium-induced energy distribution which includes full resummation over multiple scatterings (5.8). For the all-order spectrum the parameters we need to relate with the the medium temperature are  $n$  and  $\mu$ , and we decide to set the following relations

$$n(\xi) = C_1 T(\xi), \quad \text{and} \quad \mu^2(\xi) = \frac{6\pi\alpha_s}{e} T^2(\xi), \quad (5.62)$$

being  $T(\xi)$  the local medium temperature, which in this case we take directly from the hydrodynamical model in [111,112], we keep  $\alpha_s = 0.3$  constant as we have done throughout the whole thesis and  $C_1$  is our only fitting parameter. For the second relation in (5.62) we have taken the leading order HTL result for the Debye mass  $m_D$  [57] and used the leading logarithmic matching between the Yukawa and HTL cross sections employed in previous works [58,92], which results in the following relation between the screening and Debye masses  $\mu^2 = m_D^2/e$ . We could have not employed the second relation in (5.62) and simply use  $\mu^2 \propto T^2$  (as we did in chapter 4), having then a second fitting parameter to take in account in the analysis. However, as we are only employing a toy model to derive qualitative results, we do not need to achieve highly precise fitting results and we decided to keep things as simple as possible by having just one fitting parameter.

The fully resummed BDMPS-Z spectrum always needs to be computed by numerical methods, thus in this case there is no advantage in parametrizing the temperature with a particular profile which allows for analytical simplifications that relieve the computational power needed to perform the evaluation and we just take  $T(\xi)$  directly from the hydrodynamical model described in [111,112]. There is, however, one case where the hydrodynamical description is not enough to perform the computation and we need to make some assumption about the temperature. The scenario where we set  $\tau_m = 0$ , illustrated in the top picture of Fig. 5.1, has parton-medium interactions before the hydrodynamization time  $\tau_{\text{hydro}} = 1$  fm, thus, we need to extrapolate the temperature before the hydrodynamical simulation is applicable. We decided then to fit the temperature coming from the hydrodynamic model to (5.61) and employ the outcome function for  $\xi < \tau_{\text{hydro}}$ . With these considerations we are now capable of reproducing the analysis performed in the previous section: we compute the all-order medium-induced emission spectrum for the three studied scenarios in the in-plane and out-of-plane directions<sup>6</sup>, with both spectra we compute the QWs and quenching factor for both paths, which are then used to compute  $Q_{AA}$  (5.58) using  $n = 4$ .  $Q_{AA}$  is evaluated for several values of the fitting parameter  $C_1$ , and then we perform a  $\chi^2$ -fit to the ALICE  $R_{AA}$  data for the 20 – 30% centrality class in  $\sqrt{s_{NN}} = 2.76$  TeV Pb-Pb collisions [141] at the LHC. With this fit we obtain the optimal value of the free parameter  $C_1$  for every scenario described in Fig. 5.1 and, finally, we use this  $C_1$  value to obtain our proxy for the high- $p_T$  azimuthal asymmetry  $w_2$  (5.59).

<sup>6</sup>The numerical evaluation of the all-order spectrum for a longitudinally expanding medium is performed in exactly the same way as it was explained in chapter 4, without using any scaling law.

| Case                                | $C_1$ |
|-------------------------------------|-------|
| A: $\tau_p = \tau_m = 0$ fm         | 2.2   |
| B: $\tau_p = \tau_m = 1$ fm         | 4.5   |
| C: $\tau_p = 0$ and $\tau_m = 1$ fm | 2.5   |

Table 5.2:  $C_1$  values obtained by fitting eq. (5.58) to ALICE  $R_{AA}$  data [141] with  $p_T > 6$  GeV for the 20 – 30% centrality class in  $\sqrt{s_{NN}} = 2.76$  TeV Pb-Pb collisions. We present results for the three different initial stages scenarios analyzed here, which are illustrated in Fig. 5.1.

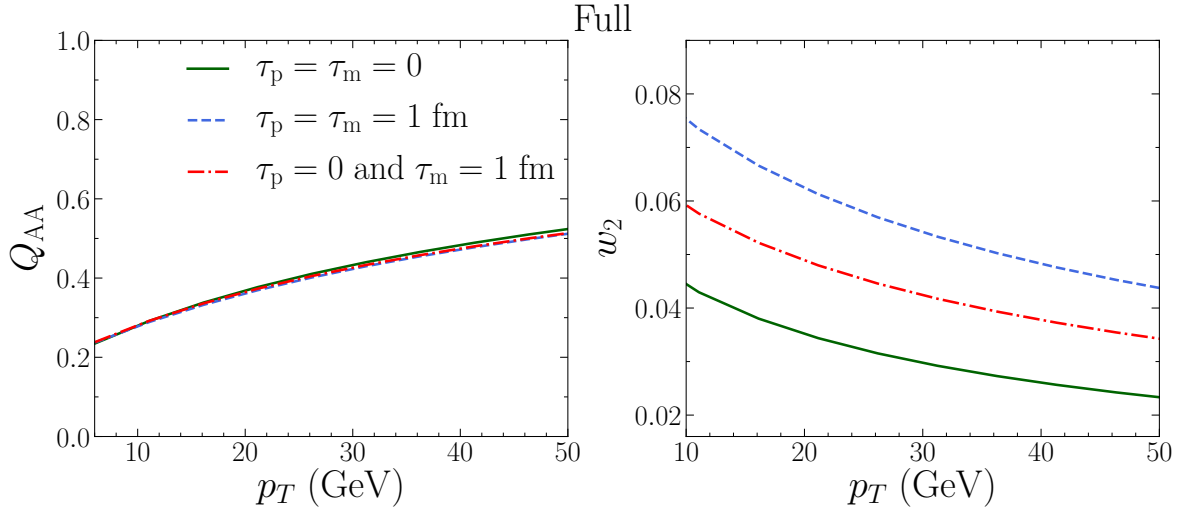


Figure 5.9: Left:  $Q_{AA}$ , our proxy for the single-inclusive particle suppression  $R_{AA}$ , as a function of  $p_T$ . Right:  $w_2$ , our proxy for the high- $p_T$  azimuthal asymmetry  $v_2$ , as a function of  $p_T$ . In each panel we show the results obtained employing the BDMPS-Z spectrum with full resummation over multiple scatterings for the three different scenarios discussed in the thesis:  $\tau_p = \tau_m = 0$  fm (green solid lines),  $\tau_p = \tau_m = 1$  fm (blue dashed lines); and  $\tau_p = 0$  fm and  $\tau_m = 1$  fm (red dashed dotted lines). All of the results are for the 20 – 30% centrality class in  $\sqrt{s_{NN}} = 2.76$  TeV Pb-Pb collisions at the LHC. Figure extracted from [4] under a Creative Commons license, CC-BY 4.0.

We show in table 5.2 and Fig. 5.9 the outcomes of our study. In table 5.2 we have listed the values of the fitting parameter  $C_1$  for every initial stages model. As in the HO case,  $C_1$  is different for each scenario so the same amount of quenching is achieved with media of different lengths. However, now the values of the fitting parameter have much smaller discrepancies, pointing out that having a better description of the energy distribution gives more robust results than relying on approximations. Fig. 5.9 contains

the results for our proxies of  $R_{AA}$  (left) and high- $p_T$   $v_2$  (right) for the three different initial stages scenarios considered as a function of  $p_T$ . Once more, the results obtained with the full spectrum are in agreement with the ones coming from the HO approach, but now the different scenarios are more differentiated in the right panel. Again, we have the same amount of energy loss (all curves agree in the left panel) but with different azimuthal asymmetry, and now the case with vacuum propagation before the QGP phase (red dashed dotted line) is closer to the scenario which neglects the radiation coming from the initial stages (blue dashed line) compared to the HO framework (see right plots of Figs. 5.8 and 5.9). This scenario also exhibits more disparities when compared to the case where the medium is created at  $\tau_m = 0$  (green solid line). These results are somehow expected if we take a look at the spectra outcomes obtained in sections 5.2 and 5.4. For the full result the separation between different scenarios is clearer than for the HO approach, having larger discrepancies spectra with different considerations regarding the initial stages propagation. Our results show for the first time the strong dependence of this set of observables on the description of the initial stages after the collision in a formalism with full resummation of multiple scatterings with a realistic medium-probe interaction. Reaching this result within a more accurate calculation of the medium-induced radiation, which does not require the usually employed approximations like HO or GLV, reveals the potential of jet quenching to constraint the dynamics of the initial stages.

This is just the first step towards a deep understanding of the description of initial stages in heavy ion collisions and more exhaustive analysis will be done in the future. For example, we used the most simple assumption regarding the propagation during the early stages. We could replace the propagation through vacuum by a detailed model of the interactions in the pre-QGP phase. Presumably, this change would have a large effect in initial stages radiation, according to recent results [121–124]. Moreover, we used a toy model to derive the  $R_{AA}$  and  $v_2$  in the different scenarios, whose results can only be employed to extract qualitative conclusions. The natural step to follow is to perform a more thorough phenomenological analysis, like in [119], using the general fully resummed energy distribution with propagation during the initial stages that we derived here, which will be done in future works.

## Summary, Results and Conclusions

In this thesis we employed jet quenching to study the Quantum Chromodynamics (QCD) collectivity. More precisely, we have improved the current knowledge about the emission process, achieving a precise description of the medium-induced gluon radiation spectrum in realistic conditions. This is essential to accurately analyze the properties of the quark gluon plasma (QGP), as many phenomenological works employ this spectrum as basic input.

After a general introduction where high-energy propagation and radiation in a background field is briefly explained, we present, in chapter 2, the new computational tool that will be employed in the rest of the chapters. This new tool allows to perform a resummation of arbitrary number of scatterings for a realistic potential. The main idea is to solve the differential equations that satisfy both the emission kernel and the momentum broadening, instead of directly computing them, which is very difficult to perform in a general scenario. This approach is very flexible, allowing to include any elastic collision rate model or medium profile, but it has the disadvantage of being highly computationally demanding.

The original contributions of the thesis began in the third chapter, where we used this framework to improve the current knowledge of the physics behind the emission process. We did so by using the medium-induced radiation spectrum with full resummation over multiple scatterings as a baseline w.r.t. which we compared several analytical approximations. As every approach relies on different assumptions regarding the physical interpretation of the emission process, the formalism which provides the most precise description of the full spectrum would be the one which better encodes the information about the gluon radiation. We divided the energy distribution in three different regions:

high, intermediate and low energy regimes.

For the the high energy region, where the gluon energy is higher than the characteristic frequency of the medium, we checked that the Gyulassy-Levai-Vitev (GLV) result is indeed the correct limit of the full spectrum. Therefore, in this regime one single hard scattering is enough to precisely describe the radiation process, and accounting for multiple scatterings is not necessary, as expected. Going a little lower in energy, we reach the multiple scattering regime. In this region it is mandatory to consider more than just one scattering, as in this regime the Landau-Pomeranchuk-Migdal (LPM) effect gains relevance and suppresses the spectrum comparing to the single radiation scenario. This is the region where we typically use the Harmonic Oscillator (HO) approximation for describing the spectrum.

However, performing quantitative comparisons between the full spectrum and the Harmonic Oscillator approximation is not possible, as when we define the jet quenching parameter in the HO formalism we introduce an arbitrary upper cut-off. Hence, the mapping between the parameters of both frameworks would depend on this arbitrary choice. Fortunately, in recent years, the Improved Opacity Expansion (IOE) has been developed, an approximation which performs an expansion of the emission kernel using the HO solution as Leading Order (LO) contribution. This approach allows us to perform quantitative comparisons, thus we decided to compare the IOE with our full result. By doing so we found out that the IOE described with high precision (in its region of applicability) the all-order spectrum, whereas the GLV result largely overpredicted it. Therefore, accounting for multiple scatterings in this region is essential to correctly describe the emission process.

Finally, in the Bethe-Heitler (BH) region there is no analytical framework which computes the energy distribution. Thus here we derived, for the first time, the low energy limit of the complete spectrum, which gives an accurate approximation of the full result for energies of the gluon small enough. This newly derived result has the advantage of having a straightforward interpretation of the emission process: in this region the spectrum is given by the single scattering approximation multiplied by a suppression factor which accounts for the probability of not having further scatterings. The no-scattering probability factor comes from the contribution of virtual terms. Removing this factor leads to a high overestimation of the spectrum, thus the first opacity result does not correctly describe the radiation process in this region, which was generally assumed before. This is the first time that the contribution of multiple scatterings to the emission process in the low energy region is described.

Moving on towards the improvement of the framework that we use to compute the spectrum, the following chapter of the thesis focused on computing the spectrum beyond the brick approximation, which assumes that the background medium is a static collection of scattering centres. As we know, this is an unrealistic approximation, as the QGP is an evolving medium well-described by hydrodynamical simulations. However, the brick assumption is commonly employed in theoretical analysis as a first step which could provide us with insightful results before performing more realistic and complicated

computations. Nevertheless, computing the spectrum in evolving media is a mandatory step to be able to implement our framework in high precision phenomenological studies which describe experimental data. As we mentioned before, our method can be applied to any medium profile, thus the brick approximation can be easily improved, but at the cost of requiring a great amount of computing time. This makes the computation unpractical for phenomenological works which need to evaluate the spectrum several times in different conditions. To overcome this limitation we explored the use of scaling laws, a simple set of relations between parameters that allow us to identify spectra computed in different conditions.

Scaling laws were used in the past with high success within the HO framework using as reference scenario the static case. Employing these scaling laws makes it possible to pre-tabulate the spectrum for an extensive range of the values of parameters, and, when we need to compute the spectrum for a specific trajectory, one can apply the scaling laws to extract the pertinent value from the pre-computed results. However, we found that the performance of the scaling laws using as reference the static scenario was not satisfactory for the all-order spectrum, so we decided to explore other options. Given that many hydrodynamical simulations that describe the QGP provide temperature profiles which decay over time as a power-law to a good approximation, we chose to employ a power-law medium profile as a reference to define a new set of scaling laws. The performance of this newly proposed set of scaling laws was excellent for all of the energy range of the spectrum, an even in extreme scenarios, including the effects of fluctuations or changing the model of the collision rate employed. Therefore, we can conclude that the tabulation of the fully resummed spectrum for this power-law expanding profile and the use of our proposed scaling laws will allow a successful implementation of this formalism in phenomenology.

After enhancing the computation of the medium-induced radiation spectrum, the concluding chapter of the thesis was devoted to study a very important topic: analyze the role played by the initial stages of the collision on jet quenching observables. Traditionally, the propagation of the high energy parton during the early stages of the collision has been neglected in the jet quenching community, arguing that the short length of the initial stages ( $\sim 0.2 - 1$  fm) compared with the total length of the QGP ( $\sim 5 - 10$  fm) should make their contribution negligible. However, a few years ago it was pointed out that when describing simultaneously two jet quenching observables ( $R_{AA}$  and high- $p_T$   $v_2$ ), different treatments of the initial stages resulted in distinct descriptions of the experimental data. Therefore, finding the correct way of accounting for the contribution of the initial stages is essential to achieve high precision results in phenomenology.

With that goal in mind, we compared three scenarios with different descriptions of the propagation of the high energy parton during the early stages. First, we considered the usual model employed in phenomenology, neglecting the whole contribution of the initial stages and setting the parton to be created at the same time as the medium after the collision. Second, we assumed, as before, that parton and medium are both formed simultaneously. However, this time we align this creation moment with the instant the collision occurs. Finally, we propose a new scenario never analyzed before where we consider

that the parton is produced with the collision and propagates, through vacuum, until the medium is created at the time the hydrodynamical simulation begins. By performing this comparison we were able to isolate the contribution of the initial stages to the medium-induced gluon radiation. Analyzing the results at the level of the spectrum we found out that neglecting the propagation during the early stages gives an smaller spectrum than for any of the other two cases. Propagating through vacuum or medium during the initial stages provided the same results for the high energy tail of the spectrum. Nevertheless, great discrepancies between these two scenarios appeared in the low energy region, which corresponds to gluons with small formation time. This result can be understood taking in account that highly energetic gluons need larger transverse momentum transfers to decorrelate from the emitter, and, therefore, they have bigger formation times. Gluons which possess a formation time larger than the formation time of the medium  $\tau_m$  can be radiated before the hydrodynamization and still undergo the effects induced by the medium before being completely decorrelated from the emitter. However, gluons with small formation times only contribute to the in-medium radiation spectrum if they are emitted after the medium is created. This is the reason why the high energy tails remain unchanged and disagreements appear in the mid-low energy region. Moreover, by computing the different spectra using analytical approaches, like the HO and GLV, we found the new energy scales that separate the region where the effects of initial stages become relevant for the radiation spectrum.

Finally, using the spectra evaluated for the three different initial stages scenarios, we computed proxies for the inclusive suppression of charged particles and the high momentum azimuthal asymmetry. With the spectrum we computed the quenching weights (QWs), which are distributions that encode the probability that the parton losses a certain amount of energy due to independent gluon emissions induced by the medium. These QWs allow us to compute the quenching factor, which is the ratio between the partonic medium and vacuum spectrum, and it can be used to estimate the inclusive suppression and azimuthal asymmetry. To achieve a simultaneous description of both observables, we computed the inclusive suppression for several values of the free parameter and fitted the results to experimental data of the  $R_{AA}$  of collisions at the Large Hadron Collider (LHC). Then, we used the optimal value extracted from the fit to estimate the azimuthal asymmetry, having in that way a simultaneous description of both observables. As we have different medium configurations for every scenario, but we always fit the results to the same energy loss (same  $R_{AA}$  value), the properties of the medium are different for each case. Consequently, the high momentum azimuthal asymmetry obtained also changes between distinct scenarios. This analysis was performed using the HO approach and the full spectrum, with similar conclusions for both frameworks, thus our results are robust. We conclude that, the treatment of the parton propagation during the initial stages is crucial in jet quenching studies. Finally, we want to remark that some of the current hot topics in the community, such as understanding the apparent lack of jet quenching in small systems [144] and the proper interpretation of jet quenching physics for intermediate A-A collisions, for instance in the upcoming O-O run at the LHC [145] and the proposed

## 6 Summary, Results and Conclusions

O-O and Ar-Ar runs at RHIC by sPHENIX [146], need a deep understanding of the jet quenching during the initial stages of the evolution of the system. The next natural step towards this goal would be to implement a more sophisticated model for the propagation of the high energy parton during the initial stages, which will be done in future works.

MARCOS GONZÁLEZ MARTÍNEZ

## Resumo

A Cromodinámica Quántica (QCD) é a teoría que describe a forza forte que media as interaccións entre as partículas con carga de cor, os quarks e os gluóns. Dende que foi proposta na segunda metade do século pasado, foi posta a proba con éxito en todo tipo de colisións, dende electrón-positrón ata núcleo-núcleo, e en todo tipo de réximes cinemáticos. A descrición dos datos é excelente nas rexións onde se poden aplicar as técnicas de expansión perturbativa grazas a unha das propiedades fundamentais da QCD, a liberdade asintótica, que fai que a constante de acoplo da teoría sexa máis pequena canto maior é a escala enerxética do proceso. Sen embargo, a diferenza doutras teorías cuánticas de campos, a QCD é unha teoría non Abeliiana, o que complica enormemente os cálculos. Os gluóns, os bosóns mediadores da forza forte, posúen a súa propia carga de cor, a diferenza de, por exemplo, o fotón, que non ten a carga eléctrica que media. Este feito complica os diagramas de Feynman que é preciso computar para caracterizar os procesos, pois os vértices fundamentais da teoría son tres: quark-quark-gluón, gluón-gluón-gluón e gluón-gluón-gluón-gluón. Son estes dous últimos os que xorden do feito de que a teoría sexa non Abeliiana, e en última instancia son os que lle confiren á QCD as súas propiedades fundamentais, como a anteriormente mentada liberdade asintótica. Ao mesmo tempo, que a QCD sexa non Abeliiana tamén a fai unha teoría moito máis rica, pois posúe fenómenos emerxentes que non son evidentes simplemente observando o seu Lagrangiano. Así pois, a QCD posúe un diagrama de fases extraordinariamente rico, con numerosos estados da materia e posibles transicións entre eles de diferente tipo, e mesmo a posible existencia dun punto crítico. En particular, este diagrama de fases inclúe un estado da materia universal para temperaturas suficientemente elevadas no cal os quarks e gluóns confinados dentro da materia nuclear ordinaria son quen de liberarse, formando o coñecido como Plasma de Quarks e Gluóns (QGP). Este QGP, ao ser un estado universal para elevadas enerxías, era o estado no que se atopaba toda a materia do universo uns intres despois do Big Bang, así pois, estudar as súas propiedades e como se produce a transición confinamento/denconfinamento aportará información moi relevante sobre o universo primixenio e como se formou a materia ordinaria. Por completitude comentamos que este non é o único estado da materia importante que atopamos no diagrama de fases da QCD, tamén temos estados de alta densidade bariónica moi relevantes para o estudo das estrelas de neutróns e outros obxectos, e mesmo un posible estado da materia análogo aos supercondutores chamado superconductor de cor.

Para estudar o QGP no laboratorio empregamos as colisións de ións pesados a altas enerxías, que consiguen acadar condicións suficientemente extremas para que os nucleóns rachen e se liberen os quarks e gluóns confinados dentro deles. Sen embargo, como o tempo de vida do QGP é extremadamente curto, na práctica precisamos empregar probas indirectas para acceder á súa información. Este tipo de probas adóitase dividir en dous tipos: brandas (soft) e duras (hard). As primeiras exploran o réximen non perturbativo de QCD, onde as técnicas habituais de cálculo de amplitudes a través dos diagramas de Feynman non é posible, e non foron consideradas nesta tese. Aquí centrarémonos nas

probas hard, que nos permiten empregar técnicas perturbativas analíticas para describir os procesos a estudar. En concreto, a nosa proba por excelencia son os jets, partóns de alta enerxía que se propagan radiando partículas dun xeito moi particular dentro dun cono. Estes conos de partículas, despois da hadronización, son detectados experimentalmente polos calorímetros. Os jets prodúcense tanto dentro dun medio (coma o QGP xerado en colisións como chumbo-chumbo ou ouro-ouro) coma no baleiro (medido no laboratorio a través de colisións protón-protón), existindo diferenzas entre os dous casos. En concreto, un jet que se propague por un medio experimentará modificacións con respecto ao caso no que se propague polo baleiro, debido ás interaccións das partículas co propio medio. A todas estas modificacións que un jet poida sufrir polo feito de se estar propagando dentro dun medio fortemente interaccionante englobámolas dentro do termo jet quenching. Nesta tese centrarémonos en estudar este jet quenching, en concreto a radición de gluóns inducida polo medio, como medio para obter información sobre o QGP. Estas emisións de gluóns que só están presentes cando o jet se propaga por un medio, se por interaccións co QGP saen fóra do cono do jet, experimentalmente detéctanse como unha perda de enerxía (energy loss) do jet con respecto á propagación en baleiro, e é unha das pegadas inconfundibles que nos informan da presenza dun medio durante o percorrido do jet.

A pesares do grande coñecemento que posuímos acerca do QGP grazas á labor desenvolta nos grandes colisionadores de ións pesados arredor de todo o mundo durante décadas, a día de hoxe aínda descoñecemos varias cuestións fundamentais. Unha das maiores preguntas abertas que temos actualmente é como é o proceso de formación deste novo estado da materia que posúe un tempo de termalización extremadamente curto, de tan só uns iocosegundos. Nos últimos anos obtivéronse resultados impactantes nos runs de colisións proton-chumbo no LHC, compatibles coa presenza dun mini-QGP do tamaño dun protón ou inferior, que trouxeron un cambio de paradigma nestes estudos. Pola outra banda, agárdase que o fenómeno da termalización e o jet quenching sigan esencialmente a mesma dinámica. Polo tanto, a ausencia de jet quenching (en concreto, a falta de energy loss) nas colisións protón-chumbo do LHC é o maior misterio destes resultados experimentais: é posible ter un medio termalizado sen que haxa jet quenching? A resposta precisa do estudo do jet quenching durante os estadíos iniciais da colisión. Así pois, un dos obxectivos máis importantes da presente tese é comprender o papel que xogan os estadíos iniciais ao describir observables de jet quenching.

Para acadar ese resultado comezaremos por mellorar a comprensión actual que temos das emisións de gluóns inducidas polo medio, que é a peza fundamental sobre a que se sustentan moitos estudos fenomenolóxicos que describen os datos experimentais. Se asumimos que o partón pai é un quark lixeiro, entón toda a enerxía que perde débese á radiación de gluóns inducidas polo medio que escapan do cono do jet, e as interaccións co medio que producen perdas enerxéticas son desprezables. Na aproximación eikonal que considera que a enerxía das partículas que se propagan é moito maior que a escala típica do medio, o cálculo admite unha formalización doada en forma de teoría efectiva na que, simplificando, (a parte escalar de) os propagadores son remprazados por integrais de camiño nas que o medio queda reducido a un campo de cor clásico. Nesta aproximación,

as partes transversa e lonxitudinal da propagación desacóplanse e a enerxía xoga o papel da masa na integral de camiño dun sistema cuántico bidimensional. No límite asintótico de enerxía infinitamente grande a integral de camiño convértese nunha liña de Wilson na traxectoria clásica (unha liña recta). A aproximación eikonal pódese relaxar no gluón radiado, o que conduce a que o partón orixinal estea descrito por unha liña de Wilson, mentres que a propagación do gluón emitido consiste nunha liña de Wilson na dirección lonxitudinal, mentres que a integral de camiño describe o seu movemento browniano no plano transversal. A expresión formal que describe este proceso de radiación obtívose a finais dos 90, e é coñecida como espectro BDMPS-Z.

Sen embargo, o feito de que esta fórmula conteña unha integral de camiño (con potencial imaxinario) dificulta a súa avaliación, pois a resolución destas integrais adoita ser laboriosa. De feito, só se coñecen solucións analíticas nuns poucos casos, coma o baleiro ou o oscilador harmónico, por exemplo. Esta limitación forzou á comunidade a empregar aproximacións para calcular o espectro, sendo dous dos exemplos máis coñecidos a expansión de opacidade, que só ten en conta un pequeno número de interaccións hard do gluon co medio; ou o oscilador harmónico, que describe múltiples interaccións soft. A situación permaneceu sen cambios ata que nos últimos anos comezaron a aparecer novos desenvolvementos teóricos que permitiron mellorar o cálculo do espectro. Nesta tese presentamos un destes novos formalismos que permite avaliar o espectro BDMPS-Z exactamente, sen necesidade de empregar aproximacións, a través de métodos numéricos. Isto acádase mediante a resolución da ecuación diferencial que describe a integral de camiño, no canto de calculala directamente, que despois se emprega para avaliar o espectro.

Despois de presentar o formalismo usarémolo para comprobar a validez das aproximacións tipicamente empregadas, xa que as asuncións feitas por cada formalismo teñen diferentes interpretacións físicas. Así pois, a aproximación que mellor describa o espectro exacto será a que posúa a correcta interpretación do proceso de emisión. Esencialmente, o espectro pódese dividir en tres rexións: altas, medias e baixas enerxías. A enerxía do gluón emitido é directamente proporcional ao seu tempo de formación, que non é instantáneo ao ser un obxecto cuántico. Polo tanto, a zona de altas enerxías correspóndese a gluóns que teñen un elevado tempo de formación, da orde da lonxitude do medio. Ao mesmo tempo, o período de formación do gluón é inversamente proporcional á inversa do cadrado do seu momento transversal. Para que sexa compatible a situación na que un gluón teña unha enerxía moi elevada e aínda así o seu tempo de formación permita que se forme dentro do medio é que experimente polo menos unha interacción hard que lle permita adquirir un gran momento transversal, moi superior á escala típica do medio. Así pois, na zona de altas enerxías a emisión do gluón é gobernada simplemente por unha única interacción hard, que é o que describe a expansión de opacidade a primeiro orde, tamén coñecida como espectro GLV. Baixando un chisco en enerxía atopámonos na rexión na que unha soa interacción non é abondo para describir correctamente a emisión. Esta zona de enerxías intermedias é a rexión na que tradicionalmente se empregou a aproximación de múltiples interaccións soft, tamén chamada oscilador harmónico (HO), para describir o espectro. Sen embargo, existen certas problemáticas ao comparar o HO con outros for-

malismos, pois neste caso un dos parámetros do espectro, o coñecido como parámetro de jet quenching  $\hat{q}$ , depende dun cut-off arbitrario que regulariza unha integral diverxente, facendo que calquer comparativa cuantitativa sexa inútil ao depender dunha selección arbitraria. Afortunadamente, existe un formalismo recente que incorpora as vantaxes da aproximación HO sen ese inconveniente chamado Improved Opacity Expansion (IOE). Esta IOE será a aproximación que empregaremos na zona de enerxías intermedias para comparar o noso espectro completamente resumado sobre interaccións múltiples, poñendo de manifesto a importancia de telas en conta nesta rexión para acadar unha descrición das emisións axeitada. Finalmente queda o réxime de baixas enerxías, tamén chamado de Bethe-Heitler (BH). Esta zona BH correspóndese a tempos de formación tan pequenos que o gluón non pode interactuar con dous centros de interacción do medio consecutivos. Este feito fixo que durante moito tempo a principal hipótese fose que o formalismo de GLV tamén debería de describir o espectro nesta rexión, pois só inclúe unha interacción, mais este resultado nunca foi probado. Nesta tese nós, por primeira vez, amosamos non só que esa elucubración non é certa, senón que acadamos a expresión que describe o límite de baixa enerxía do espectro. A interpretación física do noso resultado é que o espectro nesta rexión está descrito por a fórmula de GLV en réxime incoherente multiplicada por un factor de supresión que ten en conta que non hai interaccións posteriores. Se este factor non está presente, entón a expresión de GLV aporta resultados moi superiores ao espectro completo.

Unha vez resolta a cuestión sobre en que rexións do espectro é preciso ter en conta múltiples interaccións e onde abonda cunha soa, continuamos co seguinte paso de cara a mellorar a descrición das emisións inducidas polo medio. No seguinte capítulo da tese adicarémonos a superar unha das aproximacións máis comúns nos cálculos teóricos, a aproximación do brick. Esta consiste en asumir que o medio sobre o que se propagan as nosas partículas de alta enerxía é un conxunto de centros de interacción estáticos que non cambian co tempo. Esta é unha aproximación moi forte, pois o QGP é un medio que evoluciona e está ben descrito por simulacións hidrodinámicas, pero adóitase empregar con frecuencia como método para obter os primeiros resultados nun escenario sinxelo antes de realizar cálculos realistas que requiran de maior esforzo. Co noso formalismo no que calculamos o espectro mediante técnicas numéricas é posible empregar calquer perfil do medio que queiramos, co cal esta aproximación pódese superar doadamente. Sen embargo, o noso método require dun gran custo computacional, facendo que o tempo de avaliación do espectro sexa elevado. Isto convértese nun gran problema ao implementar o noso formalismo na fenomenoloxía, pois é habitual que os estudos feitos neste ámbito requiran da avaliación do espectro en múltiples situacións diferentes, polo que a velocidade de cómputo é primordial. Así pois, superaremos esta limitación a través do emprego das coñecidas como leis de escala (scaling laws). Estas son relacións simples entre os parámetros que nos permiten identificar dous espectros calculados en escenarios diferentes, podendo deste xeito pre-tabular o espectro nun escenario de referencia para un amplo rango de valores dos parámetros dos que depende e, no canto de ter que calcular o espectro cada vez que o precisemos, simplemente usamos as leis de escala para

ler o seu valor dos resultados tabulados. As leis de escala foron empregadas con gran éxito na aproximación HO, permitindo a súa implementación en numerosas análises fenomenolóxicas. O escenario de referencia usado neste caso para pre-computar o espectro foi o caso estático, polo que é natural considerar este como a primeira opción para a búsqueda dunhas leis de escala para o noso espectro. Sen embargo, o gran desemprego que tiveron estas leis de escala non é tan preciso no caso do noso método. Polo tanto, aquí buscamos un novo conxunto de leis de escala que nos permita obter unha aproximación do espectro completo cunha grande certeza en calquer situación que poidamos atopar na fenomenoloxía. Dado que moitos modelos hidrodinámicos que describen o comportamento do QGP proporcionan perfís de temperatura que teñen un comportamento similar a un decaemento de lei de potencias (power law) con respecto ao tempo, a nosa proposta é empregar, no canto do escenario estático como referencia, un medio que decaía seguindo unha power law para estimar o espectro obtido en traxectorias realistas descritas polas simulacións hidrodinámicas. Os resultados obtidos analizando tanto as scaling laws cun medio estático como cunha power law como referencia amosan que esta última aporta, en xeral, unha descrición do espectro moito máis precisa en todo o rango de enerxías e en calquera tipo de situación. En vista disto, a tabulación do espectro completamente resumado empregando as nosas scaling laws permitirá a súa implementación nos traballos fenomenolóxicos que describan os datos experimentais, o cal mellorará a precisión dos resultados e permitirá desenmarañar algúns dos puzzles que temos actualmente debido ao limitado coñecemento sobre o espectro de radiación inducido polo medio que se emprega nos diferentes estudos.

Unha vez acadadas todas estas melloras respecto ao cómputo actual das emisións inducidas polo medio é hora de chegar á etapa final da presente tese, a análise dos efectos que os estadíos iniciais da colisión teñen no jet quenching. Dita relación foi sinalada por primeira vez hai uns poucos anos ao describir simultaneamente datos experimentais da supresión inclusiva de partículas cargadas e da asimetría azimutal a alto momento. O pensamento máis espallado na comunidade de jet quenching respecto aos estadíos iniciais adoitaba ser que o seu efecto debería de ser desprezable, polo que simplemente se ignoraban e a propagación dos partóns altamente enerxéticos asumíase que comezaba ao mesmo tempo que se creaba o medio, no canto de no instante no que a colisión tivo lugar. A xustificación para esta consideración proviña do feito de que a lonxitude dos estadíos iniciais, comunmente situada entre 0.2 e 1 fm, é moito menor que a extensión do QGP, que se adoita atopar comprendida entre 5 e 10 fm, ou incluso máis. Sen embargo, ao estudar máis dun observable de jet quenching simultaneamente comprobouse que esta asunción era falsa, e de feito a descrición dos estadíos iniciais da propagación é crucial para os resultados finais. En particular, viuse que se se axustan os parámetros libres para describir os datos experimentais de supresión inclusiva, a variación do punto no cal comeza a energy loss do jet modificaba a asimetría azimutal obtida. Máis en concreto, canto máis tarde comeza o proceso da perda de enerxía inducida polo medio, maior é a asimetría azimutal obtida, a pesar de que a enerxía total perdida sexa sempre a mesma. Isto pódese entender doadamente se temos en conta que a maior parte de estudos fenomenolóxicos describen

os observables dun en un, axustando os resultados aos datos experimentais para poder describilos o máis precisamente posible. Sen embargo, para obter resultados similares de energy loss empregando diferentes configuracións do medio, as propiedades deste teñen que ser diferentes en cada caso. Así pois, se precisamos unha certa cantidade de energy loss para describir a supresión inclusiva cun medio dunha certa lonxitude, se acurtamos a extensión do medio, para poder obter a mesma perda enerxética nun espazo menor é necesario aumentar a súa densidade. Ao describir só un observable de cada vez isto non é relevante, pero ao describir un segundo observable despois de facer o axuste aos datos experimentais entón diferentes configuracións do medio terán distintas propiedades e aportarán resultados diferentes. Isto ocasiona que, a pesar de obter a mesma supresión inclusiva de partículas cargadas con diferentes medios, a asimetría azimutal de alto momento difire en función da configuración particular sobre os estadíos iniciais empregada en cada caso.

O método proposto nesta tese para analizar o efecto dos estadíos iniciais no jet quenching consiste en realizar un estudo similar ao explicado no parágrafo anterior: calcular a radiación inducida polo medio en diferentes escenarios con distintas consideracións respecto á descrición dos estadíos iniciais, e usar estes datos para estimar dous observables simultaneamente, analizando como varían os resultados en función da configuración empregada. Os diferentes casos que analizamos nesta tese son: a consideración habitual empregada na fenomenoloxía de ignorar por completo a propagación nos estadíos iniciais ao asumir que o partón e o medio se crean ao mesmo tempo; o mesmo caso que o anterior, pero asumindo que o intre no que se xeran o partón e o medio coincide co instante no que se produce a colisión; e un novo escenario proposto aquí por primeira vez no que consideramos que o partón se crea coa colisión e se propaga polo baleiro ata que o medio se forma un certo tempo despois, que coincide co intre no cal se pode comezar a aplicar a simulación hidrodinámica. A comparación entre estes diferentes casos permítenos aillar por completo a contribución dos estadíos iniciais á radiación de gluons inducida polo medio. Os nosos resultados amosan que ignorar por completo a propagación durante os estadíos iniciais produce un espectro de radiación menor que o obtido en calquera dos outros dous casos. Ademais, estes dous últimos escenarios aportan resultados equivalentes a altas enerxías, aparecendo discrepanzas entre eles na rexión de media-baixa enerxía. Estes resultados pódense entender doadamente en termos do tempo de formación. Os gluons de alta enerxía son os que teñen un tempo de formación máis grande, polo cal precisan de case todo o medio para se poderen emitir e non perciben se cando comezou o seu proceso de formación a propagación acontecía nun medio ou no baleiro. Pola contra, a rexión de baixa enerxía correspóndese cos tempos de formación máis pequenos, e un gluón destas características podería ser emitido tanto durante os estadíos iniciais como ao final da propagación do jet, sendo ambas situacións radicalmente diferentes para distintas consideracións sobre os estadíos iniciais. Polo tanto, na zona de baixas enerxías o modelo empregado sobre a propagación temperá do noso partón resulta en diferentes resultados para o espectro. O cómputo do espectro para os tres escenarios analizados empregando formalismos analíticos coma o GLV ou o HO permítenos obter as novas escalas de enerxía

que separan a rexión onde os efectos dos estádios iniciais son importantes para a radiación inducida polo medio da zona onde os podemos desprezar de forma segura.

Unha vez equipados cos nosos espectros, estes son empregados para estimar a supresión inclusiva de partículas cargadas e a asimetría azimutal de alto momento, de xeito simultáneo. Como explicamos previamente, ao axustar os diferentes casos aos mesmos resultados experimentais, a energy loss obtida en cada escenario é practicamente igual que para os outros dous. Sen embargo, as propiedades subxacentes dos diferentes medios son radicalmente distintas, o que ocasiona que varíe enormemente a asimetría azimutal obtida. Estes resultados poñen de manifesto que a descrición da propagación dos partóns de alta enerxía durante os estádios iniciais é crucial, e é preciso aprender a tratala correctamente se queremos realizar complexos estudos fenomenolóxicos nos que describamos múltiples datos experimentais simultaneamente con gran precisión. Ademais, algúns dos actuais temas candentes da comunidade requiren dunha profunda comprensión do jet quenching durante os estádios iniciais da evolución do sistema. Entre algúns dos exemplos máis relevantes podemos mencionar: comprender a aparente falta de jet quenching en sistemas pequenos xerados en colisións protón-protón ou protón-núcleo, e a apropiada interpretación da física do jet quenching en colisións A-A para núcleos intermedios, por exemplo, no futuro run de osíxeno-osíxeno no LHC ou nos runs de osíxeno-osíxeno e argón-argón propostos por sPHENIX en RHIC. O seguinte paso natural que podemos dar para acadar esta gran comprensión sobre o efecto dos estádios iniciais no jet quenching sería reemplazar o modelo de propagación no baleiro empregado na presente tese por outros escenarios máis realistas, o cal queda para futuros traballos.

## Publications used in the thesis

We summarize in the following pages the information about the publications produced in the thesis. Part of their content has been reproduced in chapters 3, 4 and 5.

From soft to hard radiation: the role of multiple scatterings in medium-induced gluon emissions, *J. High Energ. Phys.* **03** (2021) 102

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In this thesis we use the medium-induced gluon radiation spectrum to study the quark gluon plasma generated at the LHC. In particular, we employ a new computational algorithm that exactly resums the QCD potential without truncations — either in the opacity expansion or the perturbative tails. First, we compare the fully resummed spectrum with different analytical approaches, improving our understanding of the physics behind the emission process. Then, we find an efficient way of computing the radiation spectrum in a realistically evolving media described by a hydrodynamical simulation, a crucial step to implement the framework in phenomenology. Finally, we analyze the impact that the propagation during the initial stages of the collision, which is usually neglected, has on jet quenching observables.