Economics Letters 259 (2026) 112794

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Normalizing VES production functions: extending the supply-side system approach

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ARTICLE INFO

JEL-Codes:
E23
O33
O41

Keywords:
Variable elasticity of substitution (VES)
Normalization
Factor-Augmenting Technical Change
Identification
Capital-labour elasticity of substitution
Normalized supply-side system

ABSTRACT

This paper extends the Klump et al. (2007) normalization procedure to Variable Elasticity of Substitution (VES) production functions. Normalization addresses identification issues in VES model estimation, allowing joint estimation of substitution elasticities and factor-augmenting technical change, while offering a tractable extension of the normalized supply-side system that bridges the gap between CES and more flexible VES specifications.

1. Introduction

The capital-labour elasticity of substitution is central to macroeconomic models.[1] The Constant Elasticity of Substitution (CES) production function is widely used to estimate this elasticity across aggregate, sectoral plant levels. However, assuming Hicks-neutral technological change can bias elasticity estimates (Antràs, 2004).

Normalization of CES functions (Klump et al., 2007) expresses production relations in indexed form, defines a baseline point where CES isoquants with different elasticities of substitution are tangent (Irmen and Klump, 2009), and enables joint identification of the elasticity of substitution — which is constant and independent of the capital-labour ratio— and factor-augmenting technological change.

Variable Elasticity of Substitution (VES) production functions (Kadiyala, 1972) provide a flexible representation of production technologies, allowing the substitution elasticity to depend on input ratios. Despite their theoretical appeal, VES functions have been underused since the 1970s due to identification issues and lack of a systematic estimation framework comparable to CES functions. However, although it faces identification challenges, a VES specification can reduce — as noted by Kazi (1980, p. 169) — the upward bias in CES estimates, which "includes part of the variation due to K/L in the average product".[2] Since the elasticity of substitution shapes key macroeconomic outcomes — including income distribution, growth or convergence behaviour — its accurate estimation is essential for consistent structural interpretation (Gómez, 2018, 2023; Irmen and Klump, 2009).

This paper extends the Klump et al. (2007) CES normalization procedure to Variable Elasticity of Substitution (VES) production functions, addressing identification challenges and providing a flexible supply-side framework for applied research that enables robust joint estimation of substitution elasticities and factor-augmenting technical change. The framework offers a tractable extension of the normalized supply-side system, bridging the gap between CES and more flexible VES specifications, and lays the foundation for empirical applications of VES functions in macroeconomic and sectoral analysis.

The remainder of the article is organized as follows. Section 2 presents the normalized VES function, Section 3 discusses estimation and identification, and Section 4 concludes.

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[1] Del Río and Rebelo (2025b) provide a comprehensive review of the role that the capital-labour elasticity of substitution plays in macroeconomic analysis.
[2] Antony (2009) proposes a dual elasticity of substitution production function.

https://doi.org/10.1016/j.econlet.2025.112794
Received 17 October 2025; Received in revised form 12 December 2025; Accepted 16 December 2025
Available online 17 December 2025

2. Normalization of the general VES production function

Following Kadiyala (1972), the general VES production function is:

$$y = \left[\beta_{11}\hat{k}^{\psi} + \beta_{12}\hat{k}^{\upsilon\psi}\hat{l}^{(1-\upsilon)\psi} + \beta_{22}\hat{l}^{\psi}\right]^{\frac{1}{\psi}}, \quad (1)$$

where $\beta_{11},\ \beta_{12},\ \beta_{22} > 0,\ 0 \le \upsilon \le 1,\ \psi < 1,\ y$ is output, $\hat{k}$ is efficiency-adjusted capital, and $\hat{l}$ is efficiency-adjusted labour.

This specification encompasses several known cases: $\beta_{22} = 0$, yields the Liu-Hildebrand (1965) / Nerlove (1967), and Lu-Fletcher (1968) forms; $\beta_{22} = 0$ and $\upsilon = 1 - 1/\psi$ lead to Sato-Hoffman (1968) and Revankar (1971); and $\beta_{11} = 0$ corresponds to Del Río and Lores (2019, 2021, 2023a, 2023b).

Under perfect competition, with rental prices $\hat{r}$ for efficiency-adjusted capital and $\hat{w}$ for efficiency-adjusted labour, the first-order conditions yield the following capital, $\pi = \hat{r}\frac{\hat{k}}{y}$, and labour shares, $1 - \pi = \hat{w}\frac{\hat{l}}{y}$:

$$\pi = \left(\frac{y}{\hat{k}}\right)^{-\psi}\left[\beta_{11} + \upsilon\beta_{12}\hat{k}^{(\upsilon-1)\psi}\hat{l}^{(1-\upsilon)\psi}\right] \quad (2)$$

and

$$1 - \pi = \left(\frac{y}{\hat{l}}\right)^{-\psi}\left[(1-\upsilon)\beta_{12}\hat{k}^{\upsilon\psi}\hat{l}^{-\upsilon\psi} + \beta_{22}\right] \quad (3)$$

The VES function faces identification problems, as distinct parameter sets $(\beta_{11},\ \beta_{22},\ \beta_{12},\ \psi,\ \upsilon)$ can produce observationally equivalent outcomes. It collapses to CES when: (i) $\upsilon = 1$, (ii) $\upsilon = 0$, or (iii) $\beta_{12} = 0$. The Cobb-Douglas specification arises either when $\beta_{11} = \beta_{22} = 0$ or as $\psi \to 0$. In these cases, the same observed output–input relationships can be rationalized by multiple structural parameterizations, implying that the model is not globally identified (Gujarati and Porter, 2009). This non-uniqueness has long been noted in the VES literature (e.g., Kadiyala, 1972; Kazi, 1980; Revankar, 1971).

Normalization mitigates this indeterminacy. By fixing a baseline point and imposing consistency restrictions on factor shares and the elasticity of substitution, the parameter space is reduced, and structural identification becomes feasible — at least locally and conditional on the choice of the benchmark elasticity. In this respect, the challenge is analogous to the identification issues emphasized in the normalized CES literature (Klump et al., 2007; León-Ledesma et al., 2010), although the VES setting is more demanding due to its additional flexibility. Nevertheless, empirical robustness ultimately depends on the calibration of the baseline, which must be carried out with care to avoid misspecification in applied work.

As with CES production functions, a family of normalized VES production functions can be defined as a set of VES production functions that share the same baseline levels of efficiency-adjusted capital, $\hat{k}$, efficiency-adjusted labour, $\hat{l}$, and output, y. At the normalization point (denoted 0), $\hat{k}_0 = \hat{l}_0 = y_0 = 1$, and the first-order conditions yield

$$\pi_0 = \beta_{11} + \upsilon\beta_{12}, \quad (4)$$

and

$$1 - \pi_0 = (1-\upsilon)\beta_{12} + \beta_{22} \quad (5)$$

Imposing these restrictions on the production function we have

$$y = \left[(\pi_0 - \upsilon\beta_{12})\hat{k}^{\psi} + \beta_{12}\hat{k}^{\upsilon\psi}\hat{l}^{(1-\upsilon)\psi} + [(1-\pi_0) - (1-\upsilon)\beta_{12}]\hat{l}^{\psi}\right]^{\frac{1}{\psi}} \quad (6)$$

which still suffers from the structural identification problem highlighted earlier. An additional restriction is required for identification, which we obtain from the definition of the capital–labour elasticity. When $\beta_{12} = 0$, the model reduces to CES and the first-order conditions suffice (Klump et al., 2007).

The capital-labour elasticity of substitution is

$$\sigma\left(\frac{\hat{k}}{\hat{l}}\right) = \left[1 - \left(\frac{\beta_{11}(1-\upsilon)\psi}{\beta_{11} + \beta_{12}\upsilon\left(\frac{\hat{k}}{\hat{l}}\right)^{-(1-\upsilon)\psi}} + \frac{\beta_{22}\upsilon\psi}{\beta_{22} + \beta_{12}(1-\upsilon)\left(\frac{\hat{k}}{\hat{l}}\right)^{\upsilon\psi}}\right)\right]^{-1} \quad (7)$$

which varies with the efficiency-adjusted input ratio along the same isoquant. Since $\frac{\hat{k}}{\hat{l}} > 0,\ \psi < 0$ implies that capital and labour are gross complements $(\sigma < 1)$, and $0 < \psi < 1$ implies that they are gross substitutes $(\sigma < 1)$. When $\beta_{11} = 0$ or $\beta_{22} = 0$, the elasticity of substitution displays opposite patterns: decreasing or increasing with $\frac{\hat{k}}{\hat{l}}$, respectively.

From (7), at the normalization point,

$$\frac{\sigma_0 - 1}{\sigma_0} = \psi\left[\frac{(1-\upsilon)\beta_{11}}{\pi_0} + \frac{\upsilon\beta_{22}}{1-\pi_0}\right] \quad (8)$$

and from (4), (5) and (7), it follows that

$$\beta_{12} = \left(1 - \frac{\sigma_0 - 1}{\sigma_0}\frac{1}{\psi}\right)\frac{\pi_0(1-\pi_0)}{\upsilon(1-\upsilon)} \equiv \mathrm{B}(\psi, \upsilon; \sigma_0) \quad (9)$$

Finally, we derive the normalized VES production function by substituting Eq. (9) into (6):

$$y = \left[(\pi_0 - \upsilon\mathrm{B}(\psi,\upsilon;\sigma_0))\hat{k}^{\psi} + \mathrm{B}(\psi,\upsilon;\sigma_0)\hat{k}^{\upsilon\psi}l^{(1-\upsilon)\psi} + [(1-\pi_0) - (1-\upsilon)\mathrm{B}(\psi,\upsilon;\sigma_0)]\hat{l}^{\psi}\right]^{\frac{1}{\psi}} \quad (10)$$

The function above defines a family of normalized VES production functions that share the same factor shares and elasticity of substitution at the normalization point but vary in their specific combination of technological parameters υ and ψ. With the constraints imposed by normalization, the model is locally identified.

3. Estimation

We now turn to the estimation of the normalized VES specification, beginning with the standard case, which follows directly from Section 2, and then discussing estimation when allowing for technological change or when imposing restricted VES structures designed to mitigate potential weak-identification issues.

3.1. The standard case

Under the normalization conditions established in Section 2 — Eqs. (4), (5) and (9) — the first-order conditions are

$$\pi = \left(\frac{y}{\hat{k}}\right)^{-\psi}\left[(\pi_0 - \upsilon\mathrm{B}(\psi,\upsilon;\sigma_0)) + \upsilon\mathrm{B}(\psi,\upsilon;\sigma_0)\hat{k}^{(\upsilon-1)\psi}\hat{l}^{(1-\upsilon)\psi}\right] \quad (11)$$

and

$$1 - \pi = \left(\frac{y}{\hat{l}}\right)^{-\psi}\left[(1-\upsilon)\mathrm{B}(\psi,\upsilon;\sigma_0)\hat{k}^{\upsilon\psi}\hat{l}^{-\upsilon\psi} + [(1-\pi_0) - (1-\upsilon)\mathrm{B}(\psi,\upsilon;\sigma_0)]\right] \quad (12)$$

The estimating system consists of the normalized production function (10) and the two first-order conditions for capital and labour, expressed in logarithmic form. Provided π_0 and σ_0, parameters υ and ψ can then be estimated from the three equations applying standard nonlinear estimation techniques.

As highlighted by Klump et al. (2012), empirical applicability requires a clearly defined normalization point; a condition that carries over from the CES case to the VES setting. In this context, an additional parameter, σ_0, must be specified at the normalization point in order to ensure consistency with the data.

Since the baseline values of the production variables are typically calibrated using sample averages, the most straightforward approach is to set σ_0 based on the estimated elasticity from the time-invariant case. In this context, the CES production function provides a natural reference due to its constant elasticity of substitution, simplifying the calibration and allowing a direct estimation of the elasticity at the normalization point.

By adopting the CES production function as a baseline, we align the model with empirical data, ensuring that the calibrated baseline value of σ_0 reflects realistic production relationships. This approach also facilitates comparisons across different parametric specifications of the production function, contributing to a more robust analysis of the substitution elasticity and its evolution over time.

3.2. Incorporating factor-augmenting technological change

Thus far, we have not explicitly considered technological progress in the analysis. However, it can be incorporated by defining $\widehat{k} = a_k k,\ \widehat{l} = a_l l,\ r = a_k \widehat{r}$ and $w = a_l \widehat{w}$, where a_k and a_l are the normalized factor efficiency levels ($a_{k,0} = a_{l,0} = 1$). It is generally assumed that they grow at a constant rate: $\ln(a_k) = \gamma_k t$ and $\ln(a_l) = \gamma_l t,$ where t is time. However, some studies also adopt assumptions involving variable rates of change, such as the Box-Cox formulation (Serrano-Quintero, 2023; Del Río and Rebelo, 2025a).

The logarithmic forms of the normalized VES production function and the first-order conditions for profit maximization are:

$$\ln(y) = \frac{1}{\psi}\ln\left[(\pi_0 - \upsilon \mathrm{B}(\psi,\upsilon;\sigma_0))(a_k k)^{\psi} + \mathrm{B}(\psi,\upsilon;\sigma_0)(a_k k)^{\upsilon\psi}(a_l l)^{(1-\upsilon)\psi}\right.$$
$$\left. + [(1-\pi_0) - (1-\upsilon)\mathrm{B}(\psi,\upsilon;\sigma_0)](a_l l)^{\psi}\right] \tag{13}$$

$$\ln\left(\frac{y}{k}\right) = \frac{\psi}{\psi-1}\ln(a_k) + \frac{1}{1-\psi}\ln(r) + \frac{1}{\psi-1}ln\left[\upsilon \mathrm{B}(\psi,\upsilon;\sigma_0)\left(\frac{a_k}{a_l}\frac{k}{l}\right)^{(\upsilon-1)\psi} + \pi_0 - \upsilon \mathrm{B}(\psi,\upsilon;\sigma_0)\right] \tag{14}$$

and

$$\ln\left(\frac{y}{l}\right) = \frac{\psi}{\psi-1}\ln(a_l) + \frac{1}{1-\psi}\ln(w) + \frac{1}{\psi-1}ln\left[(1-\upsilon)\mathrm{B}(\psi,\upsilon;\sigma_0)\left(\frac{a_k}{a_l}\frac{k}{l}\right)^{\upsilon\psi} + 1 - \pi_0 - (1-\upsilon)\mathrm{B}(\psi,\upsilon;\sigma_0)\right] \tag{15}$$

It is worth noting that we have formulated the system of equations for estimation using normalized variables. To express it in terms of non-normalized variables, we need only to consider that $l = \frac{L}{L_0}$, $k = \frac{K}{K_0}$, $y = \frac{Y}{Y_0}$, $r = R\frac{Y_0}{K_0}$ and $w = W\frac{Y_0}{L_0}$ and perform the corresponding substitutions in the equation system (13–15).

The general normalized VES specification involves up to four parameters to estimate ($\upsilon,\ \psi,\ \gamma_k$ and γ_l). Estimation may suffer from weak identification due to nonlinear interactions or data limitations (León-Ledesma et al., 2010), requiring additional parametric restrictions to ensure identifiability, either by constraining the form of technological change or by restricting the structure of the production function.

We may impose restrictions on technological change — for instance, assuming it is Hicks-neutral ($\gamma_k = \gamma_l$), capital-augmenting ($\gamma_l = 0$), or labour-augmenting ($\gamma_k = 0$). Each reduces the parameter space to three estimable parameters: υ, ψ, and the corresponding technological change rate. When allowing for variable rates of technical progress, as in Box-Cox formulations, additional curvature parameters arise; it is therefore necessary to impose sufficient parametric restrictions to avoid weak identification, ensuring that only a limited set of parameters governing technological progress remains unrestricted.

3.3. Restricted VES specifications

Instead of constraining the form of technological progress, we could restrict the structure of the production function to enable estimation of only three parameters: $\psi,\ \gamma_k$ and γ_l. Three distinct possibilities arise in this context:

i). $\beta_{12} = \mathrm{B}(\psi,\upsilon;\sigma_0) = 0$, CES. In this case, $\psi = \frac{\sigma_0 - 1}{\sigma_0}$ and the VES specification collapses to the standard CES form, with a constant elasticity of substitution:

$$y = [\pi_0 \widehat{k}^{\psi} + (1-\pi_0)\widehat{l}^{\psi}]^{\frac{1}{\psi}}$$

ii). $\beta_{22} = 0$, Liu-Hildebrand / Nerlove / Lu-Fletcher VES (elasticity increasing in $\widehat{k}/\widehat{l}$). In this case, $\beta_{12} = \mathrm{B}(\psi,\upsilon;\sigma_0) = \frac{1-\pi_0}{1-\upsilon}$:

$$y = \left[\frac{\pi_0 - \upsilon}{1-\upsilon}\widehat{k}^{\psi} + \frac{1-\pi_0}{1-\upsilon}\widehat{k}^{\upsilon\psi}\widehat{l}^{(1-\upsilon)\psi}\right]^{\frac{1}{\psi}}$$

with $\upsilon < \pi_0$, to ensure $\beta_{11} > 0$. The elasticity of substitution is

$$\sigma\left(\frac{\widehat{k}}{\widehat{l}}\right) = \left[1 - \frac{(1-\upsilon)\psi}{1 + \frac{1-\pi_0}{\pi_0 - \upsilon}\upsilon\left(\frac{\widehat{k}}{\widehat{l}}\right)^{-(1-\upsilon)\psi}}\right]^{-1}$$

At the normalization point, $\sigma_0 = \left[1 - \frac{\psi(\pi_0 - \upsilon)}{\pi_0}\right]^{-1}$, which implies that $\upsilon = \pi_0 - \frac{\pi_0}{\psi}\frac{\sigma_0 - 1}{\sigma_0}$.

iii). $\beta_{11} = 0$, Del Río and Lores VES (elasticity decreasing in $\widehat{k}/\widehat{l}$). In this case, $\beta_{12} = \mathrm{B}(\psi,\upsilon;\sigma_0) = \frac{\pi_0}{\upsilon}$:

$$y = \left[\frac{\pi_0}{\upsilon}\widehat{k}^{\upsilon\psi}\widehat{l}^{(1-\upsilon)\psi} + \left(1 - \frac{\pi_0}{\upsilon}\right)\widehat{l}^{\psi}\right]^{\frac{1}{\psi}}$$

with $\upsilon > \pi_0$ to ensure $\beta_{22} > 0$. The elasticity of substitution is

$$\sigma\left(\frac{\widehat{k}}{\widehat{l}}\right) = \left[1 - \frac{\upsilon\psi}{1 + \frac{(1-\upsilon)\pi_0}{\upsilon - \pi_0}\left(\frac{\widehat{k}}{\widehat{l}}\right)^{\upsilon\psi}}\right]^{-1}$$

At the normalization point, $\sigma_0 = \left[1 - \frac{\psi(\upsilon - \pi_0)}{1-\pi_0}\right]^{-1}$, implying that $\upsilon = \pi_0 + \frac{(1-\pi_0)}{\psi}\frac{\sigma_0 - 1}{\sigma_0}$.

These restricted VES specifications are useful because the standard VES case may be prone to multicollinearity or weak identification. By reducing the dimensionality of the parameter vector, they facilitate estimation while retaining the flexibility to capture non-constant substitution elasticities. Nevertheless, these formulations impose a strictly monotonic relationship between the elasticity of substitution and the adjusted capital–labor ratio: an increasing relationship in case (ii) and a decreasing one in case (iii).

4. Conclusion

This paper extends CES normalization to a broad class of Variable Elasticity of Substitution production functions, offering a methodological framework for estimating capital-labour elasticities of substitution and factor-augmenting technical change with improved empirical robustness. The proposed normalization resolves structural identification challenges and enables precise parameter estimation when elasticity varies with input ratios.

Nevertheless, identification in the normalized VES framework remains conditional on the choice of the benchmark elasticity, and empirical robustness depends on careful calibration of this normalization point. Moreover, the added flexibility of the VES specification comes at the cost of increased complexity in estimation, raising potential challenges of multicollinearity and weak identification in applied work.

Future research directions include applying normalized VES to macro and sectoral data, conducting simulation studies on estimator properties, and leveraging the normalized VES production function for analyzing technological change, factor substitution, and long-run growth dynamics.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

We acknowledge the financial support of the Spanish Ministry of Sciences and Innovation (PID2023-147667NB-I00) and Xunta de Galicia (GRC GI-2060 ED431C2023/05)

Data availability

No data was used for the research described in the article.

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