Subgame perfection and the rule of k names*

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Abstract

In this paper we revisit the rule of k names from a game theoretic perspective. This rule can be described as follows. Given a set of candidates for a position, a committee (formed by the proposers) selects k elements of that set using a screening rule; then a single individual from outside the committee (the chooser) chooses for the position one of the k selected candidates. In this context we first give conditions for the existence of a subgame perfect equilibrium. Then we provide conditions for the existence of subgame perfect q-strong equilibria when the screening rule is π-majoritarian. Finally, we show that when the chooser can strategically appoint a delegate to choose on behalf of him, the conditions for the existence of subgame perfect q-strong equilibria are weaker.

Key words. Rule of k names, screening rule, subgame perfect equilibrium, strong equilibrium, delegation.

1 Introduction

In this paper we revisit the rule of k names first studied from a game theoretic perspective in Barberà and Coelho (2010). This rule can be described as follows. Given a set of candidates for a position, a committee (formed by the *proposers*) selects k elements of this set; then a single individual from outside the committee (the *chooser*) chooses for the position one of the k selected candidates. The chooser and also each proposer have their own preferences over the set of candidates. The rule

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of k names is in fact a family of rules, because it does not specify the procedure (the *screening rule*) used by the committee to select the k candidates; several screening rules associated with the rule of k names are used by different institutions around the world. For instance, Chapter 3 of Colomer (1995) describes how the rule of k-names with a sophisticated screening rule was used to choose Adolfo Suárez for the Presidency of the Spanish Government in July 1976, during the Spanish transition to democracy. The rule of k names has also been analyzed in other papers. For instance, Barberà and Coelho (2008) and Pérez et al. (2012) study several aspects of set valued screening rules, that are one of the main ingredients of the rule of k names. In Barberà and Coelho (2017) a specific family of rules of k names is studied, the v-rules of k-names, and the impact of the choice of parameters v and k upon the distribution of power among the proposers and the chooser is investigated.

Barberà and Coelho (2010) provide a game theoretical analysis of the rule of k names using two strategic games: the *constrained game* in which the chooser is assumed to choose always his best option within the list provided by the committee, and the *unconstrained game* in which the latter assumption is dropped. They say to offer the analysis of both games (not only of the constrained game, more natural from the point of view of rationality) *because the ability of the chooser to commit may be different in different real-life cases*. Also, they use the strong equilibrium concept to capture *the mix of threats and cooperation*, as they write, which is present in these voting situations. They mainly deal with *majority screening rules*, which are those in which a majority of choosers can impose the election of any set of k candidates.

In this paper we go further in the game theoretical analysis of the rule of k names. We separate from Barberà and Coelho in the following points:

- They mainly deal with the strong equilibrium concept. In this paper we use several equilibrium concepts. We stress that the rule of k names is in fact a two-step rule in which the chooser is perfectly informed of the set of k candidates chosen by the committee in the first step. Consequently, we pay special attention to the subgame perfect equilibrium concept and its variations.
- We make a non-cooperative analysis of the rule of k names in a strict sense, i.e., we do not consider any commitment abilities of the chooser or the proposers unless we explicit model them. For instance, in Section 4 we study how the analysis of a k-names game changes when the chooser has a specific commitment ability. For that purpose, we explicitly model this ability defining a particular three-stage game and, then, we analyze the resulting game.
- We deal with several families of screening rules, not only with majority or weak majority screening rules.

The structure and main results of this paper are as follows. In Section 2 we formally introduce the non-cooperative model that we analyze: the k-names games. We first deal with no-veto screening rules, which are those for which no proposer has the ability of vetoing a candidate. We prove that a screening rule is no-veto if and only if, in each k-names game with this screening rule, there exists a subgame perfect equilibrium whose associated outcome is the best candidate for the chooser. The class of no-veto screening rules is a wide one and includes, for instance, all the majority rules. By the other hand, since the rule of k names is in fact a two-step rule, subgame perfectness is necessary to warrant strategic stability (see, for instance, Selten (1975)).

In Section 3 we use a more restrictive equilibrium concept: the subgame perfect q-strong equilibrium, based on the strong equilibrium introduced in Aumann (1959). While the Nash concept of stability defines equilibrium only in terms of unilateral deviations, strong equilibrium allows for deviations by every conceivable coalition; particularly, q-strong equilibrium allows for deviations by every conceivable coalition of proposers. In the context of k-names games it seems to be more appropriate to use the q-strongness equilibrium concept than the strongness equilibrium concept, because while the possible deviation of a group of proponents in the first step is plausible, the possible deviation of a group that includes some proposers and the chooser (in the first and second stages) is not credible unless they have a commitment device. In Section 3 we give a sufficient condition and a necessary condition for the existence of subgame perfect q-strong equilibria in k-names games based on π-majoritarian screening rules, where $\pi \in [1/2, 1)$ indicates the "degree" of majority required by the rule. We finish this section proving that within the class of majority rules, subgame perfect q-strongness and subgame perfect strongness are somewhat equivalent conditions.

Finally, in Section 4 we show that, within the class of majoritarian screening rules, if the chooser can strategically appoint a delegate to choose on behalf of him then the conditions for the existence of subgame perfect q-strong equilibria are weaker (under certain assumptions). According to our result we could say that the capacity of the chooser to appoint a delegate can increase the stability of the game. In Section 5 we enumerate the main conclusions of this paper.

2 The class of k-names games

To start with, we introduce the *k-names games* that we use to analyze the rule of k names. Our model is essentially the same as in Barberà and Coelho (2010) although we stress that we are dealing with the strategic game corresponding to a two-stage game.

Definition 2.1. *A k-names game G_k is given by the tuple $(N, A, X, S_k, X_0, \{\succeq_i\}_{i \in N})$ where:*

- *$N = \{0, 1, \ldots, n\}$ is the set of players; 0 is the chooser and $N_p = \{1 \ldots, n\}$ is the set of proposers.*
- *A is the finite set of candidates. We denote by A_k the collection of subsets of A with cardinal k. Since k is the number of candidates selected by the proposers, k can vary from one to $|A|$.*
- *X is the finite set of strategies of each proposer and, thus, X^n is the set of profiles of strategies of the proposers. We take a general point of view and do not specify the process through which the proposers select their proposals. We just consider that this process can be modeled as a movement in which proposers choose simultaneously strategies of a common set and, depending on the chosen profile of strategies, a screening rule known in advance by the proposers makes a selection in A_k.*
- *S_k is the screening rule. So, $S_k : X^n \to A_k$ selects a set of k candidates $S_k(\mathbf{x})$ for every $\mathbf{x} = (x_1, \ldots, x_n) \in X^n$.*
- *X_0 is the set of strategies of the chooser. In this two-stage game the chooser is perfectly informed when making his choice and thus he makes a plan for every set in A_k possibly reached after the first stage. Formally,*

$$X_0 = \{x_0 : A_k \to A \mid x_0(B) \in B, \text{ for all } B \in A_k\}.$$

- *For all $i \in N$, $\succeq_i$ is a binary relation over A that describes the preference of i over A. We assume that $\succeq_i$ is reflexive, antisymmetric, transitive and complete. The antisymmetry assumption implies that all agents' preferences are strict.*

Now, we give the formal definition of Nash equilibrium and subgame perfect equilibrium in this context. As we have already mentioned, since the k-names games can be seen as two-stage games, subgame perfectness is necessary to warrant strategic stability in this context and, thus, in this paper we mainly concentrate on equilibrium concepts that include subgame perfection.

Definition 2.2. *Take a k-names game $G_k = (N, A, X, S_k, X_0, \{\succeq_i\}_{i \in N})$. A Nash equilibrium of G_k is a pair $(\mathbf{x}, x_0) \in X^n \times X_0$ such that:*

1. *$x_0(S_k(\mathbf{x})) \succeq_0 x_0'(S_k(\mathbf{x}))$, for all $x_0' \in S_0$.*
2. *$x_0(S_k(\mathbf{x})) \succeq_i x_0(S_k(\mathbf{x}_{-i}, x_i'))$, for all $x_i' \in X$ and for all $i \in N_p$, where $(\mathbf{x}_{-i}, x_i')$ denotes the profile $(x_1, \ldots, x_{i-1}, x_i', x_{i+1}, \ldots, x_n)$.*

Definition 2.3. *Take a k-names game $G_k = (N, A, X, S_k, X_0, \{\succeq_i\}_{i \in N})$. A subgame perfect equilibrium of G_k is a pair $(\mathbf{x}, x_0) \in X^n \times X_0$ such that:*

1. $x_0(S_k(\bar{\mathbf{x}})) \succeq_0 a$, *for all* $a \in S_k(\bar{\mathbf{x}})$, *and for all* $\bar{\mathbf{x}} \in X^n$.
2. $x_0(S_k(\mathbf{x})) \succeq_i x_0(S_k(\mathbf{x}_{-i}, x_i'))$, *for all* $x_i' \in X$ *and for all* $i \in N_p$.

Notice that our model allows for all kind of screening rules and, thus, is remarkably general. As a consequence of this, it is easy to see that the set of Nash equilibria of G_k may be empty. The following example illustrates this fact.

Example 2.1. *Take* $G_2 = (N, A, X, S_2, X_0, \{\succeq_i\}_{i \in N})$ *with* $N_p = \{1, 2, 3\}$, $A = \{a, b, c\}$, $X = \{\alpha, \beta\}$, *where* α *means "I vote for* $\{a, c\}$*" and* β *means "I vote for* $\{b, c\}$*". To define the screening rule denote by* $N(\{a, c\}, \mathbf{x})$ *the number of votes that* $\{a, c\}$ *receives according to* $\mathbf{x}$*. The screening rule is now given by:*

$$S_2(\mathbf{x}) = \begin{cases} \{\, a,c\} & \text{if } N(\{a,c\},\mathbf{x}) \text{ equals } 3 \text{ or } 1, \\ \{\, b,c\} & \text{if } N(\{a,c\},\mathbf{x}) \text{ equals } 2 \text{ or } 0. \end{cases}$$

Finally, the preferences of the proposers and of the chooser are as follows:

Proposer 1	*Proposer 2*	*Proposer 3*	*Chooser*
a	b	c	a
b	c	a	b
c	a	b	c

Let us check that G_2 *does not have Nash equilibria. In fact, it is clear that the result corresponding to a Nash equilibrium cannot be c (because in that case, the chooser is better off by deviating). Suppose that there exists a Nash equilibrium* $(\mathbf{x}, x_0)$ *whose corresponding result is a. Then* $N(\{a, c\}, \mathbf{x})$ *equals 3 or 1 but, in both cases, Proposer 2 would gain by deviating, so* $(\mathbf{x}, x_0)$ *cannot be a Nash equilibrium. Suppose now that there exists a Nash equilibrium* $(\mathbf{x}, x_0)$ *whose corresponding result is b. Then* $N(\{a, c\}, \mathbf{x})$ *equals 2 or 0 but, in both cases, Proposer 3 would gain by deviating, so* $(\mathbf{x}, x_0)$ *cannot be a Nash equilibrium.*

The next theorem provides a class of screening rules that guarantee the existence of a subgame perfect equilibrium in all their associated k-names games: the class of no-veto screening rules. In words, a screening rule is said to be no-veto if there does not exist a proposer with the capacity of vetoing a candidate. Moreover, the theorem characterizes this class as the unique one for which the chooser's best candidate is always a subgame perfect equilibrium outcome. Below we give the formal definition of no-veto rule and the result.

Definition 2.4. *A screening rule* $S_k : X^n \to A_k$ *is said to be* no-veto *if for every* $a \in A$ *there exists* $\mathbf{x} \in X^n$ *such that*

$$a \in S_k(\mathbf{x}_{-i}, y)$$

for all $y \in X$ *and all* $i \in N_p$.

Theorem 2.1. *Take a screening rule S_k. S_k is no-veto if and only if the best candidate of the chooser is a subgame perfect equilibrium outcome of G_k, for all G_k whose screening rule is S_k.*

Proof. Take a no-veto S_k and $G_k = (N, A, X, S_k, X_0, \{\succeq_i\}_{i\in N})$. Denote by x_0 the strategy that selects for every $B \in A_k$ the best candidate for the chooser in B. Now denote by a_{G_k} the best candidate for the chooser in G_k and take $\mathbf{x} \in X^n$ such that for all $y \in X$ it holds that $a_{G_k} \in S_k(\mathbf{x}_{-i}, y)$ for all $i \in N_p$. It is clear that $(\mathbf{x}, x_0)$ is a subgame perfect equilibrium of G_k and that $x_0(S_k(\mathbf{x})) = a_{G_k}$. Conversely, take a screening rule S_k such that for all $G_k = (N, A, X, S_k, X_0, \{\succeq_i\}_{i\in N})$ there exists $(\mathbf{x}, x_0)$ a subgame perfect equilibrium of G_k with $x_0(S_k(\mathbf{x})) = a_{G_k}$, a_{G_k} being the best candidate for the chooser in G_k. Assume that S_k fails to be no-veto. Then, there exists $a \in A$ such that, for all $\mathbf{x} \in X^n$,

$$a \notin S_k(\mathbf{x}_{-i}, y) \tag{1}$$

for some $y \in X$ and $i \in N_p$. Notice that, according to Definition 2.4, a does not depend on $\{\succeq_i\}_{i\in N}$. Now take G_k such that a is the best candidate for the chooser and a is the worst candidate for all the proposers. Then, in view of (1), there cannot exist $(\mathbf{x}, x_0)$, a Nash equilibrium of G_k, such that $x_0(S_k(\mathbf{x})) = a$, which is a contradiction (because subgame perfect implies Nash). □

Remark 2.1. *Reading the proof of Theorem 2.1, it is easy to check that its statement also holds when we write* Nash equilibrium *instead of* subgame perfect equilibrium.

Definition 2.5 below introduces the class of *dictatorial* screening rules. In general, these rules fail to be no-veto; however, it is clear that every G_k with a dictatorial S_k has a subgame perfect equilibrium.

Definition 2.5. *A screening rule $S_k : X^n \to A_k$ is said to be* dictatorial *if there exists $i \in N_p$ such that for all $\mathbf{x}, \mathbf{y} \in X^n$ and all $z \in X$ it holds that*

$$S_k(\mathbf{x}_{-i}, z) = S_k(\mathbf{y}_{-i}, z).$$

Example 2.2. *Take $N_p = \{1, 2\}$, $A = \{a, b, c\}$, and $X = A_2$. Now consider the dictatorial screening rule $S_2 : X \times X \to A_2$ given by $S_2(\mathbf{x}) = x_1$ for all $\mathbf{x} \in X \times X$. It is clear that S_2 fails to be no-veto and that every G_2 whose screening rule is S_2 has a subgame perfect equilibrium (the first proposer selects his best and his second best candidates and the chooser plans to choose his best candidate between the two selected by the first proposer). Take now $G_2 = (N, A, X, S_2, X_0, \{\succeq_i\}_{i\in N})$ with the preferences of the proposers and the chooser as follows:*

Proposer 1	*Proposer 2*	*Chooser*
a	b	c
b	c	a
c	a	b

It is easy to check that a is the unique candidate supported by a subgame perfect equilibrium in G_2 (and also by a Nash equilibrium). Notice that a is not the best candidate for the chooser.

The next two classes of screening rules were introduced in Barberà and Coelho (2010).[1] It is clear that they are included in the class of no-veto rules when $n > 2$.

Definition 2.6. *A screening rule $S_k : X^n \to A_k$ is said to be* majoritarian *if, for every $B \in A_k$, there exists $\mathbf{x} \in X^n$ such that for all $M \subset N_p$ with $|M| > \frac{n}{2}$ and all $\bar{\mathbf{x}} \in X^n$ it holds that $S_k(\mathbf{x}_M, \bar{\mathbf{x}}_{N_p \setminus M}) = B$.*[2]

Definition 2.7. *A screening rule S_k is said to be* weakly majoritarian *if, for every $a \in A$, there exists $\mathbf{x} \in X^n$ such that for all $M \subset N_p$ with $|M| > \frac{n}{2}$ and all $\bar{\mathbf{x}} \in X^n$ it holds that $a \in S_k(\mathbf{x}_M, \bar{\mathbf{x}}_{N_p \setminus M})$.*

3 Quasi-Strongness

We have already seen that the chooser's best candidate can always be selected in subgame perfect equilibrium when the screening rule is no-veto, and that the class of no-veto screening rules is a wide one; for example it includes the majority rules and the weakly majority rules. However, in the model we are considering, it seems to be appropriate to use an equilibrium concept that is coalition-proof in the first step, in which the proposers can typically discuss before choosing their strategies. The concept we use is the *subgame perfect q-strong equilibrium* which is a refinement of the *q-strong equilibrium*. The q-strong equilibrium is a variation of Aumann's concept of strong equilibrium for the particular problem we are dealing with in this paper. Specifically, a q-strong equilibrium is a strategy profile that is stable against deviations by every possible group of proposers and against unilateral deviations of the chooser. The strong equilibrium is a refinement of the q-strong which also allows for deviations by groups formed by the chooser and some proposers. We believe that q-strongness is a more natural concept than strongness for k-names games; however, at the end of this section we prove for k-names games

[1] In fact, the classes we define here are more general because Barberà and Coelho deal with symmetric rules and we do not.

[2] $(\mathbf{x}_M, \bar{\mathbf{x}}_{N_p \setminus M})$ denotes the vector $((x_i)_{i \in M}, (\bar{x}_j)_{j \in N_p \setminus M})$.

that, within the class of majority rules, subgame perfect q-strongness and subgame perfect strongness are somewhat equivalent conditions.

Definition 3.1. *Take a k-names game $G_k = (N, A, X, S_k, X_0, \{\succeq_i\}_{i\in N})$. A quasi-strong (abbreviated q-strong) equilibrium of G_k is a pair $(\mathbf{x}, x_0) \in X^n \times X_0$ such that:*

1. $x_0(S_k(\mathbf{x})) \succeq_0 x'_0(S_k(\mathbf{x}))$*, for all* $x'_0 \in S_0$.
2. *For every non-empty $M \subset N_p$ and every $\mathbf{x}' \in X^n$, there exists $i \in M$ such that:*

$$x_0(S_k(\mathbf{x})) \succeq_i x_0(S_k(\mathbf{x}'_M, \mathbf{x}_{N_p\setminus M})).$$

Definition 3.2. *Take a k-names game $G_k = (N, A, X, S_k, X_0, \{\succeq_i\}_{i\in N})$. A subgame perfect q-strong equilibrium of G_k is a pair $(\mathbf{x}, x_0) \in X^n \times X_0$ such that:*

1. $x_0(S_k(\bar{\mathbf{x}})) \succeq_0 a$*, for all $a \in S_k(\bar{\mathbf{x}})$, and all $\bar{\mathbf{x}} \in X^n$.*
2. *For every non-empty $M \subset N_p$ and every $\mathbf{x}' \in X^n$, there exists $i \in M$ such that:*

$$x_0(S_k(\mathbf{x})) \succeq_i x_0(S_k(\mathbf{x}'_M, \mathbf{x}_{N_p\setminus M})).$$

The next two results provide respectively a necessary condition and a sufficient condition for the existence of subgame perfect q-strong equilibria. Before the results, some previous definitions are needed.

Definition 3.3. *Take $\pi \in [1/2, 1)$. A screening rule $S_k : X^n \to A_k$ is said to be π-*majoritarian *if, for every $B \in A_k$, there exists $\mathbf{x} \in X^n$ such that for all $M \subset N_p$ with $|M| > \pi n$ and all $\bar{\mathbf{x}} \in X^n$ it holds that $S_k(\mathbf{x}_M, \bar{\mathbf{x}}_{N_p\setminus M}) = B$.*

It is clear that if a screening rule S_k is π-majoritarian, then it is π'-majoritarian for every $\pi' \geq \pi$. In view of Definitions 2.6 and 3.3, a majoritarian rule is simply a $\frac{1}{2}$-majoritarian rule. Notice that π-majoritarian rules with $\pi > 1/2$ are not rare, since very often a qualified majority is required for making decisions.

Definition 3.4. *Take a k-names game $G_k = (N, A, X, S_k, X_0, \{\succeq_i\}_{i\in N})$ and $\pi \in [1/2, 1)$. A candidate $a \in B \subset A$ is the π-*Condorcet winner over *B if, for all $a' \in B \setminus \{a\}$, it holds that $|\{i \in N_p$ such that $a \succeq_i a'\}| > \pi n$.*

Notice that the π-Condorcet winner over B may not exist but, if it exists, then it is unique. Clearly, if a candidate $a \in B \subset A$ is the π-Condorcet winner over B, then it is the π'-Condorcet winner over B for every $\pi' \leq \pi$ and it is the π-Condorcet winner over B' for all $B' \subset B$ with $a \in B'$. The Condorcet winner over B (a standard concept in social choice) is simply the $\frac{1}{2}$-Condorcet winner over B.

Definition 3.5. *Take a k-names game $G_k = (N, A, X, S_k, X_0, \{\succeq_i\}_{i\in N})$ and $\pi \in [1/2, 1)$. A candidate $a \in B \subset A$ is π-*Condorcet undominated over *B if, for all $a' \in B \setminus \{a\}$, it holds that $|\{i \in N_p$ such that $a' \succeq_i a\}| \leq \pi n$.*

It is clear that if $a \in B \subset A$ is the π-Condorcet winner over B, then it is π-Condorcet undominated over B. It is also easy to check that the reciprocal is not true in general. Moreover, if n is odd, $a \in B \subset A$ is $\frac{1}{2}$-Condorcet undominated over B if and only if it is the $\frac{1}{2}$-Condorcet winner over B.

Theorem 3.1. *For any π-majoritarian screening rule S_k, if a candidate $a \in A$ is a subgame perfect q-strong equilibrium outcome of a k-names game $G_k = (N, A, X, S_k, X_0, \{\succeq_i\}_{i \in N})$ then a is π-Condorcet undominated over the set of the chooser's $(|A| - k + 1)$-top candidates.*

Proof. Assume that candidate a is the outcome of $(\mathbf{x}, x_0)$, a subgame perfect q-strong equilibrium of G_k. Clearly, a is the best candidate for the chooser in $S_k(\mathbf{x})$. So, a is a chooser's $(|A| - k + 1)$-top candidate[3]. Since the screening rule is π-majoritarian, there exists no other chooser's $(|A| - k + 1)$-top candidate a' that is preferred to a by more than πn choosers. Otherwise, this coalition could impose the choice of a set where a' is the best candidate for the chooser, which implies that $(\mathbf{x}, x_0)$ is not a subgame perfect q-strong equilibrium of G_k. □

Theorem 3.2. *For any π-majoritarian screening rule S_k, if a candidate $a \in A$ is the π-Condorcet winner over the set of the chooser's $(|A| - k + 1)$-top candidates in a k-names game $G_k = (N, A, X, S_k, X_0, \{\succeq_i\}_{i \in N})$, then a is a subgame perfect q-strong equilibrium outcome of G_k.*

Proof. Assume that a is the π-Condorcet winner over the set of the chooser's $(|A| - k + 1)$-top candidates. Take a set $B \in A_k$ such that a is the chooser's best candidate in this set. Since S_k is π-majoritarian there exists $\mathbf{x} \in X^n$ such that for all $M \subset N_p$ with $|M| > \pi n$ and all $\bar{\mathbf{x}} \in X^n$ it holds that $S_k(\mathbf{x}_M, \bar{\mathbf{x}}_{N_p \setminus M}) = B$. Take the strategy $x_0 \in X_0$ that selects for any $B \in A_k$ the best candidate for the chooser in B. It is clear that $x_0(S_k(\mathbf{x})) = a$. Let us check that $(\mathbf{x}, x_0)$ is a subgame perfect q-strong equilibrium of G_k. Obviously, any coalition $(1 - \pi)n$ proposers or less cannot change the outcome and thus have no incentive to deviate. Notice that any coalition with more than $(1 - \pi)n$ proposers has no incentive to deviate since a is the π-Condorcet winner over the set of the chooser's $(|A| - k + 1)$-top candidates. □

Corollary 3.1. *For any majoritarian screening rule S_k and any k-names game $G_k = (N, A, X, S_k, X_0, \{\succeq_i\}_{i \in N})$ with an odd n, a candidate $a \in A$ is a subgame perfect q-strong equilibrium outcome of G_k if and only if a is the $\frac{1}{2}$-Condorcet winner over the set of the chooser's $(|A| - k + 1)$-top candidates.*

[3] A chooser's $(|A| - k + 1)$-top candidate is anyone belonging to the set of the $|A| - k + 1$ candidates preferred by the chooser.

Proof. The *if* statement is an immediate result of Theorem 3.2. The *only if* statement is an immediate result of Theorem 3.1 taking into account that n is odd. □

Observe that Corollary 3.1 is similar to Proposition 2 in Barberà and Coelho (2010); they differ in that (a) we use q-strongness instead of strongness, and (b) the definition of majoritarian rule we use is slightly more general. One could wonder if q-strongness or strongness really makes a difference in this context. Next we prove that, within the class of majority rules, subgame perfect q-strongness and subgame perfect strongness are somewhat equivalent conditions. To start with, let us write down the definition of subgame perfect strong equilibrium.

Definition 3.6. *Take a k-names game $G_k = (N, A, X, S_k, X_0, \{\succeq_i\}_{i\in N})$. A subgame perfect strong equilibrium of G_k is a pair $(\mathbf{x}, x_0) \in X^n \times X_0$ such that:*

1. $x_0(S_k(\bar{\mathbf{x}})) \succeq_0 a$*, for all* $a \in S_k(\bar{\mathbf{x}})$*, and all* $\bar{\mathbf{x}} \in X^n$.
2. *For every non-empty* $M \subset N_p$ *and every* $\mathbf{x}' \in X^n$*, there exists* $i \in M$ *such that:*

$$x_0(S_k(\mathbf{x})) \succeq_i x_0(S_k(\mathbf{x'}_M, \mathbf{x}_{N_p\setminus M})).$$

3. *For every non-empty* $M \subset N_p$ *and every* $\mathbf{x}' \in X^n$ *and* $x_0' \in X_0$,
   - *either there exists* $i \in M$ *such that* $x_0(S_k(\mathbf{x})) \succeq_i x_0'(S_k(\mathbf{x'}_M, \mathbf{x}_{N_p\setminus M}))$,
   - *or* $x_0(S_k(\mathbf{x})) \succeq_0 x_0'(S_k(\mathbf{x'}_M, \mathbf{x}_{N_p\setminus M}))$.

From Definitions 3.2 and 3.6, it is clear that every subgame perfect strong equilibrium of a k-names game G_k is subgame perfect q-strong. Let us see a reciprocal when the screening rule is majoritarian.

Theorem 3.3. *Take a k-names game $G_k = (N, A, X, S_k, X_0, \{\succeq_i\}_{i\in N})$ with a majoritarian screening rule S_k. If $a \in A$ is a subgame perfect q-strong equilibrium outcome, then it is a subgame perfect strong equilibrium outcome.*

Proof. Take a subgame perfect q-strong equilibrium $(\mathbf{x}, x_0)$ with $a = x_0(S_k(\mathbf{x}))$. Since S_k is majoritarian there exists $\bar{\mathbf{x}}$ such that $S_k(\mathbf{x}) = S_k(\bar{\mathbf{x}}) = S_k(\bar{\mathbf{x}}_M, \hat{\mathbf{x}}_{N_p\setminus M})$ for all $M \subset N_p$ with $|M| > \frac{n}{2}$ and all $\hat{\mathbf{x}} \in X^n$. Since $x_0(S_k(\bar{\mathbf{x}})) = a$, it is enough to prove that $(\bar{\mathbf{x}}, x_0)$ is a subgame perfect strong equilibrium. Assume that it is not. Then there exists $L \subset N$ such that all the players in L prefer to deviate from $(\bar{\mathbf{x}}, x_0)$. We consider two cases.

1. Case 1, the chooser is not in L. Then there exists $\mathbf{y} \in X^n$ such that

$$x_0(S_k(\bar{\mathbf{x}}_{N_p\setminus L}, \mathbf{y}_L)) \succ_i x_0(S_k(\bar{\mathbf{x}}))$$

for all $i \in L$.[4] Notice that $|L| > \frac{n}{2}$; otherwise $S_k(\bar{\mathbf{x}}_{N_p \setminus L}, \mathbf{y}_L) = S_k(\bar{\mathbf{x}})$, which is impossible. Using again that S_k is majoritarian, there exists $\mathbf{z} \in X^n$ such that

$$S_k(\bar{\mathbf{x}}_{N_p \setminus L}, \mathbf{y}_L) = S_k(\mathbf{z}) = S_k(\mathbf{x}_{N_p \setminus L}, \mathbf{z}_L).$$

But then

$$x_0(S_k(\mathbf{x}_{N_p \setminus L}, \mathbf{z}_L)) \succ_i x_0(S_k(\bar{\mathbf{x}})) = x_0(S_k(\mathbf{x}))$$

for all $i \in L$ and $(\mathbf{x}, x_0)$ would not be subgame perfect q-strong.

2. Case 2, the chooser belongs to L. Denote $L_0 = L \setminus \{0\}$. In this case there exist $\mathbf{y} \in X^n$ and $x'_0 \in X_0$ such that

$$x'_0(S_k(\bar{\mathbf{x}}_{L_0}, \mathbf{y}_{N_p \setminus L_0})) \succ_0 x_0(S_k(\bar{\mathbf{x}})), \tag{2}$$

and

$$x'_0(S_k(\bar{\mathbf{x}}_{L_0}, \mathbf{y}_{N_p \setminus L_0})) \succ_i x_0(S_k(\bar{\mathbf{x}})) \text{ for all } i \in L_0. \tag{3}$$

(2) implies that $S_k(\bar{\mathbf{x}}_{L_0}, \mathbf{y}_{N_p \setminus L_0}) \neq S_k(\bar{\mathbf{x}})$, because otherwise $x'_0(S_k(\bar{\mathbf{x}})) \succ_0 x_0(S_k(\bar{\mathbf{x}}))$ and $(\mathbf{x}, x_0)$ cannot be subgame perfect q-strong. Then, $|L_0| > \frac{n}{2}$ since S_k is majoritarian. Now (2) and the subgame perfect q-strongness of $(\mathbf{x}, x_0)$ imply that $x'_0(S_k(\bar{\mathbf{x}}_{L_0}, \mathbf{y}_{N_p \setminus L_0}))$ is a $(|A| - k + 1)$-top candidate for the chooser and, then, there exists $B \in A_k$ such that $x'_0(S_k(\bar{\mathbf{x}}_{L_0}, \mathbf{y}_{N_p \setminus L_0}))$ is the best candidate for the chooser in B. Taking into account that S_k is majoritarian, there exists $\mathbf{t} \in X^n$ such that

$$S_k(\mathbf{x}_{N_p \setminus L_0}, \mathbf{t}_{L_0}) = B,$$

and then, in view of (3),

$$x_0(S_k(\mathbf{x}_{N_p \setminus L_0}, \mathbf{t}_{L_0})) = x'_0(S_k(\bar{\mathbf{x}}_{L_0}, \mathbf{y}_{N_p \setminus L_0})) \succ_i x_0(S_k(\bar{\mathbf{x}})) = x_0(S_k(\mathbf{x}))$$

for all $i \in L_0$. This contradicts that $(\mathbf{x}, x_0)$ is subgame perfect q-strong.

□

4 On k-names games with delegates

In this section we study how the analysis of a k-names game changes when the chooser has a specific commitment ability: he can strategically appoint a delegate to choose on behalf of him. For that purpose, we explicitly model this ability defining a particular three-stage game and, then, we analyze the resulting game.

[4] We write $a \succ_i b$ when $b \not\succeq_i a$ (for any $a, b \in A$ and any $i \in N_p$).

In Section 3 we proved Corollary 3.1 that gives a necessary and sufficient condition in order that a chooser's $(|A| - k + 1)$-top candidate can be supported in subgame perfect q-strong equilibrium when n is odd. Barberà and Coelho (2010) show that if the subgame perfectness is dropped, the necessary and sufficient condition is weaker. The main result that we obtain in this section is that Barberà and Coelho's weaker condition is again necessary and sufficient in order that a chooser's $(|A| - k + 1)$-top candidate can be supported in *subgame perfect* q-strong equilibrium when n is odd, *in the case that the chooser can strategically appoint a delegate*. In other words, we prove that when the chooser has a particular commitment ability (appointing a delegate to represent him) some more candidates can be supported in subgame perfect q-strong equilibrium (those that can be supported in q-strong equilibrium but cannot be supported in subgame perfect q-strong equilibrium in the original k-names game).

To start with, let us see how can Theorems 3.1 and 3.2 can be adapted when the subgame perfect condition is dropped.

Theorem 4.1. *For any π-majoritarian screening rule S_k, if a candidate $a \in A$ is a q-strong equilibrium outcome of a k-names game $G_k = (N, A, X, S_k, X_0, \{\succeq_i\}_{i \in N})$, then a is a chooser's $(|A| - k + 1)$-top candidate and, moreover, it is π-Condorcet undominated over some subset of A with cardinality $(|A| - k + 1)$.*

Proof. Suppose that $a \in A$ is the outcome of a q-strong equilibrium $(\mathbf{x}, x_0)$. Then a is the chooser's best candidate in $S_k(\mathbf{x})$ and thus a chooser's $(|A| - k + 1)$-top candidate. Clearly, there do not exist $B \in A_k$ such that for every $b \in B$ there exists $M^b \subset N_p$ with $|M^b| > \pi n$ and satisfying that $b \succ_i a$ for all $i \in M^b$; otherwise $x_0(B) \succ_i a$ for all $i \in M^{x_0(B)}$ and this contradicts the fact that S_k is π-majoritarian and $(\mathbf{x}, x_0)$ is q-strong. Then, no more than $k - 1$ candidates are preferred to a by a set of more than πn proposers. This implies that a is π-Condorcet undominated over some subset of A with cardinality $(|A| - k + 1)$. □

Theorem 4.2. *Take a k-names game $G_k = (N, A, X, S_k, X_0, \{\succeq_i\}_{i \in N})$ and assume that S_k is a π-majoritarian screening rule. If $a \in A$ is a chooser's $(|A| - k + 1)$-top candidate and, moreover, it is a π-Condorcet winner over some subset of A with cardinality $(|A| - k + 1)$, then a is a q-strong equilibrium outcome of G_k.*

Proof. Since a is a chooser's $(|A| - k + 1)$-top candidate, then there exists $B \in A_k$ such that $a \in B$ and a is the chooser's best result in B. Since S_k is π-majoritarian there exists $\mathbf{x} \in X^n$ such that for all $M \subset N_p$ with $|M| > \pi n$ and all $\bar{\mathbf{x}} \in X^n$ it holds that $S_k(\mathbf{x}_M, \bar{\mathbf{x}}_{N_p \setminus M}) = B$. Let x_0 be the chooser's strategy such that $x_0(\bar{B}) = a$ if $a \in \bar{B}$ and $x_0(\bar{B}) = b$ if $a \notin \bar{B}$, b being such that $|i \in N_p$ such that $b \succ_i a| \leq \pi n$ (such a b always exists in a subset of $A \setminus \{a\}$ with k elements because a is the π-

Condorcet winner over some subset of A with cardinality $(|A| - k + 1)$. It is clear that $(\mathbf{x}, x_0)$ is a q-strong Nash equilibrium of G_k and $a = x_0(S_k(\mathbf{x}))$. □

Similarly as in Section 3, the next corollary follows from the two theorems above; again, it is similar to Proposition 1 in Barberà and Coelho (2010).

Corollary 4.1. *For any majoritarian screening rule S_k and any k-names game $G_k = (N, A, X, S_k, X_0, \{\succeq_i\}_{i \in N})$ with an odd n, a candidate $a \in A$ is a q-strong equilibrium outcome of G_k if and only if a is a chooser's $(|A| - k + 1)$-top candidate and, moreover, it is the $\frac{1}{2}$-Condorcet winner over some subset of A with cardinality $(|A| - k + 1)$.*

Proof. The *if* statement is an immediate result of Theorem 4.2. The *only if* statement is an immediate result of Theorem 4.1 taking into account that n is odd. □

In non-cooperative game theory, subgame perfection is needed to warrant strategic stability in a multi-stage game, and notice that a k-names game is in fact a two-steps game. Although Theorems 4.1 and 4.2 and Corollary 4.1 provide conditions for a candidate to be a q-strong equilibrium outcome, they do not guarantee subgame perfection and thus their value is hardly significant from a non-cooperative point of view; in other words, they can only be relevant if the chooser has some kind of ability to make commitments. But non-cooperative game theory assumes that players do not have commitment abilities unless they are explicitly included as formal moves in the game (see, for instance the introduction of van Damme (2002)).

In the remainder of this section we provide a model that explicitly incorporates a particular chooser's ability of commitment. In this novel model we assume that the chooser can appoint a delegate to play on behalf of him, and show that the weaker conditions in Corollary 4.1 guarantee that a candidate can be supported in a *subgame perfect* q-strong equilibrium in the novel model.

Notice that the ability to delegate is not something strange in economic theory. There is a strand of literature analyzing how delegation affects equilibrium; a pioneering paper within this strand is Fershtman et al. (1991). A recent paper dealing with delegation is Winter et al. (2017). Here, by appointing delegates we mean the following. In a previous step the chooser announces the delegate designated by him; then, the proposers select a set of k candidates; finally, the appointed delegate chooses one of the k selected candidates. Notice that appointing a delegate is in fact designating a binary relation d over A, and that the delegate makes his choice in the final step according to his preferences over A given by d.

Formally, take a k-names game $G_k = (N, A, X, S_k, X_0, \{\succeq_i\}_{i \in N})$ and denote by $\mathcal{R}(A)$ the set of reflexive, antisymmetric, transitive and complete binary relations over A. The *k-names game with delegation* G_k^* is the three-step game described below.

Step 1 The chooser selects a delegate, i.e. an element d of $\mathcal{R}(A)$.

Step 2 The proposers, knowing the chooser's selection in Step 1, select a set of k candidates using the screening rule.

Step 3 The delegate corresponding to the chooser's selection in Step 1 chooses one of the k selected candidates in Step 2.

A strategy profile in G_k^* is a pair $(d, (\mathbf{x}^e, x_o^e)_{e \in \mathcal{R}(A)})$, such that $d \in \mathcal{R}(A)$ and, for all $e \in \mathcal{R}(A)$, $(\mathbf{x}^e, x_o^e)$ is a strategy profile in G_k^e, G_k^e being a k-names game identical to G_k with the only exception that the chooser's preference is e. Let us give now the equilibrium concept that we use in this context.

Definition 4.1. *Take a k-names game $G_k = (N, A, X, S_k, X_0, \{\succeq_i\}_{i \in N})$ and its corresponding k-names game with delegation G_k^*. A subgame perfect q-strong equilibrium of G_k^* is a pair $(d, (\mathbf{x}^e, x_o^e)_{e \in \mathcal{R}(A)})$ such that:*

1. $x_0^d(S_k(\mathbf{x}^d)) \succeq_0 x_0^e(S_k(\mathbf{x}^e))$*, for all $e \in \mathcal{R}(A)$.*
2. $(\mathbf{x}^d, x_o^d)$ *is a subgame perfect q-strong equilibrium of G_k^d.*
3. *For every $e \in \mathcal{R}(A)$, $(\mathbf{x}^e, x_o^e)$ is a subgame perfect equilibrium of G_k^e.*

Condition 1 states that the chooser does not gain by changing the delegate. Condition 2 assures that the proposers and the chooser play according to a subgame perfect q-strong equilibrium in the equilibrium path. Condition 3 warrants that the proposers and the chooser make reasonable plans outside the equilibrium path. At first sight, Condition 3 may seem to be too weak; maybe, it would have been more natural to ask for $(\mathbf{x}^e, x_o^e)$ being a subgame perfect q-strong equilibrium of G_k^e. However, notice that e is any element in $\mathcal{R}(A)$ and that for many of those e the corresponding G_k^e does not have q-strong equilibria; on the contrary, if S_k is no-veto, G_k^e has subgame perfect equilibria for all $e \in \mathcal{R}(A)$ and, thus, Definition 4.1 is senseful.

Let us now provide some results that indicate under what conditions a candidate can be selected in subgame perfect q-strong equilibrium in G_k^* when the screening rule is majoritarian. We start with two preliminary lemmas.

Lemma 4.1. *Take a k-names game $G_k = (N, A, X, S_k, X_0, \{\succeq_i\}_{i \in N})$ with a majoritarian screening rule S_k and with $n > 2$. Take $d \in \mathcal{R}(A)$ and $a \in A$. Then, a is a delegate d's $(|A| - k + 1)$-top candidate if and only if a is a subgame perfect equilibrium outcome of G_k^d.*

Proof. The "if" part is obvious. To prove the "only if" part take $B \in A_k$ such that a is the best option for delegate d in B. Since S_k is majoritarian, there exists $\mathbf{x}^B \in X^n$ such that for all $M \subset N_p$ with $|M| > \frac{n}{2}$ and all $\bar{\mathbf{x}} \in X^n$ it holds that $S_k(\mathbf{x}_M^B, \bar{\mathbf{x}}_{N_p \setminus M}) = B$. Then, if x_0^d chooses d's best option in each possible $\bar{B} \in A_k$, it is clear that $(\mathbf{x}^B, x_0^d)$ is a subgame perfect equilibrium of G_k^d and that $x_0^d(S_k(\mathbf{x}^B)) = a$ (notice that it is necessary $n > 2$; if $n = 2$ a one-player deviation can result in an element of A_k different from B and, thus, the final result might be preferred by the deviating player). □

Lemma 4.2. *Take a k-names game $G_k = (N, A, X, S_k, X_0, \{\succeq_i\}_{i \in N})$ with a majoritarian screening rule S_k and with $n > 2$, and take $a \in A$. Then, a is a chooser's $(|A| - k + 1)$-top candidate if and only if for all $d \in \mathcal{R}(A)$ there exists $b \in A$, a subgame perfect equilibrium outcome of G_k^d, such that $a \succeq_0 b$.*

Proof. For any l-names game G_l, denote by $SPO(G_l)$ the set of all subgame perfect equilibrium outcomes of G_l. From Lemma 4.1,

$$\forall d \in \mathcal{R}(A), \exists b \in SPO(G_k^d) \text{ with } a \succeq_o b$$

is equivalent to

$$\forall d \in \mathcal{R}(A), \exists b \in A \text{ with } b \text{ being a } d\text{'s } (|A| - k + 1)\text{-top candidate and } a \succeq_o b.$$

The last statement is clearly equivalent to the following one:

$$\forall S \subset A \text{ with } |S| = |A| - k + 1, \ \exists b \in S \text{ such that } a \succeq_o b. \tag{4}$$

Obviously (4) implies that a is a chooser's $(|A| - k + 1)$-top candidate. Conversely, if a is a chooser's $(|A| - k + 1)$-top candidate, then (4) is true (maybe b being a itself). This completes the proof. □

Next we state and prove the main result of this section.

Theorem 4.3. *Take a k-names game $G_k = (N, A, X, S_k, X_0, \{\succeq_i\}_{i \in N})$ with a majoritarian screening rule S_k and take $a \in A$. Then, if $n > 2$ and k is odd, the next two statements are equivalent.*

1. *a is a q-strong equilibrium outcome of G_k.*
2. *a is a subgame perfect q-strong equilibrium outcome of G_k^*.*

Proof. **1 ⇒ 2.** Assume that a is a q-strong equilibrium outcome of G_k. Then, Theorem 4.1 implies that (i) a is a chooser's $(|A| - k + 1)$-top candidate, and (ii) a is $\frac{1}{2}$-Condorcet undominated over some subset of A with cardinality $(|A| - k + 1)$. Now, (i) and Lemma 4.2 imply:

$$\forall d \in \mathcal{R}(A), \ \exists b \in SPO(G_k^d) \text{ such that } a \succeq_0 b. \tag{5}$$

Since k is odd, a $\frac{1}{2}$-Condorcet undominated candidate is the same as a Condorcet winner and, thus, (ii) implies that there exists a delegate $\bar{d}$ such that a is a Condorcet winner over the set of $\bar{d}$'s $(|A| - k + 1)$-top candidates. Then, Theorem 3.2 implies that:

$$a \text{ is a subgame perfect q-strong equilibrium outcome of } G_k^{\bar{d}}. \tag{6}$$

Clearly, (5) and (6) imply that a is a subgame perfect q-strong equilibrium outcome of G_k^*.

2 $\Rightarrow$ 1. Assume that a is a subgame perfect q-strong equilibrium outcome of G_k^*. Then:

$$\forall d \in \mathcal{R}(A),\ \exists b \text{ a subgame perfect equilibrium outcome of } G_k^d \text{ such that } a \succeq_0 b, \tag{7}$$

and

$$\exists \bar{d} \in \mathcal{R}(A) \text{ such that } a \text{ is a q-strong equilibrium outcome of } G_k^{\bar{d}}. \tag{8}$$

Now, (7) implies that a is a chooser's $(|A| - k + 1)$-top candidate. Besides, (8) and Theorem 4.1 imply that a is $\frac{1}{2}$-Condorcet undominated over some subset of A with cardinality $(|A| - k + 1)$. Thus, the oddness of k and Theorem 4.2 imply that a is a q-strong equilibrium outcome of G_k. □

Theorem 4.3 shows that when the chooser can appoint delegates, then the set of candidates that can be supported by a subgame perfect q-strong equilibrium may increase. Let us illustrate it with an example.

Example 4.1. *Take $G_2 = (N, A, X, S_2, X_0, \{\succeq_i\}_{i \in N})$ with $N_p = \{1, 2, 3\}$ and $A = \{a, b, c, d\}$. S_2 is a majoritarian screening rule. The preferences of the proposers are as follows:*

Proposer 1	*Proposer 2*	*Proposer 3*
a	d	b
b	a	d
c	b	a
d	c	c

Notice that a is the Condorcet winner over $\{a, b, c\}$, b is the Condorcet winner over $\{b, c, d\}$, and d is the Condorcet winner over $\{a, c, d\}$. We consider now three cases.

Case 1: $a \succeq_0 b \succeq_0 c \succeq_0 d$. Then Corollary 3.1, Corollary 4.1 and Theorem 4.3 imply that a is the unique subgame perfect q-strong equilibrium outcome of G_2 and that the set of subgame perfect q-strong equilibrium outcomes of G_2^* is $\{a, b\}$. Since $a \succeq_0 b$, in this case the chooser's ability of appointing delegates increases the set of subgame perfect q-strong

equilibrium outcomes, but it cannot be beneficial for the chooser.
Case 2: $b \succeq_0 a \succeq_0 c \succeq_0 d$*. Then Corollary 3.1, Corollary 4.1 and Theorem 4.3 imply that* a *is the unique subgame perfect q-strong equilibrium outcome of* G_2 *and that the set of subgame perfect q-strong equilibrium outcomes of* G_2^* *is* $\{a, b\}$*. Since* $b \succeq_0 a$*, in this case the chooser's ability of appointing delegates increases the set of subgame perfect q-strong equilibrium outcomes, and it can be beneficial for the chooser.*
Case 3: $a \succeq_0 b \succeq_0 d \succeq_0 c$*. Then Corollary 3.1, Corollary 4.1 and Theorem 4.3 imply that* G_2 *does not have subgame perfect q-strong equilibria and that the set of subgame perfect q-strong equilibrium outcomes of* G_2^* *is* $\{a, b, d\}$*. In this case chooser's ability of appointing delegates increases the set of subgame perfect q-strong equilibrium outcomes, and it is beneficial for the chooser.*

To finish this section we provide an example that shows that the statement of Theorem 4.3 is not true if we drop the condition that S_k is majoritarian.

Example 4.2. *Take* $G_3 = (N, A, X, S_3, X_0, \{\succeq_i\}_{i \in N})$ *with* $N_p = \{1, 2, 3\}$ *and* $A = \{a, b, c, d, e, f\}$*. In order to define the set* X *and the screening rule* S_3 *consider the following subsets of* A*:* $B = \{a, b, d\}$*,* $C = \{c, d, e\}$*,* $D = \{a, b, e\}$*,* $E = \{d, e, f\}$*. Now define* $X = \{\beta, \gamma, \delta\}$*, where* β *means "I vote for B",* γ *means "I vote for C", and* δ *means "I vote for D". The screening rule is given by:*

$$S_3(\mathbf{x}) = \begin{cases} B & \text{if at least two proposers choose } \beta, \\ D & \text{if at least two proposers choose } \delta, \\ C & \text{if the three proposers choose different strategies,} \\ E & \text{in any other case.} \end{cases}$$

The preferences of the proposers and the chooser are as follows:

Proposer 1	*Proposer 2*	*Proposer 3*	*Chooser*
d	d	e	a
a	b	f	b
c	c	c	c
b	a	d	d
e	e	a	e
f	f	b	f

Clearly, S_3 *is not majoritarian. Let us check now that* c *is a q-strong equilibrium outcome of* G_3*. From Corollary 4.1, it is enough to prove that* c *is a chooser's* $(|A| - k + 1)$*-top candidate and a Condorcet winner over some subset of* A *with cardinality* $(|A| - k + 1)$*. Here* $(|A| - k + 1) = 4$*; observe that* c *is in fact a chooser's* 4*-top candidate and that* c *is a Condorcet winner over* $\{a, b, c, e, f\}$*. We next prove by contradiction that* c *is not a* *subgame perfect q-strong equilibrium outcome of G_3^*; this implies that Theorem 4.3 is not true if we drop the condition that S_k is majoritarian.*

Assume that $(u, (\mathbf{x}^v, x_o^v)_{v\in\mathcal{R}(A)})$ is a subgame perfect q-strong equilibrium outcome of G_3^* with $x_o^u(S_3(\mathbf{x}^u)) = c$; notice that, in particular, this implies that $u \in \mathcal{R}(A)$, i.e., that u is a reflexive, antisymmetric, transitive and complete binary relation over A. Since $x_o^u(S_3(\mathbf{x}^u)) = c$, then $S_3(\mathbf{x}^u) = C$ and $x_o^u(C) = c$. Thus the three proposers have chosen different strategies. Let us consider now three cases:

1. *$x_o^u(D) = a$. In this case $x_1^u = \delta$ (denote $\mathbf{x}^u = (x_1^u, x_2^u, x_3^u)$); otherwise Proposer 1 would prefer deviating to δ, because then S_3 would select D instead of C and the final result would be a instead of c. Now two subcases:*
   - *(a) $x_o^u(B) \in \{a, d\}$. Then Proposer 1 would prefer deviating to β, because then S_3 would select B instead of C and the final result would be a or d instead of c. Then, $x_o^u(B) \in \{a, d\}$ is not possible.*
   - *(b) $x_o^u(B) = b$. This would imply that bua (i.e., b is preferred to a by u), but this is in contradiction with $x_o^u(D) = a$ (that implies aub), because u is antisymmetric.*

   So, this case is impossible.
2. *$x_o^u(D) = b$. In this case $x_2^u = \delta$; otherwise Proposer 2 would prefer deviating to δ, because then S_3 would select D instead of C and the final result would be b instead of c. Again, two subcases:*
   - *(a) $x_o^u(B) \in \{b, d\}$. Then Proposer 2 would prefer deviating to β, because then S_3 would select B instead of C and the final result would be b or d instead of c. Then, $x_o^u(B) \in \{b, d\}$ is not possible.*
   - *(b) $x_o^u(B) = a$. This would imply that aub, but this is in contradiction with $x_o^u(D) = b$ (that implies bua), because u is antisymmetric.*

   So, this case is impossible.
3. *$x_o^u(D) = e$. In this case $x_3^u = \delta$; otherwise Proposer 3 would prefer deviating to δ, because then S_3 would select D instead of C and the final result would be e instead of c. But then Proposer 3 would prefer deviating to γ unless $x_o^u(E) = d$ (because then S_3 would select E instead of C and the final result would be an element of E instead of c; the unique element of E which is not preferred to c by Proposer 3 is d). Thus, $x_3^u = \delta$ and $x_o^u(E) = d$, but then the proposer choosing β (Proposer 1 or Proposer 2) would prefer deviating to γ, because in that case S_3 would select E instead of C and the final result would be d instead of c. So, this case is impossible.*

Hence, it is impossible that $(u, (\mathbf{x}^v, x_o^v)_{v\in\mathcal{R}(A)})$ is a subgame perfect q-strong equilibrium outcome of G_3^* with $x_o^u(S_3(\mathbf{x}^u)) = c$.

5 Conclusions

The main conclusions of this paper are the following ones.

- By definition, a screening rule is no-veto if and only if no proposer is able to veto one of the candidates. The class of no-veto screening rules is a specially important one; in fact, a screening rule S_k is no-veto if and only if for every k-names game G_k with screening rule S_k there exists a subgame perfect equilibrium whose associated outcome is the best candidate for the chooser.
- We give a sufficient condition and a necessary condition for the existence of subgame perfect q-strong equilibria in k-names games based on π-majoritarian screening rules, where $\pi \in [1/2, 1)$ indicates the "degree" of majority required by the rule.
- For the class of majority rules, subgame perfect q-strongness and subgame perfect strongness are somewhat equivalent conditions.
- Within the class of majoritarian screening rules, if the chooser can strategically appoint a delegate to choose on behalf of him, then a candidate is a q-strong equilibrium outcome of the k-names game if and only if it is a subgame perfect q-strong equilibrium outcome of the k-names with delegates. When the chooser possesses this commitment ability, new subgame perfect q-strong equilibrium outcomes may arise and this can be beneficial for the chooser.

References

Aumann R (1959) Acceptable points in general cooperative n-person games, in R Luce and H Raiffa (eds) Contributions to the Theory of Games IV, Annals of Mathematics Studies 40, Princeton University Press.

Barberà S, Coelho D (2008) How to choose a non-controversial list with k names. Social Choice and Welfare 31, 79-96.

Barberà S, Coelho D (2010) On the rule of k names. Games and Economic Behavior 70, 44-61.

Barberà S, Coelho D (2017) Balancing the power to appoint officers. Games and Economic Behavior 101, 189-203.

Colomer JM (1995) Game Theory and the Transition to Democracy. The Spanish Model. Edward Elgar.

Fershtman C, Judd KL, Kalai E (1991) Observable contracts: strategic delegation and cooperation. International Economic Review 32, 551-559.

Pérez J, Jimeno JL, García E (2012) No Show Paradox in Condorcet k-voting proce-

dures. Group Decision and Negotiation 21, 291-303.
Selten R (1975) Reexamination of the perfectness concept for equilibrium points in extensive games. International Journal of Game Theory 4, 25-55.
van Damme E, Furth D (2002) Game theory and the market, in P Borm and H Peters (eds) Chapters in Game Theory, Kluwer Academic Publishers.
Winter E, Méndez-Naya L, García-Jurado I (2017) Mental equilibrium and strategic emotions. Management Science 63, 1302-1317.