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NELIA 2011
Book of Abstracts

Workshop on Numerical Electromagnetics and Industrial Applications

Santiago de Compostela
October, 25-28, 2011

Edited by
A. Bermúdez
D. Gómez
P. Salgado

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Workshop on Numerical
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Workshop on Numerical Electromagnetics and Industrial Applications

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Preface

This volume contains the abstracts of the lectures given at NELIA 2011, the *Workshop on Numerical Electromagnetics and Industrial Applications*, held in Santiago de Compostela, Spain, from 25 to 28 October 2011.

The main goal of this workshop was to set up a discussion around the recent developments in the mathematical, numerical and computational analysis of electromagnetic models and their direct industrial applications.

The scientific program consisted of 23 invited lectures and a poster session. The format was deliberately chosen to encourage extended discussions and to establish or continue with networking relationships among the participants.

The invited talks were addressed by international specialists in each of the suggested subjects. Industry representatives also participated in order to consider applications that might need the development and resolution of electromagnetic models. In this context, the workshop intended to open new lines of research in the subject, taking into account the industrial demands nowadays.

The conference program covered a broad range of topics both in theoretical and applied electromagnetic problems: formulation of different models in electromagnetism (high and low frequencies), numerical and mathematical analysis of the formulations, numerical techniques, coupled problems and industrial applications. Selected papers from this program will be published in a special issue of the journal *Applied Numerical Mathematics*.

NELIA 2011 was organized by the Department of Applied Mathematics at the Universidade de Santiago de Compostela. The organizing committee wish to thank all the participants, and very specially the invited speakers for their contributions and attendance, without whom there would have been no conference.

Furthermore we would like to thank all the institutions that have made an investment in this event: the Ministry of Science and Innovation through its Ingenio MATHEMATICA (i-MATH) CONSOLIDER research project; the Galician regional Education Ministry through its Mathematica Consulting and Computing network; and the Faculty of Mathematics which provided us with space for this workshop.

Finally, we would like to express our gratitude to Elisa Eiroa, manager of the research group in Mathematical Engineering (mat+i) and to Adela Martínez, Lupe Parente and Teresa Sánchez, the Nodo CESGA Consulting and Computing Support Technicians, for their help organizing this event.

Santiago de Compostela, October 2011.

The Organizing Committe,

Alfredo Bermúdez
Dolores Gómez
Pilar Salgado

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Red Española de Matemática Industria

Ingénierie Mathématique et Calcul Scientifique

Ingeniería de Diseño Electrotécnico

CEDRAT Group

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CONTRIBUTED POSTERS

INVITED LECTURES

Design and analysis of electrical machines using finite element method

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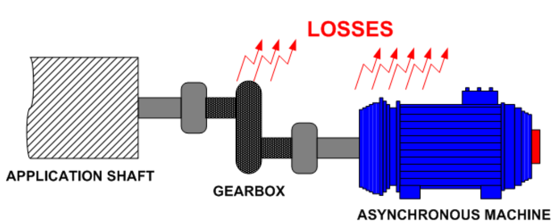
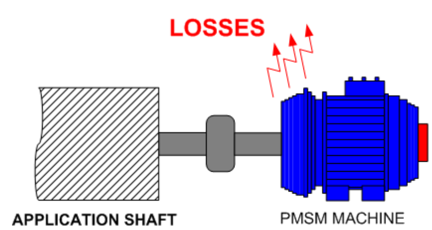
ABSTRACT

Nowadays, electrical drive technology tends to reduce life cycle cost and provide environmentally friendly solutions. Moreover, compactness is also becoming an important requirement. Within this framework, direct drive systems based on permanent magnet (PM) machines are presented as one of the best solutions to fulfil all these requirements. In direct drive applications, as it is shown in the Figure 1-B, the machine is directly connected to the application shaft, avoiding any gearbox between them. This leads to more compact, more reliable and more cost effective solutions. In most cases, the shaft of the application runs at low speeds, which means that the electric machine has to fulfil a low speed high torque characteristic. In order to obtain these requirements, a high grade of optimization in the electrical machine design may be required. The computer based numerical analysis by Finite Element Method (FEM) makes possible the optimization of the electrical machine designs.



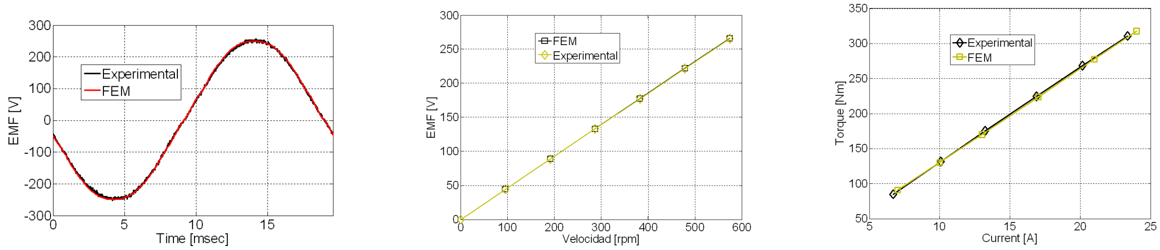
A) Conventional drive system based on a squirrel cage asynchronous machine



B) Direct-Drive system based on a PM machine

Figure 1: Comparison between a conventional drive system and a direct-drive solution

In this work, a design method based on FEM analysis of electrical machines is developed, and a real case study is presented, in which a PM motor is designed for a direct-drive elevator. Moreover, in order to validate the design, the FEM results are compared with experimental measurements. As conclusion, it can be stated that all results have a very good agreement.



A) Electromotive Force B) Electromotive Force Vs Speed C) Electromagnetic Torque

Figure 2: Comparison between FEM results and experimental measurements

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Inverse source problems for eddy current equations

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ABSTRACT

Electroencephalography (EEG) and magnetoencephalography (MEG) are two non-invasive techniques used to localize electric activity in the brain from measurements of external electromagnetic signals. EEG measures the scalp electric potential, while MEG measures the external magnetic flux. From the mathematical point of view, the goal is to solve an inverse problem for determining the source current distribution in a heterogeneous media from boundary measurements of the fields.

The frequency spectrum for electrophysiological signals in EEG and MEG is typically below 1000 Hz. For this reason most theoretical works on biomedical applications focus on the static approximation of the Maxwell equations, in which the time variation of both electric and magnetic fields is disregarded. However, some recent works investigate the localization of brain activity through the inverse source problem for the full Maxwell system of equations. In this work we analyze the inverse source problem for an alternative model: the eddy current (or low frequency) approximation of Maxwell equations where the time variation of the electric field is disregarded, while time variation of the magnetic field is kept.

We prove that a volume current source cannot be uniquely identified by the knowledge of the tangential components of the electromagnetic fields on the boundary of the conductor, and we characterize the space of non-radiating sources. On the other hand, we prove that the inverse source problem has a unique solution if the source is the sum of a finite number of dipoles or if it is supported on the boundary of a subdomain. These results are similar to those obtained for the full Maxwell system of equations. Finally we address the applicability of these results for the localization of brain activity from electroencephalography and magnetoencephalography measurements.

Keywords: inverse source problem, eddy current approximation of Maxwell equations, biomedical applications

Mathematics Subject Classifications (2010): 35R30, 35Q60

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3-D Modelling and optimisation of induction heating processes: some computational issues

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ABSTRACT

Efficient optimisation of induction heating processes requires the use of computational modelling techniques. Much work has been devoted in recent years to modelling of induction heating processes ([1]). Specific electromagnetic formulations as well as dedicated finite elements can be used ([2]). We shall introduce here a coupling procedure between electromagnetic and thermal computations ([3]) which has been extended to modelling of multiphysics couplings with solid mechanics or metallurgy. Matters regarding the conditioning of linear systems arising from this, as well as parallel computing aspects will also be presented ([4]).

Regarding optimisation, we shall introduce approaches for the case of a coupled electromagnetism-heat transfer problem. The optimisation formalism can be extended and generalised to other kinds of couplings (solid mechanics, metallurgy...). Algorithms and results are presented and discussed for first-order algorithms - computing sensitivity with adjoint state approaches ([5]) -, as well as zero-order approaches ([6]).

Keywords: numerical analysis, finite elements, optimisation, heat transfer, electromagnetism, multi-physics couplings

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Extension to non-conforming meshes of the combined current and charge integral equation

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ABSTRACT

An important issue in industrial applications in electromagnetism requiring the solution of a large scale boundary integral equation concerns the possibility of using meshes on different zones obtained independently each from the other, which thus do not comply with the usual matching requirement of finite element approximations. By bringing out some mathematical properties of the Combined Current and Charge Integral Equation (shortly C3IE) introduced by Taskinen and Ylä-Oijala [2] when it is posed on a surface without geometrical singularities, we show that this equation can be solved by a Boundary Element Method (BEM) that requires no interelement continuity. This property is crucial when using meshes on different parts of the surface obtained independently each from the other.

We show how the C3IE can be implemented by slightly modifying a usual BEM electromagnetic solver code and that the numerical behavior of this method is very similar to the usual Combined Field Integral Equation (CFIE) when dealing with smooth surfaces.

The extension to singular geometries showed that acute dihedral angles can lead to inaccuracies in the results. By considering a two-dimensional version of the approach, we have brought out that the wrong results are due to spurious oscillations concentrating around the singular points of the geometry. Noticing that the system linking the current and the charge is a saddle-point problem, we have adapted a general procedure used for stabilizing the numerical approximation of mixed formulations as the Stokes system [1], consisting here in augmenting the approximation of the charge. We show that this stabilization procedure, when coupled with a refinement of the mesh in the proximity of the geometrical singularities, obtained by a simple subdivision of the triangles, greatly reduces the effect of the spurious oscillations.

Keywords: boundary integral equation, boundary element method, current and charge equation, electromagnetic scattering

Mathematics Subject Classifications (2010): 45F15, 65R20, 78M15

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Finite element solution of nonlinear eddy current problems with periodic excitation and its industrial applications

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ABSTRACT

The most straightforward method of solving nonlinear electromagnetic field problems in the time domain by the method of finite elements is using time-stepping techniques. This requires the solution of a large nonlinear equation system at each time step and is therefore very time consuming, especially if a three-dimensional problem is being treated. If the excitations are non-periodic or if, in case of periodic excitations, the transient solution is required, one cannot avoid time-stepping. In many cases however, the excitations of the problem are periodic, and it is only the steady-state periodic solution which is needed. Then, it is wasteful to step through several periods to achieve this by the "brute force" method [1] of time stepping.

A time domain technique using the fixed-point method to decouple the time steps has been introduced in [2] and applied to two-dimensional eddy current problems described by a single component vector potential. The optimal choice of the fixed point reluctivity for such problems has been presented in [3] both in the time domain and using harmonic balance principles. The method has been applied to three-dimensional problems in terms of a magnetic vector potential and an electric scalar potential (**A**,v-**A** formulation) in [4] and, applying a current vector potential and a magnetic scalar potential ($\mathbf{T},\Phi-\Phi$ formulation) in [5] and [6].

The aim of this paper is to show the application of the method to industrial problems arising in the design of large power transformers.

Keywords: finite element methods, eddy current problems, nonlinearity, periodic solution

Mathematics Subject Classifications (2010): 35 Partial differential equations, 65 Numerical analysis, 78 Optics, electromagnetic theory

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Exterior calculus and the finite element approximation of Maxwell's eigenvalues

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ABSTRACT

Maxwell's eigenvalue problem can be seen as a particular case of the Hodge-Laplace eigenvalue problem in the framework of exterior calculus. In this context we present two mixed formulations that are equivalent to the problem under consideration and their numerical approximation. It turns out that the natural conditions for the good approximation of the eigensolutions of the mixed formulations are equivalent to a well-known discrete compactness property that has been firstly used by Kikuchi for the analysis of edge finite elements.

The result can be applied to the convergence analysis of the p-version of edge finite elements for the approximation of Maxwell's eigenvalue problem.

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Time harmonic maxwell equations with sign shifting coefficients: mathematical and numerical aspects

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ABSTRACT

We consider the time-harmonic Maxwell equations in the exotic case where the dielectric permittivity e and/or the magnetic permeability J present a change of sign across some interface Σ. This occurs for instance at the interface between the vacuum and a metal in the high frequency regime (but below the plasma frequency): the dielectric permittivity of the metal takes a negative real value, leading to the existence of the so-called plasmonic waves. More surprising is the possibility of realizing materials which exhibit negative real valued e and J in some appropriate range of frequencies: this explains the unusual negative refraction effect that is observed at the interface between a dielectric and such a metamaterial, a phenomenon which has very exciting potential applications.

The change of sign of the constants e and/or J raises a lot of original theoretical questions: indeed classical mathematical theorems proving the well-posedness of the continuous problem and the convergence of conventional numerical methods are no-longer valid in such configurations. The questions that we address are the following. Can we extend the classical theory to sign-changing coefficients? And if not, is there a new functional framework in which well-posedness and stability properties can be recovered?

We have obtained several results which strongly depend on two factors: the regularity of the interface Σ and the values of the contrasts of e and J across Σ (i.e. the ratios of the values taken by e and Jon both sides of Σ). First we show that if the boundary is regular and if the contrasts are different from -1, everything works as for positive materials. Let us point out that the case of a contrast equal to -1 (which would be optimal for applications) still raises difficult open questions. If the boundary has singularities (edges or conical points, for example), usual results are recovered if the contrasts are outside some critical interval, which always contains the value -1. Within the critical interval, we have shown in a two-dimensional configuration that a so-called "black hole" phenomenon can be observed, in the sense that there is a wave that propagates towards the singular point, taking an infinite time to reach this point.

Keywords: Maxwell equations, metamaterial, transmission problem, interface, Fredholm theory

Mathematics Subject Classifications (2010): 35Q60, 35Q61, 35J20

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Misgivings about the Maxwell tensor

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ABSTRACT

One might think that Maxwell's theory has electromagnetic forces within its ken: After all, force is a primitive concept in this theory, as witnessed by the $q(E+v\times B)$ formula, from which the interpretation of fields E and B derives. From this, one can build this object M called the Maxwell tensor (equal to $B\otimes H - B\cdot H/2$ plus a similar term in D and E), and there seems to be a widespread belief that M knows all about forces. For instance, the force field exerted on matter by the electromagnetic field would be $\operatorname{div} M$ — possibly with some caveats, such as understanding this in the sense of distributions.

This is not true. *Knowing the electromagnetic field* $\{E, D, B, H\}$ *may not be enough to know the forces*. For that, one must deal with the coupled problem (elastodynamics, say, coupled with electromagnetics) in its entirety, which requires a full knowledge of the coupled constitutive laws. Applying the virtual power principle (VPP), which is nothing else but the weak formulation of the coupled problem, will then give a satisfactory answer.

Shortly said, Maxwell's tensor is logically weaker than the VPP, from which it can be derived, and it fails for some modes of coupling. In particular, it fails when magnetostrictive effects exist. (The mathematical characterization of that will help, working backwards, to precisely define what "magnetostriction" is.) On the other hand, it does give correct answers when only shape effects are to be considered, which explains its popularity as an Engineer's tool.

Here are a few details about how to derive M, the Maxwell stress, from the VPP. Consider a domain D enclosed by a surface S, and build a virtual displacement v equal to some vector V for all points in D (the same V all over D), to 0 outside D, except for a transition layer in which v goes smoothly from 0 to V. Then take the limit of the virtual power associated with v when the width of this layer tends to zero: The result, it can be shown, is $V\cdot F(D)$, where $F(D)$ is the total force over D, as given by integration over S of M. Hence the usefulness of M. (A similar procedure would give the total torque over D.)

This derivation establishes the subordinate status of the Maxwell stress tensor. It also helps understand why, as will be explained, magnetostrictive effects stay beyond its reach: When using a piecewise constant virtual displacement field (except for the transition near S), one also considers a null virtual deformation ϵ, which forbids to capture variations of the energy due to local changes of ϵ. I'll try to present the mathematical formalism (of differential-geometric character) by which one can pinpoint the precise meaning of "local" there.

Keywords: Maxwell's tensor, elasticity, strain, stress

Mathematics Subject Classifications (2010): 78M12, 65M60, 53Z05, 74F15

Transmission eigenvalues in inverse electromagnetic scattering theory

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ABSTRACT

The transmission eigenvalue problem is a new class of eigenvalue problems that has recently appeared in inverse scattering theory for inhomogeneous media. Such eigenvalues provide information about material properties of the scattering object and can be determined from scattering data, hence can play an important role in a variety of problems in target identification. The transmission eigenvalue problem is non-selfadjoint and nonlinear which make its mathematical investigation very interesting.

In this lecture we will describe how the transmission eigenvalue problem arises in the electromagnetic scattering theory, how transmission eigenvalues can be computed from scattering data and what is known mathematically about these eigenvalues. The investigation of transmission eigenvalue problem for anisotropic media will be discussed and Faber-Krahn type inequalities for the first real transmission eigenvalue will be presented. We conclude our presentation with some recent preliminary results on transmission eigenvalues for absorbing and dispersive media, i.e. with complex valued index of refraction, as well as for anisotropic media with contrast that changes sign.

Keywords: Interior transmission problem, transmission eigenvalues, inhomogeneous medium, inverse scattering

Mathematics Subject Classifications (2010): 35R30, 35Q60, 35J40, 78A25

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Asymptotic-numerical integral equation methods for high frequency scattering

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ABSTRACT

Conventional discretisation methods (finite element, finite difference, boundary element/method of moments, ...) have costs which increase rapidly as the frequency increases because of the need for large numbers of degrees of freedom to resolve the oscillatory solution. In particular, this is an issue in boundary element/method of moments calculations for time harmonic problems, where in 3D the degrees of freedom need to increase in proportion to k^2, where k is the wave number, in order to maintain a fixed number of degrees of freedom per wavelength, required to maintain accuracy. High frequency asymptotic methods, in the other hand, based on ray tracing/solving eikonal equations, have a cost which is fixed as k increases, but have unacceptably low accuracy for many problems, except at very high k.

In this talk, we overview progress in developing numerical methods for high frequency problems which try to combine standard numerical and asymptotic approaches. In particular, we focus on boundary integral equation based methods, describing progress in generating numerical schemes which deliver an arbitrarily high requested accuracy with a number of degrees of freedom which provably (through numerical analysis theorems and numerical experiments) needs only grow logarithmically as k increases.

The methodology is to obtain knowledge of the phase structure of the solution, by rather elementary high frequency ray analysis, and to build this phase structure into the basis functions used to approximate the solution. This idea, in a simple form, dates back at least to [1], but the methodology has seen a wealth of new ideas in the last 5-10 years, see [2, 3, 4, 5, 6, 7] and the references therein, which we review.

Keywords: method of moments, boundary element methods, high frequency

Mathematics Subject Classifications (2010): 65N38, 35Q60

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Magnetic model refinements via finite element subproblems

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ABSTRACT

A subproblem method (SPM) with finite element (FE) solutions provides advantages in repetitive analyses and helps improving the solution accuracy [1, 2, 3, 4]. It allows to benefit from previous computations instead of starting a new complete FE solution for any geometrical, physical or model variation. It also allows different problem-adapted meshes and computational efficiency due to the reduced size of each SP.

A general framework allowing a wide variety of refinements is herein developed. It is defined as a SP FE approach based on canonical magnetostatic and magnetodynamic problems solved in a sequence. It splits problems into SPs of lower geometrical, physical and numerical complexity, reducing meshing operations and computational aspects. This allows a natural progression from simple to more elaborate models, while quantifying the gain given by each model refinement and justifying its utility. At each step, volume sources (VSs) and surface sources (SSs), originating from previous solutions, are applied via mesh-to-mesh projections. VSs express changes of material or volume properties. SSs express changes of boundary conditions (BCs) or interface conditions (ICs). Common and useful changes from source to reaction fields [3], ideal to real flux tubes (with leakage flux) [2], 1-D to 3-D models [3], perfect to real materials [1], statics to dynamics, linear to nonlinear models, thin to volume models [4], can all be defined and coupled through combinations of VSs and SSs. Approximate problems with ideal flux tubes can be accurately corrected when accounting for leakage fluxes and material changes, and particular source inductors, up to 3-D. Also, reference solutions related to limit behaviors of regions (perfectly conductive or magnetic nature) can be followed by accurate calculation of the field distribution in real materials and the ensuing losses. This allows efficient parameterized analyses on the electric and magnetic characteristics of regions in a wide range, covering various skin depths.

The developments are performed for the magnetic vector potential FE formulation. All the constraints involved in the SPs are carefully defined in the associated FE formulations and circuit relations, respecting their inherent strong and weak natures. As a result, local fields and global quantities, i.e. flux, MMF, reluctance, voltage, current, resistance, are efficiently and accurately calculated.

Keywords: finite-element method (FEM), model refinement, subdomain method

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Operator preconditioning

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ABSTRACT

Ill-conditioned linear systems of equations arising from finite element or boundary element Galerkin discretizations on fine meshes are often encountered in computational electromagnetism. Applying iterative solvers will incur slow convergence, unless powerful preconditioners are available. The very general policy of operator preconditioning can provide them in various settings.

Abstract operator preconditioning tackles an isomorphism $A : V \to V'$, where V is a Banach space with dual V'. It enlists another isomorphism $B : W \to W'$, where the spaces V and W are connected by an inf-sup stable sesqui-linear form $d \in L(V \times W, \mathbb{C})$. Writing $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{D}$ for Galerkin matrices associated with A, B, and d, we find that the spectral condition number of $\mathbf{D}^{-1}\mathbf{B}\mathbf{D}^{-T}\mathbf{A}$ depends only on the norms and inf-sup constants of A, B, and d. These can often be bounded independently of the trial and test spaces, which yields a "mesh-independent" preconditioner $\mathbf{D}^{-1}\mathbf{B}\mathbf{D}^{-T}$ for $\mathbf{A}$.

Operator preconditioning can be applied to a wide array of finite element methods, in particular to mixed methods that lead to saddle point problems. A particular example is provided by stabilized variational formulations for time-harmonic Maxwell's equations at low frequencies. Another important application is preconditioners for coupled boundary element and finite element schemes for eddy current problems.

Operator preconditioning has also become a key technique in the efficient implementation of low-order boundary element methods for electromagnetic scattering. In this context it is known as Calderón preconditioning and the operators A and B are discrete boundary integral operators with complementary mapping properties. Stable duality pairings d can found for pairs of boundary element spaces built upon primal and dual surface meshes.

Keywords: operator preconditioning, boundary elements, stable duality pairing

Mathematics Subject Classifications (2010): 65N38

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Mathematical and numerical modelling of piezoelectric sensors for non destructive testing by ultrasounds

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ABSTRACT

In this work, we address mathematical and numerical questions related to the simulation of non destructive testing experiments using piezoelectric devices. In particular, we focus on the modelling of piezoelectric sensors that are used to generate and record ultrasonic waves in a solid material : such waves are typically used to investigate in a non invasive way the possible presence of defects in manufactured items. Such an issue has already been tackled in the engineering literature [1] but not, to our knowledge, by way of rigorous applied mathematics. The equations of piezoelectricity [2] couple Maxwell's equations with linear elastodynamics equations which correspond to a coupled hyperbolic system. This system presents quite different time scales due to the very large ratio between the speed of light and the sound speed, which makes it impossible to treat by a direct numerical approach. To overcome this problem, we give a rigorous justification, via asymptotic analysis, of the so-called quasi-static approximation model in which the electric unknowns are reduced to a scalar electric potential : the reduced model appears as a coupled elliptic-hyperbolic system. We next justify the reduction of the computation of this electric potential to the piezoelectric parts of the computational domain. Finally, a particular attention is devoted to the modelling of the electric supply process: the different boundary conditions used to model the emission and reception regimes as well as the modelling of the coaxial cable connecting the sensor to the electric generator. Concerning the numerical approximation, an energy preserving finite element / finite differences numerical scheme is developed. Its stability is analyzed and numerical results in academic or more realistic situations will be presented. More details can be found in [3].

Keywords: piezoelectricity, ultrasonic sensors, quasi-static approximation, asymptotic analysis, electric supply modelling, finite element approximation, energy preserving schemes

Mathematics Subject Classifications (2010): 35L05, 35A35, 73R05, 35A40

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Time dependent integral equations: numerical methods and inverse problems

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ABSTRACT

We present two applications of time domain integral equations in scattering theory. The first application is to the analysis of a near field time domain inverse scattering algorithm for the wave equation, and the second is to provide a marching on in time algorithm for a boundary integral equation for Maxwell's equations.

For the inverse problem we seek to determine the shape and location of a perfectly conducting scatterer from measurements of the acoustic velocity potential at points in the near field. The method used to solve this problem is a new version of the Linear Sampling Method of Colton and Kirsch [4] in the time domain and is joint work with Q. Chen, A. Lechleiter and H. Haddar [2]. We use the Laplace transform domain approach of [1] to derive the time domain mapping properties of the near field operator and hence provide theoretical support for the method. Some numerical results will also be shown.

The second problem is to prove convergence of the Convolution Quadrature approach of Lubich [5] to discretizing the time domain Electric Field Integral Equation (EFIE) which arises as a first kind integral equation to determine the electromagnetic field in the exterior of a scatterer. To apply Lubich's theory we need to determine coercivity properties of the EFIE operator [6]. We provide an error analysis and some numerical results. This is joint work with Q. Chen and D. Weile [3].

Keywords: time domain integral equations, Laplace transform approach. inverse scattering, EFIE

Mathematics Subject Classifications (2010): 65R20, 65R32, 65M15

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The EPGS industrial process: numerical simulation and experimental validation

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ABSTRACT

EPGS stands for Electromagnetic Pump Green Sand and it is an existing industrial process that was developed from scratch by the Light Alloys Division of CIDAUT [1]. The goal of this process is to manufacture aluminum automotive components with high mechanical capabilities. Lately, the automotive industry has tried to fulfill two apparently contradictory demands. Higher fuel efficiency and greener cars are demanded (this means lighter vehicles), but society is also concerned with car safety and also demands more comfort (which means more weight). These two contradictory trends can be achieved if lighter materials, as aluminum, replace heavier actual manufacturing materials. But aluminum shows some features that make it difficult to obtain components with excellent mechanical properties using molding processes. Molten aluminum is prone to oxidize (if oxides are included in the final component due to a too fast mold filling, mechanical properties are very low) and it is a very corrosive material, what hinders handling it in injection processes. In the EPGS process, these drawbacks were overcome using an electromagnetic pump, immersed in an aluminum bath, for counter-gravity sand mold filling, which permits to control the velocity of the advancing free surface inside the mold, and therefore, no turbulence is produced during the mold filling and oxides are not included in the final car component. The EPGS process also includes the aluminum melting, holding and treatment. Almost all stages of this process were subjected to a numerical simulation and experimental validation methodology. In relation to the electromagnetic pump, simulation and validation works involved electromagnetism, magnetohydrodynamics [2], vibration [3], and thermal and cooling aspects. All these works will be presented in the workshop and others not mentioned. In CIDAUT, we believe that any numerical simulation tool, whenever possible, must be experimentally validated because it is essential in order to guarantee its reliability and that the results reproduce reality with an error within a certain range.

Keywords: numerical simulation, experimental validation **Mathematics Subject Classifications (2010)**: 78-05

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Numerical simulation of metallurgical processes wich arises in silicon industry

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ABSTRACT

Ferroatlántica I+D coordinates the research activity of the Spanish Group Ferroatlántica, which is devoted to the production of ferroalloys and, in particular, of silicon. In the last years, the company has used the numerical simulation as a powerful tool to understand and develop different technological innovations in silicon production. In this work, we will present the numerical simulation of metallurgical electrodes and induction heating furnaces. Both applications require the solution of multiphysics problems which involve an electromagnetic model based on the eddy current equations.

The numerical simulation of metallurgical electrodes is focused on the computation of the distribution of temperature and the current density under different operation conditions [1]. We will describe the physical problem involved in an electric furnace, the role of metallurgical electrodes and the key points of its numerical simulation. In particular, we will present some numerical results obtained for a compound electrode patented by the company, the ELSA electrode. On the other hand, the numerical simulation of an induction heating system requires the solution of a thermo-magneto-hydrodynamic model [2] with change of phase. We will present the physical problem, the aims of its numerical simulation and some numerical results. The simulation of induction heating systems is being used by the company to analyze the possibilities of producing silicon of photovoltaic quality via the metallurgical via [3].

This research work has been done in collaboration with the group in Mathematical Engineering of the University of Santiago de Compostela and opened interesting research lines in the field of mathematical modeling and numerical analysis.

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Trefftz-discontinuous Galerkin methods for the time-harmonic Maxwell equations

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ABSTRACT

Several finite element methods used in the numerical discretization of wave problems in frequency domain are based on incorporating a priori knowledge about the differential equation into the local approximation spaces by using Trefftz-type basis functions, namely functions which belong to the kernel of the considered differential operator. These methods differ form one another not only for the type of Trefftz basis functions used in the approximating spaces, but also for the way of imposing continuity at the interelement boundaries: partition of unit, least squares, Lagrange multipliers or discontinuous Galerkin techniques.

In this talk, Trefftz-discontinuous Galerkin methods for the time-harmonic Maxwell equations, as introduced in [1], are considered. The construction of such methods, together with their abstract p–version error analysis, following [2], will be presented. This analysis, which follows the same lines as the one developed in [3] for the Helmholtz problem, requires new stability estimates and regularity results for the continuous problem which can be of interest on their own; these results are contained in [4]. The particular case where the approximating Trefftz spaces are made of plane waves will be considered, and explicit error estimates will be given.

Keywords: time-harmonic Maxwell's equation, discontinuous Galerkin methods, Trefftz methods, p–version error analysis, plane waves

Mathematics Subject Classifications (2010): 65N15, 65N30, 35Q61

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Analysis through numerical simulations of the feasibility of plastic moulding with heat induction systems

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ABSTRACT

The aim of this work is to assess the feasibility of using induction heating techniques for plastic moulding. For that purpose a set of numerical simulations were performed to analyse the effect of different design parameters of the induction system on the energy efficiency and mould temperature pattern throughout the induction heating process. The numerical modelling was validated with experiments carried out by Kunststoff Institut of Germany in its laboratories.

The main design parameters analysed in this study are: coil-mould distance, frequency, coil geometry/configuration, mould material and the use of flux concentrators. The efficiency of the mould heating and its temperature profile are the main outputs considered in this study to evaluate the suitability of a particular induction system design.

In addition, the computed voltage and current of the coil, for a given design and different powers supplied to the system coil-mould, are used to determine the electrical characteristics and dimensions of the generator that would be required in each case. Moreover, the computed temperature profile and power dissipated in the coil can be used for the design of its cooling system.

This research work has been done in collaboration with the Applied Mathematics department of Universidade de Santiago de Compostela and is funded by the EC through grant number 243607 of the FP7-SME-2008 program.

Keywords: numerical simulation, induction heating, plastic moulding

Mathematics Subject Classifications (2010): 65N30, 65Z05

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Comments on the Galilean limits of Maxwell's equations

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ABSTRACT

Maxwell's equations are fundamental for the description of electromagnetic phenomena and valid over a wide range of spatial and temporal scales. The static limit of the theory is well defined and much easier. The electric and magnetic fields are given by the laws of Coulomb and Biot-Savart. As soon as there is any time dependence, we should in principle use the full set of Maxwell's equations with all their complexity. However, a broad range of important applications are described by some particular models, as the ones in the low frequency range, emerging from neglecting particular couplings of electric and magnetic field related quantities. These applications include motors, sensors, power generators, transformers and micromechanical systems. Note also that the quasi-static models are useful for a better understanding of both low frequency electrodynamics and the transition from statics to electrodynamics. We thus present a wider frame to treat the quasi-static (QS) limit of Maxwell equations. Following [1, 2, 3], we discuss the fact that there exists not one but indeed two dual Galilean limits (called "electric" or EQS, and "magnetic" or MQS limits). As a consequence, one has to be careful when investigating non-relativistic limits. We start by a re-examination of the gauge conditions and their compatibility with Lorentz and Galilean covariance. By means of an adimensional analysis, first on the fields and secondly on the potentials, we emphasize the correct scaling yielding the two (limit) sets of Maxwell equations. With this particular point of view, the gauge conditions of Classical Electromagnetism are continuity equations whose range of validity depend on the Relativistic or Galilean nature of the underlying phenomenon and have little to do with mathematical closure assumptions taken without physical motivations.

Keywords: Maxwell's equations, quasi-static approximations, gauge conditions, dimensional analysis

Mathematics Subject Classifications (2010): 78A25, 78M34, 65Z05

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Certified reduced basis method for radar cross section computation

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ABSTRACT

We study nontrivial applications of the reduced basis method (RBM)[4, 2] to the computation of radar cross section (RCS)[3]. The method is explained with two examples. The first one has frequency, incident angle and measurement angle as parameters. The second one models the shape of a specific scatterer - a Pacman. With appropriate applications of the empirical interpolation method [1], transformation of the domain, configuration of perfectly matched layer, the exponential convergence of the RB solution over the whole parameter domain is achieved. Moreover, it allows efficient capturing of the critical shape that produces minimal reflection of the radar signal for the Pacman scattering problem, a rather interesting phenomenon deserving further studies.

Keywords: reduced basis method, electromagnetic scattering, radar cross section, empirical interpolation method

Mathematics Subject Classifications (2010): 65N15, 65N30, 78A25

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Spectral approximation of the curl operator

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ABSTRACT

Vector fields $\boldsymbol{H}$ satisfying $\mathbf{curl}\,\boldsymbol{H} = \lambda\boldsymbol{H}$, with λ being a scalar field, are called *force-free fields*. This name arises from magnetohydrodynamics, since a magnetic field of this kind induces a vanishing Lorentz force: $\boldsymbol{F} := \boldsymbol{J} \times \boldsymbol{B} = \mathbf{curl}\,\boldsymbol{H} \times (\mu\boldsymbol{H})$. In 1958 Woltjer [6] showed that the lowest state of magnetic energy density whithin a closed system is attained when λ is spatially constant. In such a case $\boldsymbol{H}$ is called a *linear* force-free field and its determination is naturally related with the spectral problem for the curl operator. The eigenfunctions of this problem are known as *free-decay fields* and play an important role, for instance, in the study of turbulence in plasma physics.

The spectral problem for the curl operator, $\mathbf{curl}\,\boldsymbol{H} = \lambda\boldsymbol{H}$, has a longstanding tradition in mathematical physics. A large measure of the credit goes to Beltrami [1], who seems to be the first who considered this problem in the context of fluid dynamics and electromagnetism. This is the reason why the corresponding eigenfunctions are also called *Beltrami fields*. On bounded domains, the most natural boundary condition for this problem is $\boldsymbol{H} \cdot \boldsymbol{n} = 0$, which corresponds to a field confined within the domain. Analytical solutions of this problem are only known under particular symmetry assumptions. The first one was obtained in 1957 by Chandrasekhar and Kendall [4] in the context of astrophysical plasmas arising in modeling of the solar crown.

More recently, some numerical methods have been introduced to compute force-free fields in domains without symmetry assumptions [2, 3]. In this work, we propose a variational formulation for the spectral problem for the curl operator which, after discretization, leads to a well-posed generalized eigenvalue problem. We propose a method for its numerical solution based on Nédélec finite elements of arbitrary order. We prove spectral convergence, optimal order error estimates and that the method is free of spurious-modes. Finally we report some numerical experiments which confirm the theoretical results and allow us to assess the performance of the method.

Keywords: finite elements, spectrum of the curl operator, Beltrami fields, linear force-free fields

Mathematics Subject Classifications (2010): 65N25, 65N30, 76M10, 78M10

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A symmetric BEM–FEM method for an axisymmetric eddy current problem

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ABSTRACT

In this work we analyze a time-harmonic eddy current model in an axisymmetric domain which includes bounded conducting domains and unbounded dielectric regions. This electromagnetic problem arises in the modeling of induction heating systems (see, for instance, [1]). For this problem, we propose a symmetric FEM and BEM coupling method in terms of a magnetic vector potential. We write a weak formulation in suitable weighted Sobolev spaces and follow some techniques of [1] and [2] to prove that the problem is well posed. We also propose a discretization that leads to a Galerkin scheme, that we show is convergent and has an optimal order of convergence.

We underline that [2] deals with the problem in the bounded case, whereas both [1] and this work consider an unbounded situacion. Besides, on the contrary to [1], the coupling procedure that we propose is of symmetric kind, fact that allows ourselves to analyze the BEM–FEM formulation and its discretization even for realistic (Lipschitz continuous) boundaries. The implementation of our scheme and numerical results will be described in a forthcoming work.

Keywords: eddy currents, axisymmetric domain, BEM-FEM coupling

Mathematics Subject Classifications (2010): 35Q60, 35Q61, 65N30, 65N38

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Arbitrary high-order Maxwell solvers based on spline discrete differential forms

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ABSTRACT

Since the pioneering work of Alain Bossavit (see [2] for an overview), the formulation of Maxwell's equations in the language of differential geometry has had a great success for finding appropriate discretizations for numerical simulations. In particular the Whitney forms yield the now classical De Rham complex for the lowest order Finite Element spaces. Such a De Rham complex of Finite Element spaces exists for arbitrary order [5]. However the association of the higher order elements with discrete differential forms is not straightforward. A specific construction of high order Whitney forms has recently been proposed by Rapetti and Bossavit [7].

B-Splines (see [3] for a comprehensive introduction), yield a natural way of defining arbitrary high order discrete differential forms on a tensor product mesh. We shall explain this construction that we recently developed in [1]. B-Spline discrete differential forms then provide a natural discretization of Maxwell's equations written in the language of differential geometry where the degrees of freedom can either be the spline coefficients or the geometrical degrees of freedom (value at vertex, edge integral, ... depending on the degree of the form). The Hodge operator as well as other operators from differential geometry can be discretized either using a dual mesh or a weak formulation yielding the so-called Finite Element Hodge.

Using a Finite Element Hodge leads to spline Finite Elements which have a close link to the concept of isogeometric analysis [4] which has been an area of intensive investigation recently [6]. We shall also present this approach which has recently been published in [8].

Keywords: Maxwell, B-Splines, discrete differential forms

Mathematics Subject Classifications (2010): 65M60

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CONTRIBUTED POSTERS

Validation of thermal simplified models for induction heat forming applied for shipbuilding industry

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ABSTRACT

Induction heating process is known to produce controllable heat on a conductive work piece. When the induction heating process is applied in association with automatic inductor-handling equipment and a heating line generation pattern, the productivity of the curved plate forming process is expected to improve greatly. This process is an important production process that can be widely used to produce various curved thick plate for shipbuilding industry.

Therefore, the induction heating process should be precisely modeled to evaluate its feasibility in plate forming. However, modeling of this whole and complex physical process involves a very high calculation time which does not fits the daily shipyard work as the heat forming process requires many heating lines to achieve the complex desired shapes. For this reason, the present work is focused on the development of simplified models that can give enough accuracy to predict the deformation.

One of the main process parameter of heat forming is the gradient of temperature between the up and down face of the plate. Therefore, simplified models based on the equivalence of the heat input of a high-frequency inductor varying the penetration in the section and initial conditions of the thermal calculation are developed.

This first approach to the simplified method was to keep constant the induction parameters and coil geometry for all the sample test. In the simulation process, the research was focus on the variation of the heat input size and the heat input penetration, by carrying out a multiparameter variation of initial conditions. The thermal and mechanical results obtained in the simulation are in good agreement with those measured in the experimental trials.

The future work will be focuses on the development of more complex models to predict the induction process. The model will consider the variations of the properties of the material when the Curie temperature is achieved. With this model the prediction of heat penetration rate is expected to be improved.

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Radiation condition for a non-smooth interface between a dielectric and a metamaterial

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ABSTRACT

We study a problem of electromagnetism in harmonic regime set in a bounded domain constituted of a classical dielectric and a negative metamaterial. At a given frequency, this metamaterial is modeled by a homogeneous medium with real strictly negative permittivity and permeability. From a practical point of view, it is of course illusive to consider such a model because of the losses. However, for the applications, the goal is to design low dissipative media and we proved that the behavior of the solution to the associated problem when the dissipation tends to zero is closely linked with the nature of the non-dissipative problem. Thus, it is crucial to understand the ideal problem, which is an original transmission problem because of the sign-changing coefficients. Here, we do not study the case of the whole Maxwell system but instead focus on an academic version in 2-D:

$$\left|\begin{array}{l}\text{Find } u \in H^1_0(\Omega) \text{ such that}\\ \mathrm{div}(\varepsilon\nabla u) = f \in H^{-1}(\Omega).\end{array}\right. \tag{1}$$

When the interface between the dielectric and the metamaterial has a corner, according to the ratio of the values of ε (contrast), problem (1) can be ill-posed (not Fredholm) in H^1. This is due to the degeneration of the two dual singularities associated with the corner which then behave like $r^{\pm i\eta} = e^{\pm i\eta \ln r}$ with $\eta \in \mathbb{R}^*$. This apparition of propagative singularities is very similar to the apparition of propagative modes in an unbounded waveguide for the classical Helmholtz equation with Dirichlet boundary condition, the contrast playing the role of the frequency. In this work, we derive for our problem a functional framework by adding to H^1 one of these propagative singularities. Well-posedness is then obtained by imposing a radiation condition, justified by means of a limiting absorption principle, at the corner between the two media. In this poster, we also point out some original questions which appear when one studies problem (1) set in a domain Ω_δ with a slightly rounded corner.

Keywords:transmission problem, metamaterial, corner, limiting absorption principle

Mathematics Subject Classifications (2010): 35Q60, 35Q61, 35J20, 78A40, 78A50

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Modelling of non-homogeneous lossy coaxial cable for time domain simulation

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ABSTRACT

In this work, we focus on the time-domain simulation of the propagation of electromagnetic waves in non-homogeneous lossy coaxial cables. The complete modeling of electrical devices (piezoelectric sensors for instance, see [1]) often require an accurate modeling of the supply process which includes the propagation of current and electric potential through coaxial cables. The full 3D Maxwell equations, that described the propagation of current and electric potential, are classically not tackled directly, but instead a 1D scalar model known as the telegraphist's model is used. Such an issue has already been dealt with in the engineering literature or in the applied mathematics literature using systematically a modal approach in the Fourier domain. We aim at justifying, by means of error estimates, a time-domain "homogenized" telegraphist's model. This model is obtained via asymptotic analysis, for lossy coaxial cable whose cross-section is not homogeneous, it also includes non-local in time operator. Numerical results in academic situations are presented. More details can be found in [2].

Keywords: Coaxial cables, telegraphist's model, asymptotic analysis, Maxwell equations, electric supply modelling.

Mathematics Subject Classifications (2010): 35L05, 35A35, 73R05, 35A40.

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Eddy current tomography of deposits in steam generators

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ABSTRACT

Eddy current testing (ECT) using coils is widely practised in in-service inspection of steam generators in nuclear power plants of pressurized water reactor type. In this poster, we consider the shape estimate problem of magnetic deposits given some ECT signals. The non-linearity and the ill-posedness of this inverse problem make it quite difficult. We focus on the axisymmetric case and build a PDE-based direct model with Dirichlet-to-Neumann boundary operators to describe the relationship between observed data and the inspected component. With this direct model, we propose an inverse algorithm of gradient descent shape optimization type involving a regularization technique by boundary differential operators. First numerical experiments show some reconstruction results and open further topics for discussion.

Keywords: eddy current testing, shape derivative, boundary regularization

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Numerical simulation of an induction heating system

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ABSTRACT

In this work we are interested in the thermo-electrical simulation of a plastic moulding system consisting of a cylindrical mould surrounded by a copper inductor coil, connected to a power-supply and carrying an alternating electric current.

From the mathematical point of view, the full problem consists of a coupled system of partial differential equations arising from the thermo-electrical modelling. More precisely, the electromagnetic model is an eddy current problem and the thermal one consists of the transient heat transfer equation. The models are coupled because the physical parameters depend on the temperature and the heat source in the thermal problem is the Joule effect. Nevertheless, in this work we will consider that the physical properties are independent of temperature. This will allow us to solve the electromagnetic and thermal problems independently.

In order to state the problem in an axisymmetric setting, the induction coil has to be replaced by parallel rings having toroidal geometry. This will allow us to simplify the numerical simulation.

Our goal is to understand the influence on the system performance of certain geometrical parameters such as the number of induction coils and their relative position to the mould, and also physical parameters such as the frequency or the off/on cycle of the coil. In particular, we are interested in computing the distribution of heat in the mould caused by the eddy currents and also in reducing the time.

This report is the result of a master thesis project carried out for Tecnologías Avanzadas Inspiralia S.L. in the framework of the Master Degree in Mathematical Engineering of the Universities of Santiago de Compostela, A Coruña and Vigo.

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Thermo-magneto hydrodynamic simulation of industrial induction furnaces

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ABSTRACT

In this work we summarize some of the main results obtained from the thermo-magneto-hydrodynamic simulation of melting induction furnaces with cylindrical symmetry, and which can be found in the references [1, 2, 3, 4].

The induction furnace consists of a helical coil surrounding a cylindrical crucible charged with the material to be melted. The inductors are supplied with alternating current which induces eddy currents inside the component being heated due to Faraday's law. The overall process is very complex owing to the coupling of the three different physical phenomena that take part in it: an electromagnetic model, a thermal model with change of phase, and a hydrodynamic model for the molten region of the material. The models are coupled because physical parameters depend on temperature, heat source in the thermal problem is Joule effect and the liquid domain of the hydrodynamic model depends on temperature. On the other hand, buoyancy forces and Lorentz force appear in the hydrodynamic model, and the velocity of molten metal appears in the convective term of the heat equation.

The problem is formulated in a radial section of the domain by assuming cylindrical symmetry.

For the numerical approximation, the electromagnetic problem is discretized using a finite/boundary element method, whereas the thermal and the hydrodynamic problems are approximated using Lagrange-Galerkin methods. To deal with the coupling between the models and the non-linearities we employ different iterative fixed point algorithms. Finally, some numerical results obtained in the simulation of an industrial furnace are shown.

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Numerical simulation using COMSOL of a linear induction electromagnetic pump to drive molten aluminium

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ABSTRACT

The control of molten metal in all its stages is fundamental in the metallurgy industry, mainly in those where it is in motion. Different types of pumps are currently used for the movement and agitation of molten metals. When the industry works with high aggressive metals from the chemical viewpoint, as molten aluminium, the use of mechanical pumps is restricted because their components will be degraded with the use and it would be necessary their early replacement. Material from the pump, due to the molten metal corrosion, contaminates the molten metal itself, which is another negative side effect related to the use of this type of pumps. One way to avoid this situation is the use of electromagnetic pumps, which move liquids without contact using electromagnetic fields and therefore, their main advantage is the lack of moving parts in contact with the molten metal. Moreover, it is avoided the pump corrosion and consequently the contamination of the melt [1]. Many types of electromagnetic pumps are available, including linear induction pumps.

The experimental study of an electromagnetic pump for high temperature liquid metals, (as aluminum which is molten beyond 680ï¿½C), has many technical inconveniences and economical disadvantages. Simulation can be a more reasonable alternative from the scientific and cost-effective point of view. But this is only possible, if the numerical simulation is considered reliable, for this fact it is mandatory a complete experimental validation. In Cidaut, a computational and experimental investigation is being carried out about electromagnetically stirred molten aluminum. A laboratory experiment has been set up where molten aluminum is contained in a cylindrical vessel made of stainless steel and is driven by a Linear Induction Electromagnetic Pump. It has already been made a successfully multiphysics simulation of this experiment using ANSYS and FLUENT programs [2]. But recently, CIDAUT has switched to COMSOL as our multiphysics simulation platform, and again, this tool might be also validated (for this purpose it has been used the previous validated simulation works). In this work it will be explained the experimental setup, the previous simulation conclusions, and it will be presented the initial results we are getting using COMSOL as a simulation tool. To this point, CIDAUT have made the electromagnetic simulation in 3D of the Linear Induction Electromagnetic Pump and have also simulated the drive of the molten aluminum inside the vessel, but without considering the free surface. So far, the results obtained with COMSOL are comparable to those provided by ANSYS and FLUENT and are in good agreement with the experimental results.

Keywords: numerical simulation, experimental validation

Mathematics Subject Classifications (2010): 78-05

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An eddy current problem with a nonlinear boundary condition

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ABSTRACT

We consider an eddy current problem in a bounded domain with a nonlinear approximate degenerate boundary condition of the type $\boldsymbol{\nu} \times \boldsymbol{E} = \boldsymbol{\nu} \times \partial_t \boldsymbol{a}(\boldsymbol{H} \times \boldsymbol{\nu})$. This dissipative boundary condition corresponds to a non-perfect contact at the boundary.

Approximate boundary conditions in electromagnetics has been mainly studied in frequency domain, but when dealing with nonlinearities one has to work in time domain. There are several approaches how to pose the problem in time domain. We follow the work [3], where impedance-type interface conditions are used. These conditions were derived for time-harmonic electromagnetic field to model the behaviour of a thin conducting and ferromagnetic shell. A similar analysis can be performed for non-time-harmonic fields.

We formulate the problem in an appropriate functional setting. The techniques developed in the book [2] and monotone operator theory are used to establish the existence and uniqueness of a solution. Time and space error estimates are also derived. To support obtained convergence results we perform some numerical experiments. For more details we refer the reader to [1].

Keywords: nonlinear evolution boundary condition, quasi-static model, discretization, error estimates

Mathematics Subject Classifications (2010): 35D30, 35Q61, 65M15, 65M60

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CURSOS E CONGRESOS
Nº 210

This volume contains the abstracts of the lectures given at NELIA 2011, the *Workshop on Numerical Electromagnetics and Industrial Applications*, held in Santiago de Compostela, Spain, from 25 to 28 October 2011. The main goal of this workshop was to set up a discussion around the recent developments in the mathematical, numerical and computational analysis of electromagnetic models and their direct industrial applications. The scientific program consisted of 23 invited lectures and a poster session. Selected papers from this program will be published in a special issue of the journal Applied Numerical Mathematics.

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